CONSUMABLE WORKBOOKS  Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th>MHID</th>
<th>ISBN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook 0-07-660292-3</td>
<td>978-0-07-660292-6</td>
</tr>
<tr>
<td>Homework Practice Workbook 0-07-660291-5</td>
<td>978-0-07-660291-9</td>
</tr>
<tr>
<td>Spanish Version Homework Practice Workbook 0-07-660294-X</td>
<td>978-0-07-660294-0</td>
</tr>
</tbody>
</table>

Answers For Workbooks  The answers for Chapter 11 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED  All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

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Chapter 11 Resource Masters

The Chapter 11 Resource Masters includes the core materials needed for Chapter 11. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 11-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the Chapter 11 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 12 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- Form 1 contains multiple-choice questions and is intended for use with below grade level students.
- Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Forms 2C and 2D contain free-response questions aimed at on-grade level students. These tests are similar in format to offer comparable testing situations.
- Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 11. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>asymptote</td>
<td></td>
<td></td>
</tr>
<tr>
<td>complex fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>excluded values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>extraneous solutions ehk·STRAY·nee·uhs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inverse variation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>least common denominator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>least common multiple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mixed expression</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on the next page)
<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>product rule</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rate problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rational equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rational expression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rational function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>work problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Anticipation Guide

## Rational Expressions and Equations

### Step 1 Before you begin Chapter 11
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>Step 1</th>
<th>A, D, or NS</th>
<th>Statement</th>
<th>Step 2</th>
<th>A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>Since a direct variation can be written as ( y = kx ), an inverse variation can be written as ( y = \frac{x}{k} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>A rational expression is an algebraic fraction that contains a radical.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>To multiply two rational expressions, such as ( \frac{2xy^2}{3c} ) and ( \frac{3c^2}{5y} ), multiply the numerators and the denominators.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>When solving problems involving units of measure, dimensional analysis is the process of determining the units of the final answer so that the units can be ignored while performing calculations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>To divide ((4x^2 + 12x)) by (2x), divide (4x^2) by (2x) and (12x) by (2x).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>To find the sum of ( \frac{2a}{(3a - 4)} ) and ( \frac{5}{(3a - 4)} ), first add the numerators and then the denominators.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>A</td>
<td>The least common denominator of two rational expressions will be the least common multiple of the denominators.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>A complex fraction contains a fraction in its numerator or denominator.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>A</td>
<td>The fraction ( \frac{a}{b} ) can be rewritten as ( \frac{ac}{bd} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>Extraneous solutions are solutions that can be eliminated because they are extremely high or low.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Step 2 After you complete Chapter 11
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
### 11 Ejercicios preparatorios

**Expresiones y ecuaciones racionales**

#### Paso 1

**Antes de comenzar el Capítulo 11**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D, o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Dado que una variación directa se puede escribir como ( y = kx ), una variación inversa se puede escribir como ( y = \frac{x}{k} ).</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Una expresión racional es una fracción algebraica que contiene un radical.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Para multiplicar dos expresiones racionales, como ( \frac{2xy^2}{3c} ) y ( \frac{3c^2}{5y} ), multiplica los numeradores y los denominadores.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Al resolver problemas con unidades de medida, el análisis dimensional es el proceso de determinar las unidades de la respuesta final, de manera que las unidades pueden ignorarse mientras se desarrollan los cálculos.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Para dividir ((4x^2 + 12x)) entre (2x), divide (4x^2) entre (2x) y (12x) entre (2x).</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Para sumar (\frac{2a}{(3a-4)}) y (\frac{5}{(3a-4)}), primero suma los numeradores y luego los denominadores.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>El mínimo común denominador de dos expresiones racionales será el mínimo común múltiplo de los denominadores.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Una fracción compleja contiene una fracción en su numerador o denominador.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>La fracción (\frac{\frac{a}{b}}{\frac{c}{d}}) se puede volver a plantear como (\frac{ac}{bd}).</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Las soluciones extrínsecas son soluciones que se pueden eliminar porque son extremadamente altas o bajas.</td>
<td></td>
</tr>
</tbody>
</table>

#### Paso 2

**Después de completar el Capítulo 11**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
Identify and Use Inverse Variations  An inverse variation is an equation in the form of \( y = \frac{k}{x} \) or \( xy = k \). If two points \((x_1, y_1)\) and \((x_2, y_2)\) are solutions of an inverse variation, then \( x_1 \cdot y_1 = k \) and \( x_2 \cdot y_2 = k \).

**Product Rule for Inverse Variation**
\[ x_1 \cdot y_1 = x_2 \cdot y_2 \]

From the product rule, you can form the proportion \( \frac{x_1}{x_2} = \frac{y_2}{y_1} \).

**Example**  If \( y \) varies inversely as \( x \) and \( y = 12 \) when \( x = 4 \), find \( x \) when \( y = 18 \).

**Method 1**  Use the product rule.
\[
\begin{align*}
4 \cdot 12 &= x_2 \cdot 18 \\
48 &= x_2 \\
\frac{48}{18} &= x_2 \\
\frac{8}{3} &= x_2
\end{align*}
\]

**Method 2**  Use a proportion.
\[
\begin{align*}
\frac{x_1}{x_2} &= \frac{y_2}{y_1} \\
\frac{4}{x_2} &= \frac{12}{18} \\
48 &= 18x_2 \\
\frac{8}{3} &= x_2
\end{align*}
\]

Both methods show that \( x_2 = \frac{8}{3} \) when \( y = 18 \).

**Exercises**

Determine whether each table or equation represents an inverse or a direct variation. Explain.

1. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
3 & 6 \\
5 & 10 \\
8 & 16 \\
12 & 24 \\
\hline
\end{array}
\]

2. \( y = 6x \)

3. \( xy = 15 \)

Assume that \( y \) varies inversely as \( x \). Write an inverse variation equation that relates \( x \) and \( y \). Then solve.

4. If \( y = 10 \) when \( x = 5 \), find \( y \) when \( x = 2 \).

5. If \( y = 8 \) when \( x = -2 \), find \( y \) when \( x = 4 \).

6. If \( y = 100 \) when \( x = 120 \), find \( x \) when \( y = 20 \).

7. If \( y = -16 \) when \( x = 4 \), find \( x \) when \( y = 32 \).

8. If \( y = -7.5 \) when \( x = 25 \), find \( y \) when \( x = 5 \).

9. **DRIVING**  The Gerardi family can travel to Oshkosh, Wisconsin, from Chicago, Illinois, in 4 hours if they drive an average of 45 miles per hour. How long would it take them if they increased their average speed to 50 miles per hour?

10. **GEOMETRY**  For a rectangle with given area, the width of the rectangle varies inversely as the length. If the width of the rectangle is 40 meters when the length is 5 meters, find the width of the rectangle when the length is 20 meters.
Inverse Variation

Graph Inverse Variations Situations in which the values of $y$ decrease as the values of $x$ increase are examples of inverse variation. We say that $y$ varies inversely as $x$, or $y$ is inversely proportional to $x$.

Inverse Variation Equation

| an equation of the form $xy = k$, where $k \neq 0 |

**Example 1**

Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let $x =$ speed in miles per hour and $y =$ time in hours. Graph the variation.

The equation $xy = 200$ can be used to represent the situation. Use various speeds to make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>6.7</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>3.3</td>
</tr>
</tbody>
</table>

**Example 2**

Graph an inverse variation in which $y$ varies inversely as $x$ and $y = 3$ when $x = 12$.

Solve for $k$.

$xy = k$

$12(3) = k$

$x = 12$ and $y = 3$

$36 = k$

Simplify.

Choose values for $x$ and $y$, which have a product of 36.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>-3</td>
<td>-12</td>
</tr>
<tr>
<td>-2</td>
<td>-18</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each variation if $y$ varies inversely as $x$.

1. $y = 9$ when $x = -3$

2. $y = 12$ when $x = 4$

3. $y = -25$ when $x = 5$

4. $y = 4$ when $x = 5$

5. $y = -18$ when $x = -9$

6. $y = 4.8$ when $x = 5.4$
Determine whether each table or equation represents an inverse or a direct variation. Explain.

1. \[
\begin{array}{c|c}
  x & y \\
  \hline
  0.5 & 8 \\
  1 & 4 \\
  2 & 2 \\
  4 & 1 \\
\end{array}
\]

2. \[xy = \frac{2}{3}\]

3. \[-2x + y = 0\]

Assume that \(y\) varies inversely as \(x\). Write an inverse variation equation that relates \(x\) and \(y\). Then graph the equation.

4. \(y = 2\) when \(x = 5\)

5. \(y = -6\) when \(x = -6\)

6. \(y = -4\) when \(x = -12\)

7. \(y = 15\) when \(x = 3\)

Solve. Assume that \(y\) varies inversely as \(x\).

8. If \(y = 4\) when \(x = 8\), find \(y\) when \(x = 2\).

9. If \(y = -7\) when \(x = 3\), find \(y\) when \(x = -3\).

10. If \(y = -6\) when \(x = -2\), find \(y\) when \(x = 4\).

11. If \(y = -24\) when \(x = -3\), find \(x\) when \(y = -6\).

12. If \(y = 15\) when \(x = 1\), find \(x\) when \(y = -3\).

13. If \(y = 48\) when \(x = -4\), find \(y\) when \(x = 6\).

14. If \(y = -4\) when \(x = \frac{1}{2}\), find \(x\) when \(y = 2\).
11-1 Practice

**Inverse Variation**

Determine whether each table or equation represents an inverse or a direct variation. Explain.

1. \[
\begin{array}{c|c}
 x & y \\
 0.25 & 40 \\
 0.5 & 20 \\
 2 & 5 \\
 8 & 1.25 \\
\end{array}
\]

2. \[
\begin{array}{c|c}
 x & y \\
 -2 & 8 \\
 0 & 0 \\
 2 & -8 \\
 4 & -16 \\
\end{array}
\]

3. \( \frac{y}{x} = -3 \)

4. \( y = \frac{7}{x} \)

Assume that \( y \) varies inversely as \( x \). Write an inverse variation equation that relates \( x \) and \( y \). Then graph the equation.

5. \( y = -2 \) when \( x = -12 \)

6. \( y = -6 \) when \( x = -5 \)

7. \( y = 2.5 \) when \( x = 2 \)

8. If \( y = 124 \) when \( x = 12 \), find \( y \) when \( x = -24 \).

9. If \( y = -8.5 \) when \( x = 6 \), find \( y \) when \( x = -2.5 \).

10. If \( y = 3.2 \) when \( x = -5.5 \), find \( y \) when \( x = 6.4 \).

11. If \( y = 0.6 \) when \( x = 7.5 \), find \( y \) when \( x = -1.25 \).

12. **EMPLOYMENT** The manager of a lumber store schedules 6 employees to take inventory in an 8-hour work period. The manager assumes all employees work at the same rate.

   a. Suppose 2 employees call in sick. How many hours will 4 employees need to take inventory?

   b. If the district supervisor calls in and says she needs the inventory finished in 6 hours, how many employees should the manager assign to take inventory?

13. **TRAVEL** Jesse and Joaquin can drive to their grandparents’ home in 3 hours if they average 50 miles per hour. Since the road between the homes is winding and mountainous, their parents prefer they average between 40 and 45 miles per hour. How long will it take to drive to the grandparents’ home at the reduced speed?
1. PHYSICAL SCIENCE  The illumination $I$ produced by a light source varies inversely as the square of the distance $d$ from the source. The illumination produced 5 feet from the light source is 80 foot-candles.

$$Id^2 = k$$

$$80(5)^2 = k$$

$$2000 = k$$

Find the illumination produced 8 feet from the same source.

2. MONEY  A formula called the Rule of 72 approximates how fast money will double in a savings account. It is based on the relation that the number of years it takes for money to double varies inversely as the annual interest rate. Use the information in the table to write the Rule of 72 formula.

<table>
<thead>
<tr>
<th>Years to Double Money</th>
<th>Annual Interest Rate (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>14.4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>10.29</td>
<td>7</td>
</tr>
</tbody>
</table>

3. ELECTRICITY  The resistance, in ohms, of a certain length of electric wire varies inversely as the square of the diameter of the wire. If a wire 0.04 centimeter in diameter has a resistance of 0.60 ohm, what is the resistance of a wire of the same length and material that is 0.08 centimeter in diameter?

4. BUSINESS  In the manufacturing of a certain digital camera, the cost of producing the camera varies inversely as the number produced. If 15,000 cameras are produced, the cost is $80 per unit. Graph the relationship and label the point that represents the cost per unit to produce 25,000 cameras.

5. SOUND  The sound produced by a string inside a piano depends on its length. The frequency of a vibrating string varies inversely as its length.

a. Write an equation that represents the relationship between frequency $f$ and length $l$. Use $k$ for the constant of variation.

b. If you have two different length strings, which one vibrates more quickly (that is, which string has a greater frequency)?

c. Suppose a piano string 2 feet long vibrates 300 cycles per second. What would be the frequency of a string 4 feet long?
Direct or Indirect Variation

Fill in each table below. Then write \textit{inversely}, or \textit{directly} to complete each conclusion.

1. \[
\begin{array}{c|cccccc}
\ell & 2 & 4 & 8 & 16 & 32 \\
W & 4 & 4 & 4 & 4 & 4 \\
A & & & & & \\
\end{array}
\]

For a set of rectangles with a width of 4, the area varies \underline{\hspace{2cm}} as the length.

2. \[
\begin{array}{c|cccc}
\text{Hours} & 2 & 4 & 5 & 6 \\
\text{Speed} & 55 & 55 & 55 & 55 \\
\text{Distance} & & & & \\
\end{array}
\]

For a car traveling at 55 mi/h, the distance covered varies \underline{\hspace{2cm}} as the hours driven.

3. \[
\begin{array}{c|ccc}
\text{Oat Bran} & \frac{1}{3} \text{ cup} & \frac{2}{3} \text{ cup} & 1 \text{ cup} \\
\text{Water} & 1 \text{ cup} & 2 \text{ cup} & 3 \text{ cup} \\
\text{Servings} & 1 & 2 & \\
\end{array}
\]

The number of servings of oat bran varies \underline{\hspace{2cm}} as the number of cups of oat bran.

4. \[
\begin{array}{c|cccc}
\text{Hours of Work} & 128 & 128 & 128 \\
\text{People Working} & 2 & 4 & 8 \\
\text{Hours per Person} & & & \\
\end{array}
\]

A job requires 128 hours of work. The number of hours each person works varies \underline{\hspace{2cm}} as the number of people working.

5. \[
\begin{array}{c|cccc}
\text{Miles} & 100 & 100 & 100 & 100 \\
\text{Rate} & 20 & 25 & 50 & 100 \\
\text{Hours} & 5 & & & \\
\end{array}
\]

For a 100-mile car trip, the time the trip takes varies \underline{\hspace{2cm}} as the average rate of speed the car travels.

6. \[
\begin{array}{c|cccc}
b & 3 & 4 & 5 & 6 \\
h & 10 & 10 & 10 & 10 \\
A & 15 & & & \\
\end{array}
\]

For a set of right triangles with a height of 10, the area varies \underline{\hspace{2cm}} as the base.

Use the table at the right.

7. \(x\) varies \underline{\hspace{2cm}} as \(y\).

8. \(z\) varies \underline{\hspace{2cm}} as \(y\).

9. \(x\) varies \underline{\hspace{2cm}} as \(z\).
Identify Excluded Values  The function \( y = \frac{10}{x} \) is an example of a \textit{rational function}. Because division by zero is undefined, any value of a variable that results in a denominator of zero must be excluded from the domain of that variable. These are called \textit{excluded values} of the rational function.

**Example**  State the excluded value for each function.

\[ a. \ y = \frac{3}{x} \]
The denominator cannot equal zero.
The excluded value is \( x = 0 \).

\[ b. \ y = \frac{4}{x - 5} \]
Set the denominator equal to 0.
\[ x - 5 = 0 \]
\[ x = 5 \] Add 5 to each side.
The excluded value is \( x = 5 \).

**Exercises**

State the excluded value for each function.

\[ 1. \ y = \frac{2}{x} \]
\[ 2. \ y = \frac{1}{x - 4} \]
\[ 3. \ y = \frac{x - 3}{x + 1} \]

\[ 4. \ y = \frac{4}{x - 2} \]
\[ 5. \ y = \frac{x}{2x - 4} \]
\[ 6. \ y = -\frac{5}{3x} \]

\[ 7. \ y = \frac{3x - 2}{x + 3} \]
\[ 8. \ y = \frac{x - 1}{5x + 10} \]
\[ 9. \ y = \frac{x + 1}{x} \]

\[ 10. \ y = \frac{x - 7}{2x + 8} \]
\[ 11. \ y = \frac{x - 5}{6x} \]
\[ 12. \ y = \frac{x - 2}{x + 11} \]

\[ 13. \ y = \frac{7}{3x + 21} \]
\[ 14. \ y = \frac{3x - 4}{x + 4} \]
\[ 15. \ y = \frac{x}{7x - 35} \]

**16. DINING**  Mya and her friends are eating at a restaurant. The total bill of $36 is split among \( x \) friends. The amount each person pays \( y \) is given by \( y = \frac{36}{x} \), where \( x \) is the number of people. Graph the function.
11-2 Study Guide and Intervention (continued)

Rational Functions

Identify and Use Asymptotes Because excluded values are undefined, they affect the graph of the function. An asymptote is a line that the graph of a function approaches. A rational function in the form \( y = \frac{a}{x - b} + c \) has a vertical asymptote at the \( x \)-value that makes the denominator equal zero, \( x = b \). It has a horizontal asymptote at \( y = c \).

**Example** Identify the asymptotes of \( y = \frac{1}{x - 1} + 2 \). Then graph the function.

**Step 1** Identify and graph the asymptotes using dashed lines.
- vertical asymptote: \( x = 1 \)
- horizontal asymptote: \( y = 2 \)

**Step 2** Make a table of values and plot the points. Then connect them.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.5</td>
<td>1</td>
<td>3</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Exercises** Identify the asymptotes of each function. Then graph the function.

1. \( y = \frac{3}{x} \)
2. \( y = \frac{-2}{x} \)
3. \( y = \frac{4}{x} + 1 \)
4. \( y = \frac{2}{x} - 3 \)
5. \( y = \frac{2}{x + 1} \)
6. \( y = \frac{-2}{x - 3} \)
11-2 Skills Practice

Rational Functions

State the excluded value for each function.

1. \( y = \frac{6}{x} \)

2. \( y = \frac{2}{x - 2} \)

3. \( y = \frac{x}{x + 6} \)

4. \( y = \frac{x - 3}{x + 4} \)

5. \( y = \frac{3x - 5}{x + 8} \)

6. \( y = \frac{-5}{2x - 14} \)

7. \( y = \frac{x}{3x + 21} \)

8. \( y = \frac{x - 1}{9x - 36} \)

9. \( y = \frac{9}{5x + 40} \)

Identify the asymptotes of each function. Then graph the function.

10. \( y = \frac{1}{x} \)

11. \( y = \frac{3}{x} \)

12. \( y = \frac{2}{x + 1} \)

13. \( y = \frac{3}{x - 2} \)

14. \( y = \frac{2}{x + 1} - 1 \)

15. \( y = \frac{1}{x - 2} + 3 \)
11-2 Practice

Rational Functions

State the excluded value for each function.

1. \( y = \frac{-1}{x} \)  
2. \( y = \frac{3}{x + 5} \)  
3. \( y = \frac{2x}{x - 5} \)

4. \( y = \frac{x - 1}{12x + 36} \)  
5. \( y = \frac{x + 1}{2x + 3} \)  
6. \( y = \frac{1}{5x - 2} \)

Identify the asymptotes of each function. Then graph the function.

7. \( y = \frac{1}{x} \)  
8. \( y = \frac{3}{x} \)  
9. \( y = \frac{2}{x - 1} \)

10. \( y = \frac{2}{x + 2} \)  
11. \( y = \frac{1}{x - 3} + 2 \)  
12. \( y = \frac{2}{x + 1} - 1 \)

13. AIR TRAVEL  Denver, Colorado, is located approximately 1000 miles from Indianapolis, Indiana. The average speed of a plane traveling between the two cities is given by \( y = \frac{1000}{x} \), where \( x \) is the total flight time. Graph the function.
11-2 Word Problem Practice

Rational Functions

1. BULLET TRAINS The Shinkansen, or Japanese bullet train network, provides high-speed transportation throughout Japan. Trains regularly operate at speeds in excess of 200 kilometers per hour. The average speed of a bullet train traveling between Tokyo and Kyoto is given by \( y = \frac{515}{x} \), where \( x \) is the total travel time in hours. Graph the function.

2. DRIVING Peter is driving to his grandparents’ house 40 miles away. During the trip, Peter makes a 30-minute stop for lunch. The average speed of Peter’s trip is given by \( y = \frac{40}{x + 0.5} \), where \( x \) is the total time spent in the car. What are the asymptotes of the function?

3. ERROR ANALYSIS Nicolas is graphing the equation \( y = \frac{20}{x + 3} - 6 \) and draws a graph with asymptotes at \( y = 3 \) and \( x = -6 \). Explain the error that Nicolas made in his graph.

4. USED CARS While researching cars to purchase online, Ms. Jacobs found that the value of a used car is inversely proportional to the age of the car. The average price of a used car is given by \( y = \frac{17,900}{x + 1.2} + 100 \), where \( x \) is the age of the car. What are the asymptotes of the function? Explain why \( x = 0 \) cannot be an asymptote.

5. FAMILY REUNION The Gaudet family is holding their annual reunion at Watkins Park. It costs $50 to get a permit to hold the reunion at the park, and the family is spending $8 per person on food. The Gaudets have agreed to split the cost of the event evenly among all those attending.

   a. Write an equation showing the cost per person \( y \) if \( x \) people attend the reunion.

   b. What are the asymptotes of the equation?

   c. Now assume that the family wants to let a long-lost cousin attend for free. Rewrite the equation to find the new cost per paying person \( y \).

   d. What are the asymptotes for the new equation?
Inequalities involving rational functions can be graphed much like those involving linear functions.

**Example**

Graph \( y \geq \frac{1}{x} \).

**Step 1** Plot points and draw a smooth solid curve. Because the inequality involves a greater than or equal to sign, solutions that satisfy \( y = \frac{1}{x} \) will be a part of the graph.

**Step 2** Plot the asymptotes, \( x = 0 \) and \( y = 0 \), as dashed lines.

**Step 3** Begin testing values. A value must be tested between each set of lines, including asymptotes.

- **Region 1** Test \((-1, 1)\). This returns a true value for the inequality.
- **Region 2** Test \((-1, -0.5)\). This returns a true value for the inequality.
- **Region 3** Test \((-1, -2)\). This returns a false value for the inequality.
- **Region 4** Test \((1, 2)\). This returns a true value for the inequality.
- **Region 5** Test \((1, 0.5)\). This returns a false value for the inequality.
- **Region 6** Test \((1, -1)\). This returns a false value for the inequality.

**Step 4** Shade the regions where the inequality is true.

**Exercises**

Graph each inequality.

1. \( y \leq \frac{1}{x + 1} \)
2. \( y > \frac{2}{x} \)
3. \( y \leq \frac{1}{x + 1} - 1 \)
Simplifying Rational Expressions

Identify Excluded Values

Because a rational expression involves division, the denominator cannot equal zero. Any value of the denominator that results in division by zero is called an **excluded value** of the denominator.

**Example 1**

State the excluded value of \( \frac{4m - 8}{m + 2} \).

Exclude the values for which \( m + 2 = 0 \).

- \( m + 2 = 0 \) The denominator cannot equal 0.
- \( m + 2 - 2 = 0 - 2 \) Subtract 2 from each side.
- \( m = -2 \) Simplify.

Therefore, \( m \) cannot equal \(-2\).

**Example 2**

State the excluded values of \( \frac{x^2 + 1}{x^2 - 9} \).

Exclude the values for which \( x^2 - 9 = 0 \).

- \( x^2 - 9 = 0 \) The denominator cannot equal 0.
- \( (x + 3)(x - 3) = 0 \) Factor.
- \( x + 3 = 0 \) or \( x - 3 = 0 \) Zero Product Property
- \( x = -3 \) or \( x = 3 \)

Therefore, \( x \) cannot equal \(-3\) or \(3\).

**Exercises**

State the excluded values for each rational expression.

1. \( \frac{2b}{b^2 - 8} \)
2. \( \frac{12 - a}{32 + a} \)
3. \( \frac{x^2 - 2}{x^2 + 4} \)
4. \( \frac{m^2 - 4}{2m^2 - 8} \)
5. \( \frac{2n - 12}{n^2 - 4} \)
6. \( \frac{2x + 18}{x^2 - 16} \)
7. \( \frac{x^2 - 4}{x^2 + 4x + 4} \)
8. \( \frac{a - 1}{a^2 + 5a + 6} \)
9. \( \frac{k^2 - 2k + 1}{k^2 + 4k + 3} \)
10. \( \frac{m^2 - 1}{2m^2 - m - 1} \)
11. \( \frac{25 - n^2}{n^2 - 4n - 5} \)
12. \( \frac{2x^2 + 5x + 1}{x^2 - 10x + 16} \)
13. \( \frac{n^2 - 2n - 3}{n^2 + 4n - 5} \)
14. \( \frac{y^2 - y - 2}{3y^2 - 12} \)
15. \( \frac{k^2 + 2k - 3}{k^2 - 20k + 64} \)
16. \( \frac{x^2 + 4x + 4}{4x^2 + 11x - 3} \)
Simplifying Rational Expressions

Simplify Expressions Factoring polynomials is a useful tool for simplifying rational expressions. To simplify a rational expression, first factor the numerator and denominator. Then divide each by the greatest common factor.

**Example 1**

Simplify \( \frac{54z^3}{24yz} \).

\[
\frac{54z^3}{24yz} = \frac{(6z)(9z^2)}{(6z)(4y)} = \frac{4(9z^2)}{4y} = \frac{9z^2}{4y}
\]

The GCF of the numerator and the denominator is 6z.

Divide the numerator and denominator by 6z.

Simplify.

**Example 2**

Simplify \( \frac{3x - 9}{x^2 - 5x + 6} \). State the excluded values of \( x \).

\[
\frac{3x - 9}{x^2 - 5x + 6} = \frac{3(x - 3)}{(x - 2)(x - 3)} = \frac{3}{x - 2}
\]

Factor.

Divide by the GCF, \( x - 3 \).

Simplify.

Exclude the values for which \( x^2 - 5x + 6 = 0 \).

\( x^2 - 5x + 6 = 0 \)

\( (x - 2)(x - 3) = 0 \)

\( x = 2 \) or \( x = 3 \)

Therefore, \( x \neq 2 \) and \( x \neq 3 \).

**Exercises**

Simplify each expression. State the excluded values of the variables.

1. \( \frac{12ab}{a^2b^2} \)
2. \( \frac{7n^3}{21n^6} \)
3. \( \frac{x + 2}{x^2 - 4} \)
4. \( \frac{m^2 - 4}{m^2 + 6m + 8} \)
5. \( \frac{2n - 8}{n^2 - 16} \)
6. \( \frac{x^2 + 2x + 1}{x^2 - 1} \)
7. \( \frac{x^2 - 4}{x^2 + 4x + 4} \)
8. \( \frac{a^2 + 3a + 2}{a^2 + 5a + 6} \)
9. \( \frac{k^2 - 1}{k^2 + 4k + 3} \)
10. \( \frac{m^2 - 2m + 1}{2m^2 - m - 1} \)
11. \( \frac{n^2 - 25}{n^2 - 4n - 5} \)
12. \( \frac{x^2 + x - 6}{2x^2 - 8} \)
13. \( \frac{n^2 + 7n + 12}{n^2 + 2n - 8} \)
14. \( \frac{y^2 - y - 2}{y^2 - 10y + 16} \)
11-3 Skills Practice

Simplifying Rational Expressions

State the excluded values for each rational expression.

1. \( \frac{2p}{p - 7} \)
2. \( \frac{4n + 1}{n + 4} \)
3. \( \frac{k + 2}{k^2 - 4} \)
4. \( \frac{3x + 15}{x^2 - 25} \)
5. \( \frac{y^2 - 9}{y^2 + 3y - 18} \)
6. \( \frac{b^2 - 2b - 8}{b^2 + 7b + 10} \)

Simplify each expression. State the excluded values of the variables.

7. \( \frac{21bc}{28bc^2} \)
8. \( \frac{12m^2r}{24mr^3} \)
9. \( \frac{16x^3y^2}{36x^5y^3} \)
10. \( \frac{8a^2b^3}{40a^3b} \)
11. \( \frac{n + 6}{3n + 18} \)
12. \( \frac{4x - 4}{4x + 4} \)
13. \( \frac{y^2 - 64}{y + 8} \)
14. \( \frac{y^2 - 7y - 18}{y - 9} \)
15. \( \frac{z + 1}{z^2 - 1} \)
16. \( \frac{x + 6}{x^2 + 2x - 24} \)
17. \( \frac{2d + 10}{d^2 - 2d - 35} \)
18. \( \frac{3h - 9}{h^2 - 7h + 12} \)
19. \( \frac{t^2 + 5t + 6}{t^2 + 6t + 8} \)
20. \( \frac{a^2 + 3a - 4}{a^2 + 2a - 8} \)
21. \( \frac{x^2 + 10x + 24}{x^2 - 2x - 24} \)
22. \( \frac{b^2 - 6b + 9}{b^2 - 9b + 18} \)
11-3 Practice

Simplifying Rational Expressions

State the excluded values for each rational expression.

1. \(\frac{4n - 28}{n^2 - 49}\)  
2. \(\frac{p^2 - 16}{p^2 - 13p + 36}\)  
3. \(\frac{a^2 - 2a - 15}{a^2 + 8a + 15}\)

Simplify each expression. State the excluded values of the variables.

4. \(\frac{12a}{48a^3}\)  
5. \(\frac{6xyz^3}{3x^2y^2z}\)  
6. \(\frac{36k^3np^2}{20k^2np^5}\)  
7. \(\frac{5c^3d^4}{40cd^2 + 5c^4d^2}\)  
8. \(\frac{p^2 - 8p + 12}{p - 2}\)  
9. \(\frac{m^2 - 4m - 12}{m - 6}\)  
10. \(\frac{m + 3}{m^2 - 9}\)  
11. \(\frac{2b - 14}{b^2 - 9b + 14}\)  
12. \(\frac{x^2 - 7x + 10}{x^2 - 2x - 15}\)  
13. \(\frac{y^2 + 6y - 16}{y^2 - 4y + 4}\)  
14. \(\frac{r^2 - 7r + 6}{r^2 + 6r - 7}\)  
15. \(\frac{t^2 - 81}{t^2 - 12t + 27}\)  
16. \(\frac{r^2 + r - 6}{r^2 + 4r - 12}\)  
17. \(\frac{2x^2 + 18x + 36}{3x^2 - 3x - 36}\)  
18. \(\frac{2y^2 + 9y + 4}{4y^2 - 4y - 3}\)

19. ENTERTAINMENT  Fairfield High spent \(d\) dollars for refreshments, decorations, and advertising for a dance. In addition, they hired a band for $550.

   a. Write an expression that represents the cost of the band as a fraction of the total amount spent for the school dance.

   b. If \(d\) is $1650, what percent of the budget did the band account for?

20. PHYSICAL SCIENCE  Mr. Kaminski plans to dislodge a tree stump in his yard by using a 6-foot bar as a lever. He places the bar so that 0.5 foot extends from the fulcrum to the end of the bar under the tree stump. In the diagram, \(b\) represents the total length of the bar and \(t\) represents the portion of the bar beyond the fulcrum.

   a. Write an equation that can be used to calculate the mechanical advantage.

   b. What is the mechanical advantage?

   c. If a force of 200 pounds is applied to the end of the lever, what is the force placed on the tree stump?
11-3 Word Problem Practice

Simplifying Rational Expressions

1. **PHYSICAL SCIENCE** Pressure is equal to the magnitude of a force divided by the area over which the force acts.
   \[ P = \frac{F}{A} \]

   Gabe and Shelby each push open a door with one hand. In order to open, the door requires 20 pounds of force. The surface area of Gabe's hand is 10 square inches, and the surface area of Shelby's hand is 8 square inches. Whose hand feels the greater pressure?

2. **GRAPHING** Recall that the slope of a line is a ratio of the vertical change to the horizontal change in coordinates for two given points. Write a rational expression that represents the slope of the line containing the points at \((p, r)\) and \((7, -3)\).

3. **AUTOMOBILES** The force needed to keep a car from skidding out of a turn on a particular road is given by the formula below. What force is required to keep a 2000-pound car traveling at 50 miles per hour on a curve with radius of 750 feet on the road? What value of \(r\) is excluded?
   \[ f = \frac{0.0672ws^2}{r} \]
   \(f = \) force in pounds
   \(w = \) weight in pounds
   \(s = \) speed in mph
   \(r = \) radius in feet

4. **PACKAGING** In order to safely ship a new electronic device, the distribution manager at Data Products Company determines that the package must contain a certain amount of cushioning on each side of the device. The device is shaped like a cube with side length \(x\), and some sides need more cushioning than others because of the device's design. The volume of a shipping container is represented by the expression \((x^2 + 6x + 8)(x + 6)\). Find the polynomial that represents the area of the top of the box if the height of the box is \(x + 2\).

5. **SCHOOL CHOICE** During a recent school year, the ratio of public school students to private school students in the United States was approximately 7.6 to 1. The students attending public school outnumbered those attending private schools by 42,240,000.
   a. Write a rational expression to express the ratio of public school students to \(x\) private school students.
   b. How many students attended private school?
Shannon's Juggling Theorem

Mathematicians look at various mathematical ways to represent juggling. One way they have found to represent juggling is Shannon's Juggling Theorem. Shannon's Juggling Theorem uses the rational equation

\[ \frac{f + d}{v + d} = \frac{b}{h} \]

where \( f \) is the flight time, or how long a ball is in the air, \( d \) is the dwell time, or how long a ball is in a hand, \( v \) is the vacant time, or how long a hand is empty, \( b \) is the number of balls, and \( h \) is the number of hands (either 1 or 2 for a real-life situation, possibly more for a computer simulation).

So, given the values for \( f, d, v, \) and \( h \), it is possible to determine the number of balls being juggled. If the flight time is 9 seconds, the dwell time is 3 seconds, the vacant time is 1 second, and the number of hands is 2, how many balls are being juggled?

\[
\frac{9 + 3}{1 + 3} = \frac{b}{2} \quad \text{Original equation}
\]

\[
12 = 4b \quad \text{Simplify.}
\]

\[
6 = b \quad \text{Divide.}
\]

So, the number of balls being juggled is 6.

Given the following information, determine the number of balls being juggled.

1. flight time = 6 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2

2. flight time = 13 seconds, vacant time = 1 second, dwell time = 5 seconds, number of hands = 1

3. flight time = 4 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2

4. flight time = 16 seconds, vacant time = 1 second, dwell time = 2 seconds, number of hands = 2

5. flight time = 18 seconds, vacant time = 3 seconds, dwell time = 2 seconds, number of hands = 1
Multiplying and Dividing Rational Expressions

Multiply Rational Expressions

To multiply rational expressions, you multiply the numerators and multiply the denominators. Then simplify.

Example 1

Find \( \frac{2c^2f}{5ab^2} \cdot \frac{a^2b}{3cf} \).

\[
\frac{2c^2f}{5ab^2} \cdot \frac{a^2b}{3cf} = \frac{2a^2bc^2f}{15ab^2cf} \quad \text{Multiply.}
\]

\[
= \frac{(abef)(2ac)}{(abef)(15b)} \quad \text{Simplify.}
\]

\[
= \frac{2ac}{15b} \quad \text{Simplify.}
\]

Example 2

Find \( \frac{x^2 - 16}{2x + 8} \cdot \frac{x + 4}{x^2 + 8x + 16} \).

\[
\frac{x^2 - 16}{2x + 8} \cdot \frac{x + 4}{x^2 + 8x + 16} = \frac{(x - 4)(x + 4)}{2(x + 4)} \cdot \frac{x + 4}{(x + 4)(x + 4)} \quad \text{Factor.}
\]

\[
= \frac{(x - 4)(x + 4)}{2(x + 4)} \cdot \frac{x + 4}{(x + 4)(x + 4)} \quad \text{Simplify.}
\]

\[
= \frac{x - 4}{2x + 8} \quad \text{Multiply.}
\]

Exercises

Find each product.

1. \( \frac{6ab}{a^2b^2} \cdot \frac{a^2}{b} \)

2. \( \frac{mp^2}{3} \cdot \frac{4}{mp} \)

3. \( \frac{x + 2}{x - 4} \cdot \frac{x - 4}{x + 1} \)

4. \( \frac{m - 5}{8} \cdot \frac{16}{m - 5} \)

5. \( \frac{2n - 8}{n} \cdot \frac{2n + 4}{n - 4} \)

6. \( \frac{x^2 - 64}{2x + 16} \cdot \frac{x + 8}{x^2 + 16x + 64} \)

7. \( \frac{8x + 8}{x^2 - 2x + 1} \cdot \frac{x - 1}{2x + 2} \)

8. \( \frac{a^2 - 25}{a + 2} \cdot \frac{a - 4}{a - 5} \)

9. \( \frac{x^2 + 6x + 8}{2x + 9x + 4} \cdot \frac{2x^2 - x - 1}{x^2 - 3x + 2} \)

10. \( \frac{m^2 - 1}{2m^2 - m - 1} \cdot \frac{2m + 1}{m^2 - 2m + 1} \)

11. \( \frac{n^2 - 1}{n^2 - 7n + 10} \cdot \frac{n^2 - 25}{n^2 + 6n + 5} \)

12. \( \frac{3p - 3r}{10pr} \cdot \frac{20p^2r^2}{p^2 - r^2} \)

13. \( \frac{a^2 + 7a + 12}{a^2 + 2a - 8} \cdot \frac{a^2 + 3a - 10}{a^2 + 2a - 8} \)

14. \( \frac{v^2 - 4v - 21}{3v^2 + 6v} \cdot \frac{v^2 + 8v}{v^2 + 11v + 24} \)
Chapter 11  

NAME ___________________________ DATE ___________ PERIOD ___________ 

11-4 Study Guide and Intervention (continued)  

Multiplying and Dividing Rational Expressions 

Divide Rational Expressions  To divide rational expressions, multiply by the reciprocal of the divisor. Then simplify. 

Example 1  

Find \( \frac{12c^2f}{5a^2b^2} \div \frac{c^2f^2}{10ab} \). 

\[
\frac{12c^2f}{5a^2b^2} \div \frac{c^2f^2}{10ab} = \frac{12c^2f \times 10ab}{5a^2b^2 \times c^2f^2} = \frac{120acf}{5a^2b^2c^2f^2} = \frac{24}{abf}
\]

Example 2  

Find \( \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^2 + x - 2} \). 

\[
\frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^2 + x - 2} = \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \times \frac{x^2 + x - 2}{x - 3} = \frac{(x + 9)(x - 3)}{(x + 9)(x + 2)} \times \frac{(x + 2)(x - 1)}{x - 3} = \frac{(x + 9)(x - 3)}{(x + 9)(x + 2)} \times \frac{(x + 2)(x - 1)}{x - 3} = x - 1
\]

Exercises 

Find each quotient. 

1. \( \frac{12ab}{a^2b^2} \div \frac{b}{a} \) 
2. \( \frac{n}{4} \div \frac{n}{p} \) 
3. \( \frac{3xy^2}{8} \div 6xy \) 
4. \( \frac{m - 5}{8} \div \frac{m - 5}{16} \) 
5. \( \frac{2n - 4}{2n} \div \frac{n^2 - 4}{n} \) 
6. \( \frac{y^2 - 36}{y^2 - 49} \div \frac{y + 6}{y + 7} \) 
7. \( \frac{x^2 - 5x + 6}{5} \div \frac{x - 3}{15} \) 
8. \( \frac{a^2b^2c}{3r^2t} \div \frac{6a^2bc}{8rt^2u} \) 
9. \( \frac{x^2 + 6x + 8}{x^2 + 4x + 4} \div \frac{x + 4}{x + 2} \) 
10. \( \frac{m^2 - 49}{m} \div \frac{m^2 - 13m + 42}{3m^2} \) 
11. \( \frac{n^2 - 5n + 6}{n^2 + 3n} \div \frac{3 - n}{4n + 12} \) 
12. \( \frac{p^2 - 2pr + r^2}{p + r} \div \frac{p^2 - r^2}{p + r} \) 
13. \( \frac{a^2 + 7a + 12}{a^2 + 3a - 10} \div \frac{a^2 - 9}{a^2 - 25} \) 
14. \( \frac{a^2 - 9}{2a^2 + 13a - 7} \div \frac{a + 3}{4a^2 - 1} \)
11-4 Skills Practice

Multiplying and Dividing Rational Expressions

Find each product.

1. \( \frac{14}{c^2} \cdot \frac{c^5}{2c} \)

2. \( \frac{3m^2}{2t} \cdot \frac{t^2}{12} \)

3. \( \frac{2a^2b}{b^2c} \cdot \frac{b}{a} \)

4. \( \frac{2x^2y}{3x^2y} \cdot \frac{3xy}{4y} \)

5. \( \frac{3(4m - 6)}{18r} \cdot \frac{9r^2}{2(4m - 6)} \)

6. \( \frac{4(n + 2)}{n(n - 2)} \cdot \frac{n - 2}{n + 2} \)

7. \( \frac{(y - 3)(y + 3)}{4} \cdot \frac{8}{y + 3} \)

8. \( \frac{(x - 2)(x + 2)}{x(8x + 3)} \cdot \frac{2(8x + 3)}{x - 2} \)

9. \( \frac{(a - 7)(a + 7)}{a(a + 5)} \cdot \frac{a + 5}{a + 7} \)

10. \( \frac{4(b + 4)}{(b - 4)(b - 3)} \cdot \frac{b - 3}{b + 4} \)

Find each quotient.

11. \( \frac{c^3}{d^3} \div \frac{d^3}{c^3} \)

12. \( \frac{x^3}{y^2} \div \frac{x^3}{y} \)

13. \( \frac{6a^3}{4f^2} \div \frac{2a^2}{12f^2} \)

14. \( \frac{4m^3}{rp^2} \div \frac{2m}{rp} \)

15. \( \frac{3b + 3}{b + 2} \div (b + 1) \)

16. \( \frac{x - 5}{x + 3} \div (x - 5) \)

17. \( \frac{x^2 - x - 12}{6} \div \frac{x + 3}{x - 4} \)

18. \( \frac{a^2 - 5a - 6}{3} \div \frac{a - 6}{a + 1} \)

19. \( \frac{m^2 + 2m + 1}{10m - 10} \div \frac{m + 1}{20} \)

20. \( \frac{y^2 + 10y + 25}{3y - 9} \div \frac{y + 5}{y - 3} \)

21. \( \frac{b + 4}{b^2 - 8b + 16} \div \frac{2b + 8}{b - 8} \)

22. \( \frac{6x + 6}{x - 1} \div \frac{x^2 + 3x + 2}{2x - 2} \)
11-4 Practice

Multiplying and Dividing Rational Expressions

Find each product or quotient.

1. \[ \frac{18x^2}{10y^2} \cdot \frac{15y^3}{24x} \]

2. \[ \frac{24r^2}{8r^3t^3} \cdot \frac{12r^3t^2}{36r^2t} \]

3. \[ \frac{(x + 2)(x + 2)}{8} \cdot \frac{72}{(x + 2)(x - 2)} \]

4. \[ \frac{m + 7}{(m - 6)(m + 2)} \cdot \frac{(m - 6)(m + 4)}{(m + 7)} \]

5. \[ \frac{a - 4}{a^2 - a - 12} \cdot \frac{a + 3}{a - 6} \]

6. \[ \frac{4x + 8}{x^2} \cdot \frac{x}{x^2 - 5x - 14} \]

7. \[ \frac{n^2 + 10n + 16}{5n - 10} \cdot \frac{n - 2}{n^2 + 9n + 8} \]

8. \[ \frac{3y - 9}{y^2 - 9y + 20} \cdot \frac{y^2 - 8y + 16}{y - 3} \]

9. \[ \frac{b^2 + 5b + 4}{b^2 - 20} \cdot \frac{b^2 + 5b - 6}{b^2 + 2b - 8} \]

10. \[ \frac{t^2 + 6t + 9}{t^2 - 10t + 25} \cdot \frac{t^2 - t - 20}{t^2 + 7t + 12} \]

11. \[ \frac{28a^2}{7b^2} \div \frac{21a^3}{35b} \]

12. \[ \frac{mn^2p^3}{x^2y^2} \div \frac{mp^2}{x^3y} \]

13. \[ \frac{2a}{a - 1} \div (a + 1) \]

14. \[ \frac{z^2 - 16}{3z} \div (z - 4) \]

15. \[ \frac{4y + 20}{y - 3} \div \frac{y + 5}{2y - 6} \]

16. \[ \frac{4x + 12}{6x - 24} \div \frac{2x + 6}{x + 3} \]

17. \[ \frac{b^2 + 2b - 8}{b^2 - 11b + 18} \div \frac{2b - 8}{2b - 18} \]

18. \[ \frac{3x - 3}{x^2 - 6x + 9} \div \frac{6x - 6}{x^2 - 5x + 6} \]

19. \[ \frac{a^2 + 8a + 12}{a^2 - 7a + 10} \div \frac{a^2 - 4a - 12}{a^2 + 3a - 10} \]

20. \[ \frac{y^2 + 6y - 7}{y^2 + 8y - 9} \div \frac{y^2 + 9y + 14}{y^2 + 7y - 18} \]

21. BIOLOGY  The heart of an average person pumps about 9000 liters of blood per day. How many quarts of blood does the heart pump per hour? (Hint: One quart is equal to 0.946 liter.) Round to the nearest whole number.

22. TRAFFIC  On Saturday, it took Ms. Torres 24 minutes to drive 20 miles from her home to her office. During Friday’s rush hour, it took 75 minutes to drive the same distance.

   a. What was Ms. Torres’s average speed in miles per hour on Saturday?

   b. What was her average speed in miles per hour on Friday?
### 1. JOBS
Rosa earned $26.25 for babysitting for $3\frac{1}{2}$ hours. At this rate, how much will she earn babysitting for 5 hours?

### 2. HOMEWORK
Alejandro and Ander were working on the following homework problem.

Find \( \frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} \).

#### Alejandro's Solution
\[
\frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} = \frac{2(n - 10)(n + 3)}{(n + 3)(n + 3)} = \frac{2n - 20}{n + 3}
\]

#### Ander's Solution
\[
\frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} = \frac{2(n - 10)(n + 3)}{n + 3} = 2n - 20
\]

Is either of them correct? Explain.

### 3. GEOMETRY
Suppose the rational expression \( \frac{5k^2m^3}{2ab} \) represents the area of a section in a tiled floor and \( \frac{2km}{a} \) represents the section's length. Write a rational expression to represent the section's width.

### 4. TRAVEL
Helene travels 800 miles from Amarillo to Brownsville at an average speed of 40 miles per hour. She makes the return trip driving an average of 60 miles per hour. What is the average rate for the entire trip? (Hint: Recall that \( t = \frac{d}{r} \).)

### 5. MANUFACTURING
India works in a metal shop and needs to drill equally spaced holes along a strip of metal. The centers of the holes on the ends of the strip must be exactly 1 inch from each end. The remaining holes will be equally spaced.

- **a.** If there are \( x \) equally spaced holes, write an expression for the number of equal spaces there are between holes.

- **b.** Write an expression for the distance between the end screws if the length is \( \ell \).

- **c.** Write a rational equation that represents the distance between the holes on a piece of metal that is \( \ell \) inches long and must have \( x \) equally spaced holes.

- **d.** How many holes will be drilled in a metal strip that is 6 feet long with a distance of 7 inches between the centers of each screw?
Geometric Series

A geometric series is a sum of the terms in a geometric sequence. Each term of a geometric sequence is formed by multiplying the previous term by a constant term called the common ratio.

\[ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \rightarrow \text{geometric sequence where the common ratio is } \frac{1}{2} \]

The sum of a geometric series can be represented by the rational expression 
\[ x_0 \frac{(r)^n - 1}{r - 1} \], where \( x_0 \) is the first term of the series, \( r \) is the common ratio, and \( n \) is the number of terms.

In the example above, 
\[ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 \cdot \frac{(\frac{1}{2})^4 - 1}{\frac{1}{2} - 1} \text{ or } \frac{15}{8}. \]

You can check this by entering \( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \) a calculator. The result is the same.

Rewrite each sum as a rational expression and simplify.

1. \( 9 + 3 + 1 + \frac{1}{3} + \frac{1}{9} \)

2. \( 500 + 250 + 125 + 62 \frac{1}{2} \)

3. \( 6 + 1 + \frac{1}{6} + \frac{1}{36} \)

4. \( 100 + 20 + 4 + \frac{4}{5} \)

5. \( 1000 + 100 + 10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} \)

6. \( 55 + 5 + \frac{5}{11} + \frac{5}{121} \)
Spreadsheet Activity

Revolutions per Minute

One of the characteristics that makes a spreadsheet powerful is the ability to recalculate values in formulas automatically. You can use this ability to investigate real-world situations.

**Example**

Use a spreadsheet to investigate the effect of doubling the diameter of a tire on the number of revolutions the tire makes at a given speed.

Use dimensional analysis to find the formula for the revolutions per minute of a tire with diameter of \(x\) inches traveling at \(y\) miles per hour.

\[
\frac{1 \text{ revolution}}{\pi \times x} \times \frac{y}{1} \times \frac{1}{60 \text{ minutes}} \times \frac{63,360}{1} = \frac{1056y \text{ revolutions}}{\pi \times x \text{ minutes}}
\]

**Step 1**
Use Column A of the spreadsheet for diameter of the tire in inches. Use Column B for the speed in miles per hour.

**Step 2**
Column C contains the formula for the number of rotations per minute. Notice that in Excel, \(\pi\) is entered as PI().

**Step 3**
Choose values for the diameter and speed and study the results shown in the spreadsheet. To compare the revolutions per minute for doubled diameters, keep the speed constant and change the diameters. It appears that when the diameter is doubled, the number of revolutions per minute is halved.

**Exercises**

Use the spreadsheet of revolutions per minute.

1. How is the number of revolutions affected if the speed of a wheel of a given diameter is doubled?

2. Name two ways that you can double the RPM of a bicycle wheel.
Divide Polynomials by Monomials
To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 1: Find \((4r^2 - 12r) \div (2r)\).

\[
(4r^2 - 12r) \div 2r = \frac{4r^2 - 12r}{2r}
\]

\[
= \frac{4r^2}{2r} - \frac{12r}{2r}
\]

\[
= 2r - 12r
\]

\[
= 2r - 6
\]

Example 2: Find \((3x^2 - 8x + 4) \div (4x)\).

\[
(3x^2 - 8x + 4) \div 4x = \frac{3x^2 - 8x + 4}{4x}
\]

\[
= \frac{3x^2}{4x} - \frac{8x}{4x} + \frac{4}{4x}
\]

\[
= \frac{3x}{4} - 2 + \frac{1}{x}
\]

Exercises
Find each quotient.

1. \((x^3 + 2x^2 - x) \div x\)

2. \((2x^3 + 12x^2 - 8x) \div (2x)\)

3. \((x^2 + 3x - 4) \div x\)

4. \((4m^2 + 6m - 8) \div (2m^2)\)

5. \((3x^3 + 15x^2 - 21x) \div (3x)\)

6. \((8m^2p^2 + 4mp - 8p) \div p\)

7. \((8y^4 + 16y^2 - 4) \div (4y^2)\)

8. \((16x^2y^2 + 24xy + 5) \div (xy)\)

9. \(\frac{15x^2 - 25x + 30}{5}\)

10. \(\frac{10a^2b + 12ab - 8b}{2a}\)

11. \(\frac{6x^3 + 9x^2 + 9}{3x}\)

12. \(\frac{m^2 - 12m + 42}{3m^2}\)

13. \(\frac{m^2p^2 - 5mp + 6}{m^2p^2}\)

14. \(\frac{p^2 - 4pr + 6r^2}{pr}\)

15. \(\frac{6a^2b^2 - 8ab + 12}{2a^2}\)

16. \(\frac{2x^2y^3 - 4x^2y^2 - 8xy}{2xy}\)

17. \(\frac{9x^2y^2 - 2xyz + 12x}{xy}\)

18. \(\frac{2a^3b^3 + 8a^2b^2 - 10ab + 12}{2a^2b^2}\)
Dividing Polynomials

Divide Polynomials by Binomials To divide a polynomial by a binomial, factor the dividend if possible and divide both dividend and divisor by the GCF. If the polynomial cannot be factored, use long division.

Example Find \((x^2 + 7x + 10) \div (x + 3)\).

Step 1 Divide the first term of the dividend, \(x^2\), by the first term of the divisor, \(x\).

\[
\begin{array}{c|cc}
\text{Step 1} & x^2 & + 7x & + 10 \\
\hline
x + 3 & x^2 & + 3x \\
\hline
& -x^2 & -3x \\
& 4x & \\
\end{array}
\]

Multiply \(x\) and \(x + 3\).

Subtract.

Step 2 Bring down the next term, 10. Divide the first term of \(4x + 10\) by \(x\).

\[
\begin{array}{c|cc}
\text{Step 2} & 4x & + 10 \\
\hline
x + 4 & 4x & + 12 \\
\hline
& -x & -2 \\
& & \\
\end{array}
\]

Multiply 4 and \(x + 3\).

Subtract.

The quotient is \(x + 4\) with remainder \(-2\). The quotient can be written as \(x + 4 + \frac{-2}{x + 3}\).

Exercises

Find each quotient.

1. \((b^2 - 5b + 6) \div (b - 2)\)
2. \((x^2 - x - 6) \div (x - 3)\)
3. \((x^2 + 3x - 4) \div (x - 1)\)
4. \((m^2 + 2m - 8) \div (m + 4)\)
5. \((x^2 + 5x + 6) \div (x + 2)\)
6. \((m^2 + 4m + 4) \div (m + 2)\)
7. \((2y^2 + 5y + 2) \div (y + 2)\)
8. \((8y^2 - 15y - 2) \div (y - 2)\)
9. \(\frac{8x^2 - 6x - 9}{4x + 3}\)
10. \(\frac{m^2 - 5m - 6}{m - 6}\)
11. \(\frac{x^3 + 1}{x - 2}\)
12. \(\frac{6m^3 + 11m^2 + 4m + 35}{2m + 5}\)
13. \(\frac{6a^2 + 7a + 5}{2a + 5}\)
14. \(\frac{8p^3 + 27}{2p + 3}\)
11-5 Skills Practice

Dividing Polynomials

Find each quotient.

1. \((20x^2 + 12x) \div 4x\)  
2. \((18n^2 + 6n) \div 3n\)

3. \((b^2 - 12b + 5) \div 2b\)  
4. \((8r^2 + 5r - 20) \div 4r\)

5. \(\frac{12p^3r^2 + 18p^2r - 6pr}{6p^2r}\)  
6. \(\frac{15k^2u - 10ku + 25u^2}{5ku}\)

7. \((x^2 - 5x - 6) \div (x - 6)\)  
8. \((a^2 - 10a + 16) \div (a - 2)\)

9. \((n^2 - n - 20) \div (n + 4)\)  
10. \((y^2 + 4y - 21) \div (y - 3)\)

11. \((h^2 - 6h + 9) \div (h - 2)\)  
12. \((b^2 + 5b - 2) \div (b + 6)\)

13. \((y^2 + 6y + 1) \div (y + 2)\)  
14. \((m^2 - 2m - 5) \div (m - 3)\)

15. \(\frac{2c^2 - 5c - 3}{2c + 1}\)  
16. \(\frac{2r^2 + 6r - 20}{2r - 4}\)

17. \(\frac{x^3 - 3x^2 - 6x - 20}{x - 5}\)  
18. \(\frac{p^3 - 4p^2 + p + 6}{p - 2}\)

19. \(\frac{n^3 - 6n - 2}{n + 1}\)  
20. \(\frac{y^3 - y^2 - 40}{y - 4}\)
11-5 Practice

Dividing Polynomials

Find each quotient.

1. \((6q^2 - 18q - 9) \div 9q\)

2. \((y^2 + 6y + 2) \div 3y\)

3. \(\frac{12a^2b - 3ab^2 + 42ab}{6a^2b}\)

4. \(\frac{2m^3p^2 + 56mp - 4m^2p^3}{8m^3p}\)

5. \((x^2 - 3x - 40) \div (x + 5)\)

6. \((3m^2 - 20m + 12) \div (m - 6)\)

7. \((a^2 + 5a + 20) \div (a - 3)\)

8. \((x^2 - 3x - 2) \div (x + 7)\)

9. \((t^2 + 9t + 28) \div (t + 3)\)

10. \((n^2 - 9n + 25) \div (n - 4)\)

11. \(\frac{6r^2 - 5r - 56}{3r + 8}\)

12. \(\frac{20w^2 + 39w + 18}{5w + 6}\)

13. \((x^3 + 2x^2 - 16) \div (x - 2)\)

14. \((t^3 - 11t - 6) \div (t + 3)\)

15. \(\frac{x^3 + 6x^2 + 3x + 1}{x - 2}\)

16. \(\frac{6d^3 + d^2 - 2d + 17}{2d + 3}\)

17. \(\frac{2k^3 + k^2 - 12k + 11}{2k - 3}\)

18. \(\frac{9y^3 - y - 1}{3y + 2}\)

19. **LANDSCAPING** Jocelyn is designing a bed for cactus specimens at a botanical garden. The total area can be modeled by the expression \(2x^2 + 7x + 3\), where \(x\) is in feet.

   a. Suppose in one design the length of the cactus bed is \(4x\), and in another, the length is \(2x + 1\). What are the widths of the two designs?

   b. If \(x = 3\) feet, what will be the dimensions of the cactus bed in each of the designs?

20. **FURNITURE** Teri is upholstering the seats of four chairs and a bench. She needs \(\frac{1}{4}\) square yard of fabric for each chair, and \(\frac{1}{2}\) square yard for the bench. If the fabric at the store is 45 inches wide, how many yards of fabric will Teri need to cover the chairs and the bench if there is no waste?
11-5 Word Problem Practice

Dividing Polynomials

1. TECHNOLOGY The surface area in square millimeters of a rectangular computer microchip is represented by the expression \(x^2 - 12x + 35\), where \(x\) is the number of circuits. If the width of the chip is \(x - 5\) millimeters, write a polynomial that represents the length.

2. HOMEWORK Your classmate Ava writes her answer to a homework problem on the chalkboard. She has simplified \(\frac{6x^2 - 12x}{6}\) as \(x^2 - 12x\). Is this correct? If not, what is the correct simplification?

3. CIVIL ENGINEERING Suppose 5400 tons of concrete costs \((500 + d)\) dollars. Write a formula that gives the cost \(C\) of \(t\) tons of concrete.

4. SHIPPING The Overseas Shipping Company loads cargo into a container to be shipped around the world. The volume of their shipping containers is determined by the following equation.
\[
x^3 + 21x^2 + 99x + 135
\]
The container’s height is \(x + 3\). Write an expression that represents the area of the base of the shipping container.

5. CIVIL ENGINEERING Greenshield’s Formula can be used to determine the amount of time a traffic light at an intersection should remain green.
\[
G = 2.1n + 3.7
\]
\(G\) = green time in seconds
\(n\) = average number of vehicles traveling in each lane per light cycle
Write a simplified expression to represent the average green light time per vehicle.

6. SOLID GEOMETRY The surface area of a right cylinder is given by the formula \(S = 2\pi r^2 + 2\pi rh\).

a. Write a simplified rational expression that represents the ratio of the surface area to the circumference of the cylinder.

b. Write a simplified rational expression that represents the ratio of the surface area to the area of the base.
Synthetic Division

You can divide a polynomial such as \(3x^3 - 4x^2 - 3x - 2\) by a binomial such as \(x - 3\) by a process called synthetic division. Compare the process with long division in the following explanation.

**Example**

Divide \((3x^3 - 4x^2 - 3x - 2)\) by \((x - 3)\) using synthetic division.

1. Show the coefficients of the terms in descending order.

   \[
   
   \begin{array}{cccc}
   3 & -4 & -3 & -2 \\
   
   \hline
   9 & 15 & 36 \\
   
   3 & 5 & 12 & 34 \\
   
   \end{array}
   
   \]

   \(3x^2 + 5x + 12, \text{ remainder } 34\)

2. The divisor is \(x - 3\). Since 3 is to be subtracted, write 3 in the corner.

3. Bring down the first coefficient, 3.

4. Multiply. \(3 \cdot 3 = 9\)

5. Add. \(-4 + 9 = 5\)

6. Multiply. \(3 \cdot 5 = 15\)

7. Add. \(-3 + 15 = 12\)

8. Multiply. \(3 \cdot 12 = 36\)

9. Add. \(-2 + 36 = 34\)

**Check**

Use long division.

\[
\begin{array}{c}
 3x^2 + 5x + 12 \\
\hline
  x - 3 \left| 3x^3 - 4x^2 - 3x - 2 \\
  \hline
  3x^3 - 9x^2 \\
  \hline
  5x^2 - 3x \\
  \hline
  5x^2 - 15x \\
  \hline
  12x - 2 \\
  \hline
  12x - 36 \\
  \hline
  34
\end{array}
\]

The result is \(3x^2 + 5x + 12 + \frac{34}{x - 3}\).

Divide by using synthetic division. Check your result using long division.

1. \((x^3 + 6x^2 + 3x + 1) \div (x - 2)\)

2. \((x^3 - 3x^2 - 6x - 20) \div (x - 5)\)

3. \((2x^3 - 5x + 1) \div (x + 1)\)

4. \((3x^3 - 7x^2 + 4) \div (x - 2)\)

5. \((x^3 + 2x^2 - x + 4) \div (x + 3)\)

6. \((x^3 + 4x^2 - 3x - 11) \div (x - 4)\)
11-5 TI-Nspire® Activity

Dividing Polynomials

You can use a graphing calculator with a computer algebra system (CAS) to divide polynomials with any divisor. One function is used to find the quotient, while another function is used to find the remainder.

Example

Use CAS to find \((x^4 + x^3 - 3x^2 + x) \div (x^2 + 4x + 5)\).

Step 1 Add a new Calculator page on the TI-Nspire.

Step 2 From the menu, select Algebra, Polynomial Tools and Quotient of Polynomial.

Step 3 Type the dividend, a comma, and the divisor.

The CAS indicates that \((x^4 + x^3 - 3x^2 + x) \div (x^2 + 4x + 5)\) is \(x^2 - 3x + 4\).

We need to determine whether there is a remainder.

Step 4 Use the Remainder of a Polynomial option from the Algebra, Polynomial Tools menu to determine the remainder. Then type the dividend, a comma, and the divisor.

The remainder is \(-20\). Therefore, \((x^4 + x^3 - 3x^2 + x) \div (x^2 + 4x + 5)\) is \(x^2 - 3x + 4 - \frac{20}{x^2 + 4x + 5}\).

Check Use the Expand option from the Algebra menu to confirm your answer. Type the quotient multiplied by the sum of the divisor and the remainder over the divisor. The CAS confirms that the division was correct.

Exercises

Find each quotient.

1. \((x^4 + 2x^3 - 19x^2 + 28x - 12) \div (x^2 + 5x - 6)\)

2. \((2x^4 + 3x^2 + x + 9) \div (x^2 - 2x + 3)\)

3. \((3x^4 - 20x^3 + 58x^2 - 30x + 8) \div (3x^2 - 2x + 1)\)

4. \((4x^4 - 7x^3 + 29x^2 - 3x - 40) \div (4x^2 + x + 5)\)

5. \((2x^5 + 15x^4 + 10x^3 + 4x^2 + 15x + 16) \div (x^2 + 3x + 5)\)

6. \((x^5 + 4x^4 - 5x^3 + 12x^2 + 40x + 60) \div (x^3 + x + 8)\)

7. Think About It When a polynomial is divided by \((x^2 + 3x + 5)\), the quotient is \((x^2 + 6x - 8)\) with a remainder of \(4x + 10\). What is the polynomial?
Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions with Like Denominators To add rational expressions with like denominators, add the numerators and then write the sum over the common denominator. To subtract fractions with like denominators, subtract the numerators. If possible, simplify the resulting rational expression.

Example 1
Find \( \frac{5n}{15} + \frac{7n}{15} \).

\[
\frac{5n}{15} + \frac{7n}{15} = \frac{5n + 7n}{15} \\
= \frac{12n}{15} \\
= \frac{4n}{5}
\]

Example 2
Find \( \frac{3x + 2}{x - 2} - \frac{4x}{x - 2} \).

\[
\frac{3x + 2}{x - 2} - \frac{4x}{x - 2} = \frac{3x + 2 - 4x}{x - 2} \\
= \frac{-1(x - 2)}{x - 2} \\
= \frac{-1}{1} \\
= -1
\]

Exercises

Find each sum or difference.

1. \( \frac{3}{a} + \frac{4}{a} \)
2. \( \frac{x^2}{8} + \frac{x}{8} \)
3. \( \frac{5x}{9} - \frac{x}{9} \)
4. \( \frac{11x}{15y} - \frac{x}{15y} \)
5. \( \frac{2a - 4}{a - 4} + \frac{-a}{a - 4} \)
6. \( \frac{m + 1}{2m - 1} + \frac{3m - 3}{2m - 1} \)
7. \( \frac{y + 7}{y + 6} - \frac{1}{y + 6} \)
8. \( \frac{3y + 5}{5} - \frac{2y}{5} \)
9. \( \frac{x + 1}{x - 2} + \frac{x - 5}{x - 2} \)
10. \( \frac{5a}{3b^2} + \frac{10a}{3b^2} \)
11. \( \frac{x^2 + x}{x} - \frac{x^2 + 5x}{x} \)
12. \( \frac{5a + 2}{a^2} - \frac{4a + 2}{a^2} \)
13. \( \frac{3x + 2}{x + 2} + \frac{x + 6}{x + 2} \)
14. \( \frac{a - 4}{a + 1} + \frac{a + 6}{a + 1} \)
Study Guide and Intervention (continued)

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions with Unlike Denominators  Adding or subtracting rational expressions with unlike denominators is similar to adding and subtracting fractions with unlike denominators.

**Adding and Subtracting Rational Expressions**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
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<tbody>
<tr>
<td>1</td>
<td>Find the LCD of the expressions.</td>
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<tr>
<td>2</td>
<td>Change each expression into an equivalent expression with the LCD as the denominator.</td>
</tr>
<tr>
<td>3</td>
<td>Add or subtract just as with expressions with like denominators.</td>
</tr>
<tr>
<td>4</td>
<td>Simplify if necessary.</td>
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**Example 1**

Find \( \frac{n+3}{n} + \frac{8n-4}{4n} \).

Factor each denominator.

\[
\begin{align*}
\frac{n}{n} & = n \\
\frac{4n}{4n} & = 4 \cdot n \\
\text{LCD} &= 4n
\end{align*}
\]

Since the denominator of \( \frac{8n-4}{4n} \) is already \( 4n \), only \( \frac{n+3}{n} \) needs to be renamed.

\[
\begin{align*}
\frac{n+3}{n} + \frac{8n-4}{4n} &= \frac{4(n+3)}{4n} + \frac{8n-4}{4n} \\
&= \frac{4n+12}{4n} + \frac{8n-4}{4n} \\
&= \frac{12n+8}{4n} \\
&= \frac{3n+2}{n}
\end{align*}
\]

**Example 2**

Find \( \frac{3x}{x^2-4x} - \frac{1}{x-4} \).

Factor the denominator.

\[
\begin{align*}
\frac{3x}{x(x-4)} - \frac{1}{x-4} &= \frac{3x}{x(x-4)} - \frac{1}{x-4} \\
&= \frac{3x}{x(x-4)} - \frac{1}{x-4} \cdot \frac{x}{x}
\end{align*}
\]

The LCD is \( x(x-4) \).

\[
\begin{align*}
&= \frac{3x}{x(x-4)} - \frac{x}{x(x-4)} \\
&= \frac{2x}{x(x-4)} \\
&= \frac{2}{x-4}
\end{align*}
\]

Subtract numerators.

Simplify.

**Exercises**

Find each sum or difference.

1. \( \frac{1}{a} + \frac{7}{3a} \)
2. \( \frac{1}{6x} + \frac{3}{8} \)
3. \( \frac{5}{9x} - \frac{1}{x^2} \)
4. \( \frac{6}{x^2} - \frac{3}{x^3} \)
5. \( \frac{8}{4a^2} + \frac{6}{3a} \)
6. \( \frac{4}{h+1} + \frac{2}{h+2} \)
7. \( \frac{y}{y-3} - \frac{3}{y+3} \)
8. \( \frac{y}{y-7} - \frac{y+3}{y^2-4y-21} \)
9. \( \frac{a}{a+4} + \frac{4}{a-4} \)
10. \( \frac{6}{3(m+1)} + \frac{2}{3(m-1)} \)
11. \( \frac{4}{x-2y} - \frac{2}{x+2y} \)
12. \( \frac{a-6b}{2a^2-5ab+2b^2} - \frac{7}{a-2b} \)
13. \( \frac{y+2}{y^2+5y+6} + \frac{2-y}{y^2+y-6} \)
14. \( \frac{q}{q^2-16} + \frac{q+1}{q^2+5q+4} \)
11-6 Skills Practice

Adding and Subtracting Rational Expressions

Find each sum or difference.

1. $\frac{2y}{5} + \frac{y}{5}$

2. $\frac{4r}{9} + \frac{5r}{9}$

3. $\frac{t + 3}{7} - \frac{t}{7}$

4. $\frac{c + 8}{4} - \frac{c + 6}{4}$

5. $\frac{x + 2}{3} + \frac{x + 5}{3}$

6. $\frac{g + 2}{4} + \frac{g - 8}{4}$

7. $\frac{x}{x - 1} - \frac{1}{x - 1}$

8. $\frac{3r}{r + 3} - \frac{r}{r + 3}$

Find the LCM of each pair of polynomials.

9. $4x^2y, 12xy^2$

10. $n + 2, n - 3$

11. $2r - 1, r + 4$

12. $t + 4, 4t + 16$

Find each sum or difference.

13. $\frac{5}{4r} - \frac{2}{r^2}$

14. $\frac{5x}{3y^2} - \frac{2x}{9y}$

15. $\frac{x}{x + 2} - \frac{4}{x - 1}$

16. $\frac{d - 1}{d - 2} - \frac{3}{d + 5}$

17. $\frac{b}{b - 1} + \frac{2}{b - 4}$

18. $\frac{k}{k - 5} + \frac{k - 1}{k + 5}$

19. $\frac{3x + 15}{x^2 - 25} + \frac{x}{x + 5}$

20. $\frac{x - 3}{x^2 - 4x + 4} + \frac{x + 2}{x - 2}$
Find each sum or difference.

1. \( \frac{n}{8} + \frac{3n}{8} \)

2. \( \frac{7u}{16} + \frac{5u}{16} \)

3. \( \frac{w + 9}{9} + \frac{w + 4}{9} \)

4. \( \frac{x - 6}{2} - \frac{x - 7}{2} \)

5. \( \frac{n + 14}{5} - \frac{n - 14}{5} \)

6. \( \frac{6}{c - 1} - \frac{-2}{c - 1} \)

7. \( \frac{x - 5}{x + 2} + \frac{-2}{x + 2} \)

8. \( \frac{r + 5}{r - 5} + \frac{2r - 1}{r - 5} \)

9. \( \frac{4p + 14}{p + 4} + \frac{2p + 10}{p + 4} \)

Find the LCM of each pair of polynomials.

10. \( 3a^3b^2, 18ab^3 \)

11. \( w - 4, w + 2 \)

12. \( 5d - 20, d - 4 \)

13. \( 6p + 1, p - 1 \)

14. \( x^2 + 5x + 4, (x + 1)^2 \)

15. \( m^2 + 3m - 10, m^2 - 4 \)

Find each sum or difference.

16. \( \frac{6p}{5x^2} - \frac{2p}{3x} \)

17. \( \frac{m + 4}{m - 3} - \frac{2}{m - 6} \)

18. \( \frac{y + 3}{y^2 - 16} + \frac{3y - 2}{y^2 + 8y + 16} \)

19. \( \frac{p + 1}{p^2 + 3p - 4} + \frac{p}{p + 4} \)

20. \( \frac{t + 3}{t^2 - 3t - 10} - \frac{4t - 8}{t^2 - 10t + 25} \)

21. \( \frac{4y}{y^2 - y - 6} - \frac{3y + 3}{y^2 - 4} \)

22. SERVICE  Members of the ninth grade class at Pine Ridge High School are organizing into service groups. What is the minimum number of students who must participate for all students to be divided into groups of 4, 6, or 9 students with no one left out?

23. GEOMETRY  Find an expression for the perimeter of rectangle \( ABCD \). Use the formula \( P = 2l + 2w \).
1. **TEXAS** Of the 254 counties in Texas, 4 are larger than 6000 square miles. Another 21 counties are smaller than 300 square miles. What fraction of the counties are 300 to 6000 square miles in size?

2. **SWIMMING** Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value $x$. The length and width of the pool are calculated with the diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions.

3. **EGYPTIAN FRACTIONS** Ancient Egyptians used only unit fractions, which are fractions in the form $\frac{1}{n}$. Their mathematical notation only allowed for a numerator of 1. When they needed to express a fraction with a numerator other than 1, they wrote it as a sum of unit fractions. An example is shown below.

   \[
   \frac{5}{6} = \frac{1}{3} + \frac{1}{2}
   \]

   Simplify the following expression so it is a sum of unit fractions.

   \[
   \frac{5x + 6}{10x^2 + 12x} + \frac{2x}{8x^2}
   \]

4. **INSURANCE** For a hospital stay, Paul’s health insurance plan requires him to pay $\frac{2}{5}$ the cost of the first day in the hospital and $\frac{1}{5}$ the cost of the second and third days. If Paul’s hospital stay is 3 days and cost him $420, what was the full daily cost?

5. **PACKAGE DELIVERY** Fredricksburg Parcel Express delivered a total of 498 packages on Monday, Tuesday, and Wednesday. On Tuesday, they delivered 7 less than 2 times the number of packages delivered on Monday. On Wednesday, they delivered the average number delivered on Monday and Tuesday.

   a. Write a rational equation that represents the sum of the numbers of packages delivered on Monday, Tuesday, and Wednesday.

   b. How many packages were delivered on Monday?
Sum and Difference of Any Two Like Powers

The sum of any two like powers can be written \( a^n + b^n \), where \( n \) is a positive integer. The difference of like powers is \( a^n - b^n \). Under what conditions are these expressions exactly divisible by \((a + b)\) or \((a - b)\)? The answer depends on whether \( n \) is an odd or even number.

Use long division to find the following quotients. (Hint: Write \( a^3 + b^3 \) as \( a^3 + 0a^2 + 0a + b^3 \).) Is the numerator exactly divisible by the denominator? Write yes or no.

1. \( \frac{a^3 + b^3}{a + b} \)  
2. \( \frac{a^3 + b^3}{a - b} \)  
3. \( \frac{a^3 - b^3}{a + b} \)  
4. \( \frac{a^3 - b^3}{a - b} \)  
5. \( \frac{a^4 + b^4}{a + b} \)  
6. \( \frac{a^4 + b^4}{a - b} \)  
7. \( \frac{a^4 - b^4}{a + b} \)  
8. \( \frac{a^4 - b^4}{a - b} \)  
9. \( \frac{a^5 + b^5}{a + b} \)  
10. \( \frac{a^5 + b^5}{a - b} \)  
11. \( \frac{a^5 - b^5}{a + b} \)  
12. \( \frac{a^5 - b^5}{a - b} \)  

13. Use the words odd and even to complete these two statements.

   a. \( a^n + b^n \) is divisible by \( a + b \) if \( n \) is ________, and by neither \( a + b \) nor \( a - b \) if \( n \) is ________.

   b. \( a^n - b^n \) is divisible by \( a - b \) if \( n \) is ________, and by both \( a + b \) and \( a - b \) if \( n \) is ________.

14. Describe the signs of the terms of the quotients when the divisor is \( a - b \).

15. Describe the signs of the terms of the quotient when the divisor is \( a + b \).
Mixed Expressions and Complex Fractions

Simplify Mixed Expressions

Algebraic expressions such as \(a + \frac{b}{c}\) and \(5 + \frac{x + y}{x + 3}\) are called **mixed expressions**. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

**Example 1**

Simplify \(5 + \frac{2}{n}\).

\[
5 + \frac{2}{n} = \frac{5n}{n} + \frac{2}{n} = \frac{5n + 2}{n}
\]

Therefore, \(5 + \frac{2}{n} = \frac{5n + 2}{n}\).

**Example 2**

Simplify \(2 + \frac{3}{n + 3}\).

\[
2 + \frac{3}{n + 3} = \frac{2(n + 3)}{n + 3} + \frac{3}{n + 3} = \frac{2n + 6}{n + 3} + \frac{3}{n + 3} = \frac{2n + 9}{n + 3}
\]

Therefore, \(2 + \frac{3}{n + 3} = \frac{2n + 9}{n + 3}\).

**Exercises**

Write each mixed expression as a rational expression.

1. \(4 + \frac{6}{a}\)
2. \(\frac{1}{9x} - 3\)
3. \(3x - \frac{1}{x^2}\)
4. \(\frac{4}{x^2} - 2\)
5. \(10 + \frac{60}{x + 5}\)
6. \(\frac{h}{h + 4} + 2\)
7. \(\frac{y}{y - 2} + y^2\)
8. \(4 - \frac{4}{2x + 1}\)
9. \(1 + \frac{1}{x}\)
10. \(\frac{4}{m - 2} - 2m\)
11. \(\frac{x^2 + x + 2}{x - 3}\)
12. \(a - 3 + \frac{a - 2}{a + 3}\)
13. \(4m + \frac{3p}{2t}\)
14. \(2q^2 + \frac{q}{p + q}\)
15. \(\frac{2}{y^2 - 1} - 4y^2\)
16. \(t^2 + \frac{p + t}{p - t}\)
Mixed Expressions and Complex Fractions

Simplify Complex Fractions  If a fraction has one or more fractions in the numerator or denominator, it is called a complex fraction.

| Simplifying a Complex Fraction | Any complex fraction \( \frac{\frac{a}{b}}{\frac{c}{d}} \) where \( b \neq 0 \), \( c \neq 0 \), and \( d \neq 0 \), can be expressed as \( \frac{ad}{bc} \). |

Example

Simplify \( \frac{2 + \frac{4}{a}}{a + \frac{2}{3}} \).

\[
\frac{2 + \frac{4}{a}}{a + \frac{2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{a + 2}{3}}
\]

Find the LCD for the numerator and rewrite as like fractions.

\[
= \frac{\frac{2a + 4}{a}}{\frac{a + 2}{3}}
\]

Simplify the numerator.

\[
= \frac{2a + 4}{a} \cdot \frac{3}{a + 2}
\]

Rewrite as the product of the numerator and the reciprocal of the denominator.

\[
= \frac{2(a + 2)}{a} \cdot \frac{3}{a + 2}
\]

Factor.

\[
= \frac{6}{a}
\]

Divide and simplify.

Exercises

Simplify each expression.

1. \( \frac{2 + \frac{2}{5}}{3 + \frac{3}{4}} \)
2. \( \frac{\frac{3}{x}}{\frac{4}{y}} \)
3. \( \frac{x}{y^3} \)
4. \( \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} \)
5. \( \frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} \)
6. \( \frac{\frac{x - 3}{2}}{x^2 - 9} \)
7. \( \frac{x^2 - 25}{y} - \frac{x^3}{x^2 - 5x^2} \)
8. \( \frac{x - \frac{12}{x - 1}}{x - \frac{8}{x - 2}} \)
9. \( \frac{\frac{3}{y + 2} - \frac{2}{y - 2}}{\frac{1}{y + 2} - \frac{2}{y - 2}} \)
11-7 Skills Practice

Mixed Expressions and Complex Fractions

Write each mixed expression as a rational expression.

1. \(6 + \frac{4}{h}\)
2. \(7 + \frac{6}{p}\)

3. \(4b + \frac{b}{c}\)
4. \(8q - \frac{2q}{r}\)

5. \(2 + \frac{4}{d - 5}\)
6. \(5 - \frac{6}{f + 2}\)

7. \(b^2 + \frac{12}{b + 3}\)
8. \(m - \frac{6}{m - 7}\)

9. \(2a + \frac{a - 2}{a}\)
10. \(4r - \frac{r + 9}{2r}\)

Simplify each expression.

11. \(2\frac{1}{2} - 4\frac{3}{4}\)
12. \(3\frac{2}{3} - 5\frac{2}{5}\)
13. \(\frac{r}{n^2} - \frac{r^2}{n}\)

14. \(\frac{a^2}{b^3} - \frac{a}{b}\)
15. \(\frac{x^2y}{c} - \frac{xy^3}{c^3}\)
16. \(\frac{r - 2}{r + 3} - \frac{r - 2}{3}\)

17. \(\frac{w + 4}{w} - \frac{w}{w^2 - 16}\)
18. \(\frac{x^2 - 1}{x} - \frac{x}{x - 1}\)
19. \(\frac{b^2 - 4}{b^2 + 7b + 10} - \frac{b - 2}{b}\)

20. \(\frac{k^2 + 5k + 6}{k^2 - 9} - \frac{k + 2}{k}\)
21. \(\frac{g + 12}{g + 8} - \frac{g}{g + 6}\)
22. \(\frac{p + 9}{p - 6} - \frac{p - 3}{p}\)
11-7 Practice

**Mixed Expressions and Complex Fractions**

Write each mixed expression as a rational expression.

1. \[ 14 - \frac{9}{u} \]

2. \[ 7d + \frac{4d}{c} \]

3. \[ 3n + \frac{6 - n}{n} \]

4. \[ 5b - \frac{b + 3}{2b} \]

5. \[ 3 + \frac{t + 5}{t^2 - 1} \]

6. \[ 2a + \frac{a - 1}{a + 1} \]

7. \[ 2p + \frac{p + 1}{p - 3} \]

8. \[ 4n^2 + \frac{n - 1}{n^2 - 1} \]

9. \[ (t + 1) + \frac{4}{t + 5} \]

Simplify each expression.

10. \[ \frac{3 \frac{2}{5}}{2 \frac{5}{6}} \]

11. \[ \frac{m^2}{\frac{6p}{3m}} \]

12. \[ \frac{x^2 - y^2}{\frac{x^2}{x + y}} \]

13. \[ \frac{a - 4}{\frac{a^2}{a^2 - 16}} \]

14. \[ \frac{q^2 - 7q + 12}{\frac{q^2 - 16}{q - 3}} \]

15. \[ \frac{k^2 + 6k}{\frac{k^2 + 4k - 5}{k - 8}} \]

16. \[ \frac{b^2 + b - 12}{\frac{b^2 + 3b - 4}{b - 3}} \]

17. \[ \frac{g - \frac{10}{g + 9}}{\frac{g - 5}{g + 4}} \]

18. \[ \frac{y + 6}{\frac{y - 7}{y - 6}} \]

19. **TRAVEL** Ray and Jan are on a 12 1/2-hour drive from Springfield, Missouri, to Chicago, Illinois. They stop for a break every 3 3/4 hours.

   a. Write an expression to model this situation.

   b. How many stops will Ray and Jan make before arriving in Chicago?

20. **CARPENTRY** Tai needs several 2 1/4-inch wooden rods to reinforce the frame on a futon. She can cut the rods from a 24 1/2-inch dowel purchased from a hardware store. How many wooden rods can she cut from the dowel?
1. CYCLING Natalie rode in a bicycle event for charity on Saturday. It took her \( \frac{2}{3} \) of an hour to complete the 18-mile race. What was her average speed in miles per hour?

2. QUILTING Mrs. Tantora sews and sells Amish baby quilts. She bought \( 42\frac{3}{4} \) yards of backing fabric, and \( 2\frac{1}{4} \) yards are needed for each quilt she sews. How many quilts can she make with the backing fabric she bought?

3. TRAVEL The Franz family traveled from Galveston to Waco for a family reunion. Driving their minivan, they averaged 30 miles per hour on the way to Waco and 45 miles per hour on the return trip home to Galveston. What is their average rate for the entire trip? (*Hint:* Remember that average rate equals total distance divided by total time and that time can be represented as a ratio of distance \( x \) to rate.)

4. PHYSICAL SCIENCE The volume of a gas varies directly as the Kelvin temperature \( T \) and inversely as the pressure \( P \), where \( k \) is the constant of variation. 

\[
V = k \left( \frac{T}{P} \right)
\]

If \( k = \frac{13}{157} \), find the volume in liters of helium gas at 273 degrees Kelvin and \( \frac{13}{3} \) atmospheres of pressure. Round your answer to the nearest hundredth.

5. SAFETY The Occupational Safety and Health Administration provides safety standards in the workplace to keep workers free from dangerous working conditions. OSHA recommends that for general construction there be 5 foot-candles of illumination in which to work. A foreman using a light meter finds that the illumination of a construction light on a surface 8 feet from the source is 11 foot-candles. The illumination produced by a light source varies inversely as the square of the distance from the source.

\[
I = \frac{k}{d^2}
\]

\( I \) is illumination (in foot-candles), \( d \) is the distance from the source (in feet), and \( k \) is a constant.

a. Find the illumination of the same light at a distance of \( 15\frac{3}{4} \) feet. Round your answer to the nearest hundredth.

b. Is there enough illumination at this distance to meet OSHA requirements for lighting?

c. In order to comply with OSHA, what is the maximum allowable working distance from this light source? Round your decimal answer to nearest tenth.
Continued Fractions

Continued fractions are a special type of complex fraction. Each fraction in a continued fraction has a numerator of 1.

\[
1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4}}}
\]

**Example 1** Evaluate the continued fraction above. Start at the bottom and work your way up.

Step 1 
\[
3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}
\]

Step 2 
\[
\frac{1}{3 + \frac{1}{4}} = \frac{4}{13}
\]

Step 3 
\[
2 + \frac{4}{13} = \frac{26}{13} + \frac{4}{13} = \frac{30}{13}
\]

Step 4 
\[
\frac{1}{\frac{30}{13}} = \frac{13}{30}
\]

Step 5 
\[
1 + \frac{13}{30} = 1 \frac{13}{30}
\]

Evaluate each continued fraction.

1. \[
0 + \frac{1}{2 + \frac{1}{4 + \frac{1}{6 + \frac{1}{8}}}}
\]

2. \[
0 + \frac{1}{1 + \frac{1}{2 + \frac{1}{6}}}
\]

3. \[
3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}
\]

4. \[
1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{7}}}
\]

**Example 2** Change \(\frac{13}{7}\) into a continued fraction.

Step 1 
\[
\frac{13}{7} = \frac{7}{7} + \frac{6}{7} = 1 + \frac{6}{7}
\]

Step 2 
\[
\frac{6}{7} = \frac{1}{\frac{7}{6}}
\]

Step 3 
\[
\frac{7}{6} = 1 + \frac{1}{6}
\]

Stop, because the numerator is 1.

Thus, \(\frac{13}{7}\) can be written as \(1 + \frac{1}{1 + \frac{1}{6}}\).

Change each fraction into a continued fraction.

5. \(\frac{71}{26}\)

6. \(\frac{9}{56}\)

7. \(\frac{4626}{2065}\)
Rational Equations

Solve Rational Equations  Rational equations are equations that contain rational expressions. To solve equations containing rational expressions, multiply each side of the equation by the least common denominator.

Rational equations can be used to solve work problems and rate problems.

Example 1  Solve \( \frac{x - 3}{3} + \frac{x}{2} = 4 \).

\[
6 \left( \frac{x - 3}{3} + \frac{x}{2} \right) = 6(4) \quad \text{The LCD is 6.}
\]

\[
2(x - 3) + 3x = 24 \quad \text{Distributive Property}
\]

\[
2x - 6 + 3x = 24 \quad \text{Distributive Property}
\]

\[
5x = 30 \quad \text{Simplify.}
\]

\[
x = 6 \quad \text{Divide each side by 5.}
\]

The solution is 6.

Example 2  Solve \( \frac{15}{x^2 - 1} = \frac{5}{2(x - 1)} \). State any extraneous solutions.

\[
\frac{15}{x^2 - 1} = \frac{5}{2(x - 1)} \quad \text{Original equation}
\]

\[
30(x - 1) = 5(x^2 - 1) \quad \text{Cross multiply.}
\]

\[
30x - 30 = 5x^2 - 5 \quad \text{Distributive Property}
\]

\[
0 = 5x^2 - 30x + 30 - 5 \quad \text{Add } -30x + 30 \text{ to each side.}
\]

\[
0 = 5x^2 - 30x + 25 \quad \text{Simplify.}
\]

\[
0 = 5(x^2 - 6x + 5) \quad \text{Factor.}
\]

\[
0 = 5(x - 1)(x - 5) \quad \text{Factor.}
\]

\[
x = 1 \quad \text{or} \quad x = 5 \quad \text{Zero Product Property}
\]

The number 1 is an extraneous solution, since 1 is an excluded value for \( x \). So, 5 is the solution of the equation.

Exercises

Solve each equation. State any extraneous solutions.

1. \( \frac{x - 5}{5} + \frac{x}{4} = 8 \)
2. \( \frac{3}{x} = \frac{6}{x + 1} \)
3. \( \frac{x - 1}{5} = \frac{2x - 2}{15} \)
4. \( \frac{8}{n - 1} = \frac{10}{n + 1} \)
5. \( t - \frac{4}{t + 3} = t + 3 \)
6. \( \frac{m + 4}{m} + \frac{m}{3} = \frac{m}{3} \)
7. \( \frac{q + 4}{q - 1} + \frac{q}{q + 1} = 2 \)
8. \( \frac{5 - 2x}{2} - \frac{4x + 3}{6} = \frac{7x + 2}{6} \)
9. \( \frac{m + 1}{m - 1} - \frac{m}{1 - m} = 1 \)
10. \( \frac{x^2 - 9}{x - 3} + x^2 = 9 \)
11. \( \frac{2}{x^2 - 36} - \frac{1}{x - 6} = 0 \)
12. \( \frac{4z}{z^2 + 4z + 3} = \frac{6}{z + 3} + \frac{4}{z + 1} \)
13. \( \frac{4}{4 - p} - \frac{p^2}{p - 4} = 4 \)
14. \( \frac{x^2 - 16}{x - 4} + x^2 = 16 \)
Study Guide and Intervention (continued)

Rational Equations

Use Rational Equations to Solve Problems Rational equation can be used to solve work problems and rate problems.

Example WORK PROBLEM Marla can paint Percy’s kitchen in 3 hours. Percy can paint it in 2 hours. Working together, how long will it take Marla and Percy to paint the kitchen?

In $t$ hours, Marla completes $t \cdot \frac{1}{3}$ of the job and Percy completes $t \cdot \frac{1}{2}$ of the job. So an equation for completing the whole job is $\frac{t}{3} + \frac{t}{2} = 1$.

\[
\begin{align*}
\frac{t}{3} + \frac{t}{2} &= 1 \\
2t + 3t &= 6 & \text{Multiply each term by 6.} \\
5t &= 6 & \text{Add like terms.} \\
t &= \frac{6}{5} & \text{Solve.}
\end{align*}
\]

So it will take Marla and Percy $1\frac{1}{5}$ hours to paint the room if they work together.

Exercises

1. GREETING CARDS It takes Kenesha 45 minutes to prepare 20 greeting cards. It takes Paula 30 minutes to prepare the same number of cards. Working together at this rate, how long will it take them to prepare the cards?

2. BOATING A motorboat went upstream at 15 miles per hour and returned downstream at 20 miles per hour. How far did the boat travel one way if the round trip took 3.5 hours?

3. FLOORING Maya and Reginald are installing hardwood flooring. Maya can install flooring in a room in 4 hours. Reginald can install flooring in a room in 3 hours. How long would it take them if they worked together?

4. BICYCLING Stefan is bicycling on a bike trail at an average of 10 miles per hour. Erik starts bicycling on the same trail 30 minutes later. If Erik averages 16 miles per hour, how long will it take him to pass Stefan?
11-8 Skills Practice

Rational Equations

Solve each equation. State any extraneous solutions.

1. \( \frac{5}{c} = \frac{2}{c + 3} \)

2. \( \frac{3}{q} = \frac{5}{q + 4} \)

3. \( \frac{7}{m + 1} = \frac{12}{m + 2} \)

4. \( \frac{3}{x + 2} = \frac{5}{x + 8} \)

5. \( \frac{y}{y - 2} = \frac{y + 1}{y - 5} \)

6. \( \frac{b - 2}{b} = \frac{b + 4}{b + 2} \)

7. \( \frac{3m}{2} - \frac{1}{4} = \frac{10m}{8} \)

8. \( \frac{7g}{9} + \frac{1}{3} = \frac{5g}{6} \)

9. \( \frac{2a + 5}{6} - \frac{2a}{3} = -\frac{1}{2} \)

10. \( \frac{n - 3}{10} + \frac{n - 5}{5} = \frac{1}{2} \)

11. \( \frac{c + 2}{c} + \frac{c + 3}{c} = 7 \)

12. \( \frac{3b - 4}{b} - \frac{b - 7}{b} = 1 \)

13. \( \frac{m - 4}{m} - \frac{m - 11}{m + 4} = \frac{1}{m} \)

14. \( \frac{f + 2}{f} - \frac{f + 1}{f + 5} = \frac{1}{f} \)

15. \( \frac{r + 3}{r - 1} - \frac{r}{r - 3} = 0 \)

16. \( \frac{u + 1}{u - 2} - \frac{u}{u + 1} = 0 \)

17. \( \frac{-2}{x + 1} + \frac{2}{x} = 1 \)

18. \( \frac{5}{m - 4} - \frac{m}{2m - 8} = 1 \)

19. **ACTIVISM** Maury and Tyra are making phone calls to state representatives’ offices to lobby for an issue. Maury can call all 120 state representatives in 10 hours. Tyra can call all 120 state representatives in 8 hours. How long would it take them to call all 120 state representatives together?
**11-8 Practice**

**Rational Equations**

Solve each equation. State any extraneous solutions.

1. \( \frac{5}{n+2} = \frac{7}{n+6} \)
2. \( \frac{x}{x-5} = \frac{x+4}{x-6} \)
3. \( \frac{k+5}{k} = \frac{k-1}{k+9} \)

4. \( \frac{2h}{h-1} = \frac{2h+1}{h+2} \)
5. \( \frac{4y+1}{3} = \frac{5y}{6} \)
6. \( \frac{y-2}{4} - \frac{y+2}{5} = -1 \)

7. \( \frac{2q-1}{6} - \frac{q}{3} = \frac{q+4}{18} \)
8. \( \frac{5}{p-1} - \frac{3}{p+2} = 0 \)
9. \( \frac{3t}{3t-3} - \frac{1}{9t+3} = 1 \)

10. \( \frac{4x}{2x+1} - \frac{2x}{2x+3} = 1 \)
11. \( \frac{d-3}{d} - \frac{d-4}{d-2} = \frac{1}{d} \)
12. \( \frac{3y-2}{y-2} + \frac{y^2}{2-y} = -3 \)

13. \( \frac{2}{m+2} - \frac{m+2}{m-2} = \frac{7}{3} \)
14. \( \frac{n+2}{n} + \frac{n+5}{n+3} = -\frac{1}{n} \)
15. \( \frac{1}{z+1} - \frac{6-z}{6z} = 0 \)

16. \( \frac{2p}{p-2} + \frac{p+2}{p^2-4} = 1 \)
17. \( \frac{x+7}{x^2-9} - \frac{x}{x+3} = 1 \)
18. \( \frac{2n}{n-4} - \frac{n+6}{n^2-16} = 1 \)

19. **PUBLISHING** Tracey and Alan publish a 10-page independent newspaper once a month. At production, Alan usually spends 6 hours on the layout of the paper. When Tracey helps, layout takes 3 hours and 20 minutes.

   a. Write an equation that could be used to determine how long it would take Tracey to do the layout by herself.

   b. How long would it take Tracey to do the job alone?

20. **TRAVEL** Emilio made arrangements to have Lynda pick him up from an auto repair shop after he dropped his car off. He called Lynda to tell her he would start walking and to look for him on the way. Emilio and Lynda live 10 miles from the auto shop. It takes Emilio \( 2\frac{1}{4} \) hours to walk the distance and Lynda 15 minutes to drive the distance.

   a. If Emilio and Lynda leave at the same time, when should Lynda expect to spot Emilio on the road?

   b. How far will Emilio have walked when Lynda picks him up?
11-8 Word Problem Practice

Rational Equations

1. **ELECTRICITY** The current in a simple electric circuit varies inversely as the resistance. If the current is 20 amps when the resistance is 5 ohms, find the current when the resistance is 8 ohms.

2. **MASONRY** Sam and Belai are masons who are working to build a stone wall that will be 120 feet long. Sam works from one end and is able to build one ten-foot section in 5 hours. Belai works from the other end and is able to finish a ten-foot section in 4 hours. How long will it take Sam and Belai to finish building the wall?

3. **NUMBERS** The formula to find the sum of the first \(n\) whole numbers is \(\text{sum} = \frac{n^2 + n}{2}\). In order to encourage students to show up early to a school dance, the dance committee decides to charge less for those who come to the dance early. Their plan is to charge the first student to arrive 1 penny. The second student through the door is charged 2 pennies; the third student through the door is charged 3 pennies, and so on. How much money, in total, would paid by the first 150 students?

4. **NAUTICAL** A ferry captain keeps track of the progress of his ship in the ship’s log. One day, he records the following entry.

   *With the recent spring snow melt, the current is running strong today. The six-mile trip downstream to Whyte’s landing was very quick. However, we only covered two miles in the same amount of time when we headed back upstream.*

   Write a rational equation using \(b\) for the speed of the boat and \(c\) for the speed of the stream and solve for \(b\) in terms of \(c\).

5. **HEALTH CARE** The total number of Americans waiting for kidney and heart transplants is approximately 66,500. The ratio of those awaiting a kidney transplant to those awaiting a heart transplant is about 20 to 1.

   a. How many people are on each of the waiting lists? Round your answers to the nearest hundred.

   b. These two groups make up about \(\frac{3}{4}\) of the transplant candidates for all organs. About how many organ transplant candidates are there altogether? Round your answer to the nearest thousand.
**Winning Distances**

In 1999, Hicham El Guerrouj set a world record for the mile run with a time of 3:43.13 (3 min 43.13 s). In 1954, Roger Bannister ran the first mile under 4 minutes at 3:59.4. Had they run those times in the same race, how far in front of Bannister would El Guerrouj have been at the finish?

Use \( \frac{d}{t} = r \). Since 3 min 43.13 s = 223.13 s, and 3 min 59.4 s = 239.4 s, El Guerrouj’s rate was \( \frac{5280}{223.13} \) and Bannister’s rate was \( \frac{5280}{239.4} \).

<table>
<thead>
<tr>
<th></th>
<th>( \frac{d}{t} )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Guerrouj</td>
<td>( \frac{5280}{223.13} )</td>
<td>5280 feet</td>
</tr>
<tr>
<td>Bannister</td>
<td>( \frac{5280}{239.4} )</td>
<td>( \frac{5280}{239.4} \cdot 223.13 ) or 4921.2 feet</td>
</tr>
</tbody>
</table>

Therefore, when El Guerrouj hit the tape, he would be \( 5280 - 4921.2 \), or 358.8 feet, ahead of Bannister. Let’s see whether we can develop a formula for this type of problem.

Let \( D = \) the distance raced,

\( W = \) the winner’s time,

and \( L = \) the loser’s time.

Following the same pattern, you obtain the results shown in the table at the right.

The winning distance will be \( D - \frac{DW}{L} \).

1. Show that the expression for the winning distance is equivalent to \( \frac{D(L - W)}{L} \).

Use the formula winning distance = \( \frac{D(L - W)}{L} \) to find the winning distance to the nearest tenth for each of the following Olympic races.

2. women’s 400 meter relay: East Germany 41.6 s (1980); Canada 48.4 s (1928)

3. men’s 200 meter freestyle swimming: Michael Phelps 1 min 42.96 s (2008); Yevgeny Sadovyi 1 min 46.70 s (1992)

4. men’s 50,000 meter walk: Alex Schwazer 3 h 37 min 9 s (2008); Hartwig Gauter 3 h 49 min 24 s (1980)

5. women’s 400 meter freestyle swimming relay: Netherlands 3 min 33.76 s (2008); United States 3 min 39.29 s (1996)
11 Student Recording Sheet

Use this recording sheet with pages 742–743 of the Student Edition.

**Multiple Choice**

Read each question. Then fill in the correct answer.

1. ◯ ◯ ◯ ◯
2. ◯ ◯ ◯ ◯
3. ◯ ◯ ◯ ◯
4. ◯ ◯ ◯ ◯
5. ◯ ◯ ◯ ◯
6. ◯ ◯ ◯ ◯

**Short Response/Grided Response**

Record your answer in the blank.

For grided response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

7a. __________
7b. __________
7c. __________
8. __________ (grid in)

9a. | x | y |
<table>
<thead>
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<tr>
<td>1</td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
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</tbody>
</table>

9b. __________

10. __________
11. __________

**Extended Response**

Record your answers for Question 12 on the back of this paper.
11 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 12 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>A correct solution that is supported by well-developed, accurate explanations. The sketch in part a should be clearly labeled with width $w$, height $w - 1$, and length $w + 2$. In part b, students should simplify the expression $w(w - 1)(w + 2)$ as $w^3 + w^2 - 2w$. In part c, the student will set the expression obtained in part b equal to 30 and solve for $w$ to get $w = 3$ ft. This value is then used to find the length (5 ft) and the height (2 ft).</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
Chapter 11 Quiz 1
(Lessons 11-1 and 11-2)

1. MULTIPLE CHOICE Which equation does not represent an inverse variation?
   A \( y = \frac{3}{x} \)  
   B \( 4xy = 12 \)  
   C \( x = \frac{5}{y} \)  
   D \( x = 2y \)

2. Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = 2 \) when \( x = 7 \).

3. Write an inverse variation equation that relates \( x \) and \( y \) if \( y = 6 \) when \( x = -3 \). Then find \( y \) when \( x = -9 \).

4. State the excluded value for the function \( y = \frac{5}{x + 2} \).

5. Identify the asymptotes of \( y = 2 - \frac{x}{x + 1} + \frac{1}{2} \).

Chapter 11 Quiz 2
(Lessons 11-3 and 11-4)

1. MULTIPLE CHOICE Simplify \( \frac{5x - 10}{x^2 - 4x + 4} \).
   State the excluded value of \( x \).
   A \( \frac{5}{x - 2}; 2 \)  
   B \( \frac{5}{x - 2}; -2 \)  
   C \( \frac{5}{x - 4}; 4 \)  
   D \( \frac{5}{x + 4}; -4 \)

Find each product or quotient.

2. \( \frac{10b^2}{4c^5} \cdot \frac{32c^4}{25b^2} \)  
   3. \( \frac{4r^2}{q} \cdot \frac{3q^2}{8r^4} \)

4. \( \frac{x^2 - x - 12}{x^2 - 3x - 4} \cdot \frac{x^2 - 1}{x^2 + 2x - 3} \)

5. \( \frac{xy^2}{wz^4} \div \frac{x'y}{w^3z} \)

6. \( \frac{8k^4}{j^2} \div \frac{2k^4}{5j^2} \)

7. \( \frac{m^2 + 3m + 2}{m^3} \div \frac{m^2 + m - 2}{m^2} \)

8. \( \frac{b^2 + 7b + 6}{b^2 + b - 2} \div \frac{b^2 - 36}{b^2 - 7b + 6} \)

Find the roots of each function.

9. \( f(x) = \frac{x^2 + 2x - 8}{x - 2} \)

10. \( f(x) = \frac{x^2 - 8x + 7}{x^2 + x - 2} \)
**Chapter 11 Quiz 3**  
(Lessons 11-5 and 11-6)

Find each quotient.

1. \((x^2 + 6x - 5) \div (2x)\)
2. \((5x^2 + 33x - 14) \div (x + 7)\)
3. \((6x^2 - x - 12) \div (2x - 3)\)

Find the LCM for each pair of polynomials.

4. \(8rt, 6r^2t\)
5. \(x^2 + 10x + 21, (x + 3)^2\)

Find each sum or difference.

6. \(\frac{12}{n^2 - 16} + \frac{3}{4 - n}\)
7. \(\frac{x + 16}{x - 5} + \frac{3x - 2}{x - 5}\)
8. \(\frac{11a}{6} - \frac{7a}{6}\)
9. \(\frac{4x}{x - 3} - \frac{7}{3 - x}\)
10. MULTIPLE CHOICE  
    Find \(\frac{m - 3}{m^2 + 6m + 9} - \frac{m - 2}{m^2 - 9}\).

A \(\frac{-5m + 10}{(m + 3)(m - 3)}\)
B \(\frac{-7m + 15}{(m + 3)(m - 3)}\)
C \(\frac{5m + 10}{(m + 3)^2(m - 3)}\)
D \(\frac{-7m + 15}{(m + 3)^2(m - 3)}\)

**Chapter 11 Quiz 4**  
(Lessons 11-7 and 11-8)

1. Write \(3m + \frac{1 + m}{m}\) as a rational expression.

Simplify each expression.

2. \(\frac{x^2y}{5n^3}\)
3. \(\frac{x - \frac{1}{x + 3}}{x + \frac{2}{x + 3}}\)

4. Solve \(\frac{5}{3} + \frac{3}{2x} = \frac{1}{6}\). State any extraneous solutions.

5. MULTIPLE CHOICE  
   Byron can paint a room in 3 hours.  
   Latoya can paint the same room in 2 hours 15 minutes.  
   How long would it take them if they worked together?  
   A \(\frac{4}{21}\) hour  
   B \(1\frac{2}{7}\) hours  
   C \(2\frac{5}{8}\) hours  
   D \(5\frac{1}{4}\) hours  

5. \(\frac{1}{6}\) hours
11 Chapter 11 Mid-Chapter Test
(Lessons 11-1 through 11-4)

Part I Write the letter for the correct answer in the blank at the right of each question.

1. If \( y \) varies inversely as \( x \) and \( y = -9 \) when \( x = -11 \), find \( y \) when \( x = 66 \).
   A 3  B 6  C 11  D \( \frac{3}{2} \)

2. State the excluded values of \( \frac{a^2 - 3a - 28}{a^2 + 3a - 4} \).
   F -4, 7  G -4, 1  H -4, -1  J -4, 1, 7

3. Simplify \( \frac{3x^2 - 5x + 2}{x^2 - 3x + 2} \).
   A \( \frac{3x - 2}{x - 2} \)  B \( \frac{3x + 1}{x - 1} \)  C \( \frac{3x - 5}{x - 3} \)  D \( \frac{2}{3} \)

4. Find \( \frac{m^2}{8m^2 - 8p^2} \cdot \frac{m + p}{m^2 + m^2} \).
   F 0  G \( \frac{1}{8(m - p)(m + 1)} \)  H \( \frac{1}{8m(m - p)} \)  J \( \frac{1}{(m - n)(m + 1)} \)

5. Find \( \frac{4m^2c}{p^3} \div \frac{mc}{6p^4} \).
   A 24mp  B \( \frac{1}{24mp} \)  C \( \frac{2m^2c^2}{3p^7} \)  D \( \frac{1}{10mp} \)

6. Find \( \frac{6x^2 + 5x - 4}{2x - 1} \).
   F \( 6x^2 + 3x \)  G \( 3x + \frac{5}{2} \)  H 3x + 4  J 4x + 4

Part II

7. Assume that \( y \) varies inversely as \( x \). If \( y = 7 \) when \( x = 3 \), write an inverse variation equation that relates \( x \) and \( y \).

8. State the excluded value of the function \( y = \frac{9}{x - 9} \).

9. Identify the asymptotes of \( y = \frac{3}{x} + 2 \).

10. Find the roots of \( f(x) = \frac{x^2 - x - 12}{x - 4} \).

11. Find \( \frac{x^2 - 6x - 7}{x^2 - 3x - 10} \cdot \frac{x + 2}{x - 7} \).

12. Find \( \frac{3x^2 - 12x - 36}{6x^2 + 12x - 18} \cdot \frac{x^2 - 2x - 24}{x^2 + 3x - 4} \).
Chapter 11 Vocabulary Test

Underline or circle the correct term or phrase to complete each sentence.

1. To add rational expressions with unlike denominators, rename them using their (excluded values, least common denominator, least common multiple).

2. \(\frac{7x + 6}{x - 3}\) is an example of a(n) (complex fraction, mixed expression, rational expression).

3. Suppose it takes you 3 hours to stock grocery shelves and you and your friend 1 1/4 hours. To find how long it would solve a(n) (inverse variation, complex fraction, work problem).

4. When you solve an equation and some of the numbers you get are not solutions of the original equation, these numbers are called (excluded values, extraneous solutions, work problems).

5. The equation \(y = \frac{10}{x}\) is an example of a(n) (inverse variation, rate problem, rational expression).

6. A fraction that contains one or more fractions in the numerator or the denominator is a (complex fraction, mixed expression, rational expression).

7. Values of the variable that make the denominator of a rational expression 0 are called (excluded values, extraneous solutions, least common denominator).

8. An expression that is the sum of a monomial and a rational expression is a (complex fraction, mixed expression, rational expression).

9. If you write a product using each prime factor of two polynomials the greatest number of times it occurs in the polynomial, you get the (least common denominator, least common multiple, mixed expression) for the polynomials.

10. A problem that involves uniform motion is a(n) (inverse variation, rate problem, work problem).

Define each term in your own words.

11. product rule

12. rational equation
Write the letter for the correct answer in the blank at the right of each question.

1. Identify the inverse variation that is graphed.
   A  \( y = 15x \)  C  \( xy = -15 \)
   B  \( xy = 15 \)  D  \( y = -15x \)

2. If \( y \) varies inversely as \( x \) and \( y = 18 \) when \( x = 2 \), find \( y \) when \( x = 4 \).
   F  144  G  36  H  9  J  \( \frac{9}{4} \)

3. State the excluded value(s) of \( x \).
   A  2  B  -2, 2  C  2, 4  D  4

4. Simplify \( \frac{18r^4n^2}{30rn^2} \).
   F  90r^3n^3  G  \( \frac{3r}{5n} \)  H  6rn  J  \( \frac{9rn}{15rn} \)

5. Find \( \frac{3r^2}{2p^3} \cdot \frac{4p^4}{9r} \).
   A  \( \frac{2rp}{3} \)  B  \( \frac{2r^2}{3p} \)  C  \( \frac{27r^3}{8p^7} \)  D  \( \frac{4p}{6r} \)

6. Find \( \frac{(t + 2)(t + 3)}{(t - 1)(t + 2)} \cdot \frac{(t - 1)(t + 7)}{(t + 7)(t - 6)}. \)
   F  \( \frac{t + 3}{t - 6} \)  G  \( \frac{t + 2}{t + 7} \)  H  \( \frac{(t + 3)(t - 1)}{(t - 1)(t - 6)} \)  J  \( \frac{1}{2} \)

7. Find \( \frac{4r}{r + 1} + \frac{3}{r + 1} \).
   A  \( \frac{12r}{(r + 1)^2} \)  B  \( \frac{3}{4r} \)  C  \( \frac{4r}{3} \)  D  \( \frac{4r}{3(r + 1)} \)

8. Find \( (a^2 + 4a - 5) \div (5a). \)
   F  \( a^2 + 4a - \frac{1}{a} \)  G  \( a^2 + 3a \)  H  5a + 20 - \( \frac{1}{a} \)  J  \( \frac{a}{5} + \frac{4}{5} - \frac{1}{a} \)

9. Find \( (x^2 - 5x - 14) \div (x + 2). \)
   A  \( x - 7 \)  B  -5x - 7  C  \( x + 7 \)  D  \( x^2 - 6x - 12 \)

10. Identify the asymptotes of \( y = \frac{3}{x} + 2. \)
    F  \( x = 0, y = 2 \)  G  \( x = 0, y = -2 \)  H  \( x = 2, y = 0 \)  J  \( x = -2, y = 0 \)
11. Find the roots of \( f(x) = \frac{x^2 + x - 6}{x - 2} \).
   A \(-3\)  \(\quad\) B \(2\)  \(\quad\) C \(2\) and \(-3\)  \(\quad\) D \(3\)  \(\quad\) 11._____

12. Find \( \frac{4}{x + 1} + \frac{3x}{x + 1} \).
   F \(\frac{7x}{x + 1}\)  \(\quad\) G \(\frac{3x + 4}{2x + 2}\)  \(\quad\) H \(\frac{3x + 4}{x + 1}\)  \(\quad\) J \(\frac{7x}{2x + 2}\)  \(\quad\) 12._____

13. Find \( \frac{x}{x + 1} + \frac{2}{x + 3} \).
   A \(\frac{3x + 4}{(x + 1)(x + 3)}\)  \(\quad\) B \(\frac{x + 2}{2x + 4}\)  \(\quad\) C \(\frac{x^2 + 2x + 4}{(x + 1)(x + 3)}\)  \(\quad\) D \(\frac{x^2 + 5x + 2}{(x + 1)(x + 3)}\)  \(\quad\) 13._____

14. Find \( \frac{x - 4}{8} - \frac{x - 7}{8} \).
   F \(-\frac{11}{8}\)  \(\quad\) G \(\frac{3}{8}\)  \(\quad\) H \(\frac{x - 11}{8}\)  \(\quad\) J \(\frac{x + 3}{8}\)  \(\quad\) 14._____

15. Find \( \frac{x - 1}{x + 2} - \frac{3}{x + 1} \).
   A \(\frac{x^2 - 3x - 7}{(x + 2)(x + 1)}\)  \(\quad\) B \(\frac{x - 4}{(x + 1)(x + 2)}\)  \(\quad\) C \(-\frac{3}{x + 2}\)  \(\quad\) D \(\frac{x^2 - 3x + 5}{(x + 2)(x + 1)}\)  \(\quad\) 15._____

16. Write \( 9 + \frac{6}{a} \) as a rational expression.
   F \(\frac{9a + 6}{a}\)  \(\quad\) G \(\frac{15}{a}\)  \(\quad\) H \(\frac{9a + 6a}{a}\)  \(\quad\) J \(15a\)  \(\quad\) 16._____

17. Simplify \( \frac{u^4}{x^3} \).
   A \(\frac{u^4}{x^3}\)  \(\quad\) B \(\frac{u}{x}\)  \(\quad\) C \(\frac{u^3}{x}\)  \(\quad\) D \(\frac{u^2}{x^2}\)  \(\quad\) 17._____

18. Solve \( \frac{3}{x - 2} = \frac{6}{x + 1} \).
   F \(1\)  \(\quad\) G \(\frac{5}{3}\)  \(\quad\) H \(5\)  \(\quad\) J \(-\frac{15}{2}\)  \(\quad\) 18._____

19. Which value is an extraneous solution of \( \frac{x}{x - 2} + \frac{x - 4}{x - 2} = 5? \)
   A \(0\)  \(\quad\) B \(4\)  \(\quad\) C \(1\)  \(\quad\) D \(2\)  \(\quad\) 19._____

20. Find \( \frac{200 \text{ m}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \).
   F \(300 \text{ km/hr}\)  \(\quad\) G \(12 \text{ km/hr}\)  \(\quad\) H \(300 \text{ m/hr}\)  \(\quad\) J \(12 \text{ m/hr}\)  \(\quad\) 20._____

Bonus Simplify \( \frac{1}{1 + \frac{3}{4}} \).  \(\quad\) B: _____
Write the letter for the correct answer in the blank at the right of each question.

1. Identify the inverse variation that is graphed.
   A  \( xy = 30 \)  C  \( y = -30x \)
   B  \( xy = -30 \)  D  \( y = 30x \)

2. If \( y \) varies inversely as \( x \) and \( y = 0.8 \) when \( x = 1.8 \), find \( y \) when \( x = 4.8 \).
   \( F \) 0.1  \( G \) 0.3  \( H \) 2.13  \( J \) 10.8

3. State the excluded value(s) of \( \frac{n^2 - 3n - 28}{n^2 + 3n - 4} \).
   A  \(-4, 7\)  B  \(-4, 1, 7\)  C  \(1\)  D  \(-4, 1\)

4. Simplify \( \frac{k^2 - 8k + 16}{k^2 - 16} \).
   \( F \)  \( k - 4 \)  \( G \)  \(-1\)  \( H \)  \( \frac{k - 4}{k + 4} \)  \( J \)  \( \frac{k + 4}{k - 4} \)

5. Find \( \frac{3a^2b}{6b^2c^3} \cdot \frac{12bc^2}{15a} \).
   \( A \)  \( \frac{ab}{3c} \)  \( B \)  \( \frac{2a^2b}{5c} \)  \( C \)  \( \frac{2a}{5c} \)  \( D \)  \( \frac{a^2}{3c^2} \)

6. Find \( \frac{y^2 + 4y + 4}{y} \cdot \frac{9y}{y^2 - 4} \).
   \( F \)  \( \frac{9(y + 2)}{y - 2} \)  \( G \)  \( \frac{-9}{y + 2} \)  \( H \)  \( \frac{9}{y - 2} \)  \( J \)  \( \frac{-9}{8(y - 2)} \)

7. Find \( \frac{n^2 + 3n - 10}{n^2 + 6n + 8} \div \frac{n - 2}{n^2 + 2n} \).
   \( A \)  \( \frac{n(n + 5)}{n + 4} \)  \( B \)  \( \frac{(n + 5)(n - 2)^2}{(n + 3)^3} \)  \( C \)  \( \frac{n + 5}{n + 2} \)  \( D \)  \( \frac{n(n - 5)}{(n - 2)^2} \)

8. Identify the asymptotes of \( y = \frac{4}{x} - 1 \).
   \( F \)  \( x = 4, y = -1 \)  \( G \)  \( x = 0, y = -1 \)  \( H \)  \( x = 0, y = 1 \)  \( J \)  \( x = -1, y = 0 \)

9. Find \( \frac{10x^3 + 15x^2 - 8x - 12}{2x + 3} \).
   \( A \)  \( 5x^2 + 15x + \frac{37}{2} \)  \( C \)  \( 5x^2 + 5x - \frac{13}{2} \)
   \( B \)  \( 5x^2 - 4 \)  \( D \)  \( 5x^2 - 4 - \frac{15}{2x + 3} \)

10. Find \( \frac{3t - 2}{3t - 1} + \frac{5t + 4}{3t - 1} \).
    \( F \)  \( \frac{8t + 2}{6t - 2} \)  \( G \)  \( \frac{8t + 2}{3t - 1} \)  \( H \)  \( \frac{8t - 8}{3t - 1} \)  \( J \)  \( \frac{10t}{4t} \)
11. Find \( \frac{2x + 3}{(x - 4)^2} + \frac{x + 4}{4 - x} \).
   \[ \text{A} \quad \frac{x^2 + 2x - 13}{(x - 4)^2} \quad \text{B} \quad \frac{x - 1}{x - 4} \quad \text{C} \quad \frac{x - 1}{(x - 4)^2} \quad \text{D} \quad \frac{-x^2 + 2x + 19}{(x - 4)^2} \]

12. Find \( \frac{6x}{x - 5} - \frac{4}{x - 5} \).
   \[ \text{F} \quad \frac{2x}{x - 5} \quad \text{G} \quad \frac{6x - 4}{x - 5} \quad \text{H} \quad 6x - \frac{4}{5} \quad \text{J} \quad \frac{6x - 4}{2x - 10} \]

13. Find \( \frac{10}{a - b} - \frac{6b}{a^2 - b^2} \).
   \[ \text{A} \quad \frac{4}{a - b} \quad \text{B} \quad \frac{10a - 6b}{a^2 - b^2} \quad \text{C} \quad \frac{4b}{a - b} \quad \text{D} \quad \frac{10a + 4b}{a^2 - b^2} \]

14. Write \( x + \frac{x + 1}{x + 8} \) as a rational expression.
   \[ \text{F} \quad \frac{x^2 + 9x + 1}{x + 8} \quad \text{G} \quad 2x + 1 \quad \text{H} \quad \frac{2x + 1}{x + 8} \quad \text{J} \quad \frac{x^2 + x + 1}{x + 8} \]

15. Simplify \( \frac{r^2 + 2r - 3}{r^2 + 3r} \).
   \[ \text{A} \quad -1(r - 1) \quad \text{B} \quad \frac{(r - 1)^2}{r} \quad \text{C} \quad \frac{(r + 3)^2}{r} \quad \text{D} \quad 3(r + 3) \]

16. Solve \( \frac{5x}{3x + 1} - \frac{10}{9x + 3} = \frac{7}{6} \).
   \[ \text{F} \quad 3 \quad \text{G} \quad \frac{27}{11} \quad \text{H} \quad \frac{67}{9} \quad \text{J} \quad \frac{7}{3} \]

17. Which value is an extraneous solution of \( \frac{x}{x + 1} - \frac{6}{x^2 - 4x - 5} = 1 \)?
   \[ \text{A} \quad 5 \quad \text{B} \quad 0 \quad \text{C} \quad -1 \quad \text{D} \quad 6 \]

18. Find \( \frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \).
   \[ \text{F} \quad \frac{22 \text{ ft}}{15 \text{ s}} \quad \text{G} \quad 68 \text{ ft/s} \quad \text{H} \quad 5280 \text{ mi/min} \quad \text{J} \quad 88 \text{ ft/s} \]

19. ART Vanesa is sculpting statues which require about \( \frac{2}{9} \) cubic foot of clay.
    If she has 2 cubic yards of clay, how many statues can Vanesa make?
   \[ \text{A} \quad 9 \quad \text{B} \quad 243 \quad \text{C} \quad 27 \quad \text{D} \quad 729 \]

20. PHYSICS The formula \( \frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \) represents the total resistance
    of a circuit. What is \( R_t \) if \( R_1 = 2.5 \) ohms, \( R_2 = 10 \) ohms, and \( R_3 = 5 \) ohms?
   \[ \text{F} \quad 17.5 \text{ ohms} \quad \text{G} \quad 2.5 \text{ ohms} \quad \text{H} \quad 10 \text{ ohms} \quad \text{J} \quad 7 \text{ ohms} \]

Bonus Find the value of \( k \) so that \( x + 1 \) is a factor of \( 3x^3 - 2x^2 + x + k \).  
   \[ \text{B:} \quad \underline{\phantom{0000}} \]
11 Chapter 11 Test, Form 2B

Write the letter for the correct answer in the blank at the right of each question.

1. Identify the inverse variation that is graphed.
   A $xy = -42$  C $y = -42x$
   B $xy = 42$  D $y = 42x$

2. If $y$ varies inversely as $x$ and $y = 0.4$ when $x = 2.7$, find $y$ when $x = 1.2$.
   F 8.1  G 0.9  H 0.1  J 0.18

3. State the excluded values of $n^2 - 2n - 24$. 
   A -4, 6  B -4, 2, 6  C 2  D -4, 2

4. Simplify $\frac{y^2 + 2y - 24}{y^2 - 16}$. 
   F $\frac{3}{2}$  G $\frac{y - 3}{1}$  H $\frac{y - 6}{y - 4}$  J $\frac{y + 6}{y + 4}$

5. Find $\frac{2n^2y^3}{6c^4} \cdot \frac{30nc}{14y^5}$. 
   A $\frac{5n^3}{7y^2c^3}$  B $\frac{5n^3c^3y^2}{8}$  C $\frac{n^3c^3y^2}{2}$  D $\frac{n^2}{2y^2c^3}$

6. Find $\frac{y^2 + 8y + 16}{y - 4} \cdot \frac{4y}{y^2 - 16}$. 
   F $\frac{-4}{y - 4}$  G $\frac{4}{y + 4}$  H $\frac{4(y + 4)}{y - 4}$  J $\frac{64(y + 4)^2}{y^2}$

7. Find $\frac{m^2 - 4m - 12}{m^2 + 6m + 8} ÷ \frac{m - 8}{m^2 + 4m}$. 
   A $\frac{m(m - 6)}{m - 8}$  B $\frac{(m - 6)(m - 8)}{m(m + 4)}$  C $\frac{m(m + 6)}{m - 8}$  D $\frac{m - 6}{8}$

8. Find $\frac{16u^2x^2 - 12u^3x - 20x^3}{4u^2x^2}$. 
   F $16 - \frac{12u}{x} - \frac{20x}{u^2}$  H $4 - \frac{3u}{x} - \frac{5x}{u^2}$
   G $4 - 3ux - 5u^2x$  J $16u^4x^4 - 12u^5x^3 - 20u^2x^5$

9. Find $\frac{20x^3 - 8x^2 + 15x - 6}{5x - 2}$. 
   A $\frac{4x^2 - \frac{16}{5}x + \frac{43}{25}}{}$  C $\frac{4x^2 - \frac{12}{5}x + \frac{87}{25}}{}$
   B $\frac{4x^2 + 3 - \frac{4}{5x - 2}}{}$  D $\frac{4x^2 + 3}{5x - 2}$

10. Find $\frac{2k + 5}{5k + 2} + \frac{3k - 7}{5k + 2}$. 
    F $\frac{5k - 2}{10k + 4}$  G $\frac{5k - 35}{5k + 2}$  H $-\frac{28k}{49k}$  J $\frac{5k - 2}{5k + 2}$

NAME ________________________ DATE _______ PERIOD _______
11. Find \( \frac{3m + 2}{(m - 3)^2} + \frac{m + 3}{3 - m} \).
   A \( \frac{m^2 + 3m - 6}{(m - 3)^2} \)  B \( \frac{-m^2 + 3m + 11}{(m - 3)^2} \)  C \( \frac{-3m^2 + 7m + 6}{(m - 3)^2} \)  D \( \frac{2m - 1}{(m - 3)^2} \)

12. Find \( \frac{5}{x + 9} - \frac{4x}{x + 9} \).
   F \( \frac{5 - 4x}{x + 9} \)  G \( \frac{5 - 4x}{2x + 18} \)  H \( \frac{x}{x + 9} \)  J \( \frac{5}{9} - 4x \)

13. Find \( \frac{x}{x - 3} - \frac{4}{x^2 - 9} \).
   A \( \frac{x - 4}{x - 3} \)  B \( \frac{x - 4}{x^2 - 9} \)  C \( \frac{(x + 4)(x - 1)}{(x + 3)(x - 3)} \)  D \( \frac{(x + 1)(x - 1)}{(x + 3)(x - 3)} \)

14. Write \( x + \frac{x + 2}{x + 5} \) as a rational expression.
   F \( \frac{x^2 + x + 2}{x + 5} \)  G \( 2x + 2 \)  H \( \frac{x^2 + 6x + 2}{x + 5} \)  J \( \frac{2x + 2}{x + 5} \)

15. Simplify \( \frac{x^2 - 4x - 5}{x^2 - 5x - 6} \).
   A \( \frac{1}{x} \)  B \( 5(x - 5) \)  C \( \frac{(x + 1)^2}{x} \)  D \( \frac{(x - 5)^2}{x} \)

16. Solve \( \frac{5x}{2x + 1} + \frac{8}{4x + 2} = \frac{8}{3} \).
   F \( 4 \)  G \( \frac{8}{7} \)  H \( \frac{8}{17} \)  J \( \frac{7}{3} \)

17. Which value is an extraneous solution of \( \frac{x}{x + 2} - \frac{10}{x^2 - x - 6} = 1 \)?
   A \( 3 \)  B \( -2 \)  C \( 0 \)  D \( 10 \)

18. Find \( \frac{126 \text{ kilometers}}{1 \text{ hour}} \cdot \frac{1000 \text{ meters}}{1 \text{ kilometer}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \).
   F \( 2100 \text{ m/s} \)  G \( \frac{5 \text{ m}}{18 \text{ s}} \)  H \( 35 \text{ m/s} \)  J \( 28 \text{ m/s} \)

19. GARDENING Yong is filling pots with soil. Each pot has a volume of \( \frac{3}{8} \) cubic foot. If he has 3 cubic yards of soil, how many pots can Yong fill?
   A \( 8 \)  B \( 24 \)  C \( 648 \)  D \( 216 \)

20. PHYSICS The formula \( \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \) represents the total resistance of a circuit. What is \( R_2 \) if \( R_T = 1.5 \text{ ohms}, R_1 = 6 \text{ ohms}, \) and \( R_3 = 10 \text{ ohms} \)?
   F \( 27 \text{ ohms} \)  G \( 2.5 \text{ ohms} \)  H \( 3 \text{ ohms} \)  J \( 6 \text{ ohms} \)

Bonus One factor of \( x^3 + x^2 - 14x - 24 \) is \( x - 4 \). Find the other two factors.
   B: 

Chapter 11
1. Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = -16 \) when \( x = -2 \).

2. Write an inverse variation equation that relates \( x \) and \( y \) if \( y \) varies inversely as \( x \) and \( y = 3.2 \) when \( x = 39.5 \). Find \( y \) when \( x = 31.6 \).

3. State the excluded value(s) of \( \frac{y^2 + 4y + 3}{y^2 - 2y - 15} \).

4. Simplify \( \frac{a^2 - 3a - 28}{a^2 + 3a - 4} \). State the excluded value(s) of \( a \).

Find each product.

5. \( \frac{y^2 - 9}{4} \cdot \frac{8}{y + 3} \)

6. \( \frac{r^2 + 2r - 8}{r^2 + 3r - 10} \cdot \frac{r + 5}{r^2 - 16} \)

Find each quotient.

7. \( \frac{5m}{m + 1} \div \frac{25m^2}{m^2 + 6m + 5} \)

8. \( \frac{x^2 + 5x - 14}{x^2 - 2x - 15} \div \frac{7x - 14}{6x + 18} \)

9. \( (6rt^2 + 4r^2t^2 - 9r^2t) \div (3rt) \)

10. \( (3b^2 - 17b + 22) \div (b - 4) \)

Find each sum.

11. \( \frac{2r - 3}{r - 5} + \frac{6r + 7}{r - 5} \)

12. \( \frac{2a + 6}{a^2 + 6a + 9} + \frac{1}{a + 3} \)

13. \( \frac{-2}{6 - n} + \frac{13}{n^2 - 36} \)
Find each difference.

14. \( \frac{15}{2x - 5} - \frac{6x}{2x - 5} \)
15. \( \frac{4n^2}{2n - 3} - \frac{9}{3 - 2n} \)
16. \( \frac{-9}{5y - 35} - \frac{-4}{y^2 - 7y} \)
17. Write \( a + \frac{a + 7}{a + 2} \) as a rational expression.

Simplify each expression.

18. \( \frac{r^2 + r - 6}{r} \cdot \frac{3r + 9}{r + 2} \)
19. \( \frac{x - \frac{15}{x + 2}}{x + \frac{5}{x + 6}} \)

Solve each equation. State any extraneous solutions.

20. \( \frac{2}{x + 2} = \frac{9}{8} - \frac{5x}{4x + 8} \)
21. \( \frac{x^2}{x - 4} + 3 = \frac{16}{x - 4} \)
22. Find \( 0.4 \text{ inch} \cdot \frac{1\text{ foot}}{12\text{ inches}} \cdot \frac{1\text{ yard}}{3\text{ feet}} \cdot \frac{365\text{ days}}{1\text{ year}} \).
23. **FITNESS** If Nate maintains a pace of \( \frac{2}{13} \) mile per minute, how long will it take for him to run 8 miles?
24. **CONSTRUCTION** Seth can build a doghouse in 6 hours. Violet can build a doghouse in 5 hours. How long will it take them to build a doghouse if they work together?
25. **PHYSICS** Given the formula \( \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \) for the resistance of a circuit, determine the resistance \( R_2 \) if \( R_T = 2 \text{ ohms}, R_1 = 4 \text{ ohms}, \) and \( R_3 = 3 \text{ ohms}. \)

Bonus Simplify \( \left( \frac{45 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \right) \div \frac{60 \text{ minutes}}{1 \text{ hour}} \div \frac{60 \text{ seconds}}{1 \text{ minute}} \).
1. Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = -15 \) when \( x = -6 \).

2. Write an inverse variation equation that relates \( x \) and \( y \) if \( y \) varies inversely as \( x \) and \( y = 34.5 \) when \( x = 3.2 \). Find \( y \) when \( x = 13.8 \).

3. State the excluded value(s) of \( x^2 + x - 12 \).

4. Simplify \( \frac{h^2 + 2k - 15}{h^2 - 4k + 3} \). State the excluded value(s) of \( k \).

5. Find each product.
   - \( \frac{5}{m^2 - 4} \cdot \frac{m + 2}{10} \)
   - \( \frac{r^2 + 2r - 3}{r^2 + 5r + 6} \cdot \frac{r + 2}{r^2 - 1} \)

6. Find each quotient.
   - \( \frac{2x}{x + 7} \div \frac{8x^2}{x^2 + 8x + 7} \)
   - \( \frac{x^2 - 4x - 21}{x^2 - 7x - 18} \div \frac{2x - 14}{5x + 10} \)
   - \( (12a^4b^2 - 5a^2b^3 - 15a^3b^2) \div (3a^2b) \)
   - \( (5r^2 - 13r + 2) \div (r - 3) \)

7. Find each sum.
   - \( \frac{4t - 5}{t + 6} + \frac{5t + 3}{t + 6} \)
   - \( \frac{3v - 6}{v^2 - 4v + 4} + \frac{1}{v - 2} \)
   - \( \frac{-3}{7 - p} + \frac{11}{p^2 - 49} \)
Find each difference.

14. \[
\frac{14}{4x - 7} - \frac{8x}{4x - 7}
\]
15. \[
\frac{25n^2}{5n - 4} - \frac{16}{4 - 5n}
\]
16. \[
\frac{-16}{6x - 30} - \frac{-1}{x^2 - 5x}
\]
17. Write \(n + \frac{n + 7}{n + 3}\) as a rational expression.

Simplify each expression.

18. \[
\frac{3x + 3}{5x^2 - 5x - 10}
\]
19. \[
\frac{x - 12}{x + 1} - \frac{x - 8}{x + 2}
\]

Solve each equation. State any extraneous solutions.

20. \[
\frac{2}{m + 1} - \frac{1}{3m + 3} = -\frac{5}{9}
\]
21. \[
\frac{x^2}{x - 3} - \frac{9}{x - 3} = -2
\]
22. Find \[
\frac{1.2 \text{ ounces}}{1 \text{ minute}} \cdot \frac{1 \text{ quart}}{16 \text{ ounces}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}}.
\]

23. FITNESS If Rozene maintains a pace of \(\frac{2}{45}\) mile per minute, how long will it take for her to swim 3 miles?

24. CONSTRUCTION Raquel can build a chair in 3 hours. Dekota can build a chair in 2 hours. How long will it take them to build a chair if they work together?

25. PHYSICS Given the formula \(\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\) for the resistance of a circuit, determine the resistance \(R_3\) if \(R_T = 4\) ohms, \(R_1 = 6\) ohms, and \(R_2 = 3\) ohms.

Bonus Simplify \[
\left(\frac{30 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}}\right) \div \frac{60 \text{ minutes}}{1 \text{ hour}} \div \frac{60 \text{ seconds}}{1 \text{ minute}}.
\]

B: _____________
11 Chapter 11 Test, Form 3

1. Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = 3.6 \) when \( x = -1.2 \).

2. Write an inverse variation equation that relates \( x \) and \( y \) if \( y \) varies inversely as \( x \) and \( y = 6 \) when \( x = \frac{2}{5} \). Find \( y \) when \( x = 4 \).

3. Identify the asymptotes of \( y = \frac{5}{2x - 1} \).

4. State the excluded values of \( \frac{x^4 - 10x^2 + 9}{x^3 + 4x^2 + 3x} \).

5. Simplify \( \frac{6x^2 - 7x - 3}{6x^2 - x - 1} \). State the excluded values of \( x \).

Find each product.

6. \( \frac{a^2 - 5a - 14}{48a^2} \cdot \frac{60a^3}{a^2 - 4a - 21} \)

7. \( \frac{2b^2 - b - 15}{6b^2 + 7b - 3} \cdot \frac{3b^2 + 5b - 2}{b^2 - b - 6} \)

Find each quotient.

8. \( \frac{n^2 - 5n + 6}{w^2 + 9w + 14} \div \frac{(2 - n)(3 - n)}{w + 7} \)

9. \( \frac{3x^3y^3}{x^2 - 9} \div \frac{12x^2y^5}{x + 3} \)

10. \( (3x^3 - 2x^2 - 48x + 32) \div (3x - 2) \)

Find each sum.

11. \( \frac{3 - 2w}{5w^2 - 1} + \frac{4w + 1}{5w^2 - 1} \)

12. \( \frac{2}{(2x + 7)^2} + \frac{1}{4x^2 - 49} \)
Chapter 11 Test, Form 3 (continued)

Find each difference.

13. \[
\frac{12x + 1}{4x - 1} - \frac{7x + 3}{4x - 1}
\]

14. \[
\frac{5y}{y^2 - 36} - \frac{2y + 5}{y + 6}
\]

15. Find \[
\frac{2w + 7}{w + 5} - \frac{3w}{5 + w} + \frac{8w + 5}{w + 5}.
\]

16. Find \[
\frac{2}{y - 7} - \frac{3y}{y - 7} + \frac{-5y}{7 - y}.
\]

17. Write \((p + 2) + \frac{p + 1}{p - 4}\) as a rational expression.

18. Simplify \[
\frac{x + 5}{x^2 + 2x - 8} \div \frac{x^2 + 2x - 15}{x^2 - 5x + 6}.
\]

19. What is the quotient of \[
\frac{4u^2 - 9r^3}{9r^3} \text{ and } \frac{6u^4}{5r^3}?
\]

Solve each equation. State any extraneous solutions.

20. \[
\frac{p^2}{p - 3} + \frac{5}{3 - p} = 2
\]

21. \[
\frac{x - 1}{x - 2} - \frac{7}{x + 3} = \frac{5}{x^2 + x - 6}
\]

22. **CLEANING** James can clean a car interior in 20 minutes, and Sally can clean the same car in 15 minutes. If they work together, how long will it take to clean 15 cars?

23. **POOL** Bill's pool has a hole and is losing water at a rate of \(
\frac{4}{9}\) gallon per minute. How many hours will it take for Bill to lose 100 gallons?

24. Simplify \[
\left[\frac{20 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}}\right] \div \frac{60 \text{ minutes}}{1 \text{ hour}} \div \frac{60 \text{ seconds}}{1 \text{ minute}}.
\]

25. **GEOMETRY** The volume of a box is \(2x^3 + 11x^2 + 18x + 9\). One dimension of the box is \(x + 1\). What are the other two dimensions if they are polynomials in \(x\) with integer coefficients?

**Bonus** Find the constants \(k\) and \(c\) so that \[
\frac{7x + 17}{x^2 + 4x + 3} = \frac{k}{x + 3} + \frac{c}{x + 1}.
\]
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. The time it takes to travel a certain distance varies inversely as the rate at which you travel. The equation \( rt = D \) can be used to represent a person traveling \( D \) miles. Choose a distance you might travel to go on vacation, and substitute this value in the formula for \( D \). Choose 5 possible values for \( r \) and find the corresponding values for \( t \). Draw a graph of this relation.

2. a. Write a rational expression that has a common factor of the form \( x + n \) in the numerator and denominator. Assume that \( n \) is an integer and \( n \neq 0 \). Simplify the expression. State the excluded values of the variable.

b. Write two rational expressions whose product is \( \frac{x - 2}{x + 3} \). Find the quotient of the two expressions.

c. Write two rational expressions whose difference is \( \frac{3x + 1}{4x - 7} \). Find the sum of the two expressions.

3. a. Write a rational expression in the form \( \frac{(x + m)(ax + n)}{x + m} \) where \( a, m, \) and \( n \) are integers and \( a \neq 0 \). Write the expression in the form \( \frac{ax^2 + bx + c}{x + m} \) by multiplying the two factors in the numerator. Use long division to simplify the expression.

b. Compare and contrast long division of a polynomial by a binomial with simplifying a rational expression whose denominator is a binomial.

4. **FISHING** Roshanda uses an average of \( 14 \frac{1}{5} \) inches of thread to tie a fishing fly. She has \( 18 \frac{1}{2} \) yards of thread to use for her flies. Describe how division can be used to find out how many flies Roshanda can make. How many flies can she make?

5. **FARMING** Pat and Harold own a farm. Working alone, Pat can irrigate their fields in \( x \) hours. Harold can irrigate their fields in two hours less time than Pat. When working together they can irrigate their fields in \( y \) hours.

The equation \( \frac{1}{x} + \frac{1}{x - 2} = \frac{1}{y} \) represents this situation.

a. Determine if \( x \) and \( y \) can be equal.

b. Choose a value for \( x \), and solve the equation for \( y \). For this value of \( x \), how long does it take for Pat and Harold to irrigate the fields together?

c. Choose a value for \( y \), and solve the equation for \( x \). For this value of \( y \), how long does it take for Pat to irrigate the fields when working alone?
1. Which point lies on the line given by \( y = 2x - 3 \)? (Lesson 3-1)
   - A (2, 7)
   - B (−1, −5)
   - C (0, −1)
   - D (−4, −9)
   1.

2. Solve \( x^3 - 4x = 0 \). (Lesson 8-6)
   - F \{−4, 0, 4\}
   - G \{−2, 0, 2\}
   - H \{−4, 4\}
   - J \{−2, 2\}
   2.

3. Solve \( x^2 - 8x + 16 = 25 \) by taking the square root of each side. (Lesson 9-4)
   - A \{−9, 1\}
   - B \{−1, 1\}
   - C \{−9, 9\}
   - D \{−1, 9\}
   3.

4. The District of Columbia has been experiencing a 1.6% annual decrease in population. In 1999, its population was 519,000. If the trend continues, predict the District of Columbia’s population in 2015. (Lesson 7-7)
   - F 441,691
   - G 1883
   - H 427,656
   - J 400,949
   4.

5. Find \((5 - \sqrt{14})^2\). (Lesson 10-3)
   - A 11
   - B \(29 - 29\sqrt{7}\)
   - C \(39 - 10\sqrt{14}\)
   - D \(29\sqrt{14}\)
   5.

6. Solve \( \sqrt{x + 3} = x - 3 \). (Lesson 10-4)
   - F 6
   - G 1, 6
   - H 1, 3, 6
   - J 1
   6.

7. A water ski ramp is 6 feet tall and 9 feet long. Find the measure of the angle the ramp makes with the water. (Lesson 10-6)
   - A about 33.7°
   - B about 41.8°
   - C about 48.2°
   - D about 56.3°
   7.

8. Find \((12x^2 - 16x) \div (4x)\). (Lesson 11-5)
   - F \(3x - 4\)
   - G −1
   - H \(\frac{3x - 4}{4}\)
   - J \(3x - 1\)
   8.

9. The dimensions of a rectangle are \(\frac{5}{3 - y}\) and \(\frac{7}{y^2 - 9}\). Find the perimeter of the rectangle. (Lesson 11-6)
   - A \(\frac{10y + 44}{y^2 - 9}\)
   - B \(\frac{-10y - 16}{y^2 - 9}\)
   - C \(\frac{10y + 16}{2y^2 - 18}\)
   - D \(\frac{-5y - 16}{2y^2 - 18}\)
   9.

10. Simplify \(\frac{x^2 - 9}{x - 3}\). (Lesson 11-7)
    - F \(\frac{-3}{(5x + 2)(-3)}\)
    - G \(\frac{1}{x + 2}\)
    - H \(\frac{(x - 3)^2}{x + 2}\)
    - J \(\frac{x + 3}{x^2 + 5x + 6}\)
   10.
11. Evaluate \(a + bc\) if \(a = 5\), \(b = 2.4\), and \(c = 1.2\). (Lesson 1-3)
   
   \[\text{A} \quad 2.88 \quad \text{B} \quad 14.4 \quad \text{C} \quad 7.88 \quad \text{D} \quad 8.6\]

12. The formula \(A = P(1 + r)\) gives the amount \(A\) in an account with simple interest rate \(r\) and principal \(P\). What is the interest rate as a decimal if the amount in the account is $1080 and the principal was $900? (Lesson 2-8)
   
   \[\text{F} \quad 0.1 \quad \text{G} \quad 0.2 \quad \text{H} \quad 0.3 \quad \text{J} \quad 0.4\]

13. Simplify \(\sqrt[3]{12x^2y}\). (Lesson 10-2)
   
   \[\text{A} \quad 2x\sqrt[3]{3y} \quad \text{B} \quad 6x\sqrt[3]{y} \quad \text{C} \quad 3y\sqrt{2x} \quad \text{D} \quad y\sqrt[3]{6x}\]

14. Simplify \(\frac{3x^2 - 6x}{x^2 - 4}\). State the excluded values of the variables. (Lesson 11-3)
   
   \[\text{F} \quad -1, 1 \quad \text{G} \quad -2, 2 \quad \text{H} \quad -3, 3 \quad \text{J} \quad -4, 4\]

15. Find \((x^3 + 7x + 22) \div (x + 2)\). (Lesson 11-5)
   
   \[\text{A} \quad x^2 - 2x + 11 \quad \text{B} \quad x^2 + 2x - 11 \quad \text{C} \quad x^2 + 11 \quad \text{D} \quad x^2 - 11\]

16. Solve \(\frac{2}{x + 2} + \frac{5}{3x} = \frac{4}{x}\). (Lesson 11-8)
   
   \[\text{F} \quad 12 \quad \text{G} \quad -12 \quad \text{H} \quad 14 \quad \text{J} \quad -14\]

17. The voltage \(V\) required for a circuit is given by \(V = \sqrt{PR}\), where \(P\) is the power in watts and \(R\) is the resistance in ohms. What is the voltage of a 65-watt bulb if the resistance is 110 ohms? Round your answer to the nearest tenth. (Lesson 10-2)

18. The Belmont Stakes is a 1.5-mile race for horses. The winning time in 2000 was about 2 minutes, 30 seconds. What was the average speed for the winning horse in miles per hour? (Lesson 11-4)
19. Determine whether \( \frac{x}{3} = 5 + \frac{3y}{4} \) is a linear equation. If so, write the equation in standard form. (Lesson 3-1)

20. A lighthouse in North Carolina was 1500 feet from the shoreline in 1870. About 1988, the lighthouse was 160 feet from the shoreline and had to be moved. Write the point-slope form of an equation to find the approximate distance from the shoreline for any given year \( x \) between 1870 and 1988. Use \( x = 0 \) to correspond with the year 1870. (Lesson 4-3)

21. Solve \( x - 10 > -2 \) and \( 2x - 5 \leq 10 \). Then graph the solution set. (Lesson 5-4)

22. Solve \( x^2 + 4x + 3 - x(x - 5) = 3(2x - 1) \). (Lesson 8-2)

23. Six times a number subtracted from the number squared is 0. Find the number. (Lesson 8-5)

24. Factor \( x^2 + 5x - 24 \). (Lesson 8-6)

25. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of the equation \( y = -2x^2 + 4x - 5 \). (Lesson 9-1)

26. Graph \( y = 4(2^x) \). State the \( y \)-intercept. (Lesson 7-5)

27. Determine whether the side measures 5, 17, and 19 form a right triangle. Justify your answer. (Lesson 10-5)

28. Find \( AB \). (Lesson 10-5)

29. Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = -12 \) when \( x = 4 \). (Lesson 11-1)

30. A square has an area of 121 square inches. The formula for the area \( A \) of a square with side length \( s \) is \( A = s^2 \). (Lesson 10-5)
   a. Find the length of one side of the square.
   b. Find the length of the diagonal.
11 Anticipation Guide

Rational Expressions and Equations

Before you begin Chapter 11

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Since a direct variation can be written as ( y = kx ), an inverse variation can be written as ( y = \frac{k}{x} ).</td>
<td>D</td>
</tr>
<tr>
<td>2.</td>
<td>A rational expression is an algebraic fraction that contains a radical.</td>
<td>D</td>
</tr>
<tr>
<td>3.</td>
<td>To multiply two rational expressions, such as ( \frac{2x^2}{3x} ) and ( \frac{3x^2}{5x^2} ), multiply the numerators and the denominators.</td>
<td>A</td>
</tr>
<tr>
<td>4.</td>
<td>When solving problems involving units of measure, dimensional analysis is the process of determining the units of the final answer so that the units can be ignored while performing calculations.</td>
<td>D</td>
</tr>
<tr>
<td>5.</td>
<td>To divide ( (4x^2 + 12x) ) by ( 2x ), divide ( 4x^2 ) by ( 2x ) and ( 12x ) by ( 2x ).</td>
<td>A</td>
</tr>
<tr>
<td>6.</td>
<td>To find the sum of ( \frac{2a}{3b} ) and ( \frac{5}{3} ), first add the numerators and then the denominators.</td>
<td>D</td>
</tr>
<tr>
<td>7.</td>
<td>The least common denominator of two rational expressions will be the least common multiple of the denominators.</td>
<td>A</td>
</tr>
<tr>
<td>8.</td>
<td>A complex fraction contains a fraction in its numerator or denominator.</td>
<td>A</td>
</tr>
<tr>
<td>9.</td>
<td>The fraction ( \frac{a}{b} ) can be rewritten as ( \frac{a}{b} ).</td>
<td>D</td>
</tr>
<tr>
<td>10.</td>
<td>Extraneous solutions are solutions that can be eliminated because they are extremely high or low.</td>
<td>D</td>
</tr>
</tbody>
</table>

After you complete Chapter 11

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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Inverse Variation

Graph Inverse Variations Situations in which the values of \( y \) decrease as the values of \( x \) increase are examples of inverse variation. We say that \( y \) varies inversely as \( x \), or \( y \) is inversely proportional to \( x \).

**Inverse Variation Equation**

An equation of the form \( xy = k \), where \( k \neq 0 \)

**Example 1**

Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let \( x \) = speed in miles per hour and \( y \) = time in hours. Graph the variation.

The equation \( xy = 200 \) can be used to represent the situation. Use various speeds to make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.3</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>6.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

**Example 2**

Graph an inverse variation in which \( y \) varies inversely as \( x \) and \( y = 3 \) when \( x = 12 \).

Solve for \( k \).

\[ xy = k \]

\[ 12(3) = k \]

\[ 36 = k \]

Choose values for \( x \) and \( y \), which have a product of 36.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-6</td>
</tr>
<tr>
<td>4</td>
<td>-9</td>
</tr>
<tr>
<td>-2</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each variation if \( y \) varies inversely as \( x \).

1. \( y = 9 \) when \( x = -3 \)
2. \( y = 12 \) when \( x = 4 \)
3. \( y = -25 \) when \( x = 5 \)
4. \( y = 4 \) when \( x = 5 \)
5. \( y = -18 \) when \( x = -9 \)
6. \( y = 4.8 \) when \( x = 5.4 \)
7. \( y = -4 \) when \( x = 1 \)
8. \( y = -4 \) when \( x = \frac{1}{2} \)

**Solve**. Assume that \( y \) varies inversely as \( x \).

9. If \( y = 7 \) when \( x = 3 \), find \( y \) when \( x = -3 \). \( xy = -21; 7 \)
10. If \( y = -6 \) when \( x = -2 \), find \( y \) when \( x = 4 \). \( xy = 12; 3 \)
11. If \( y = -24 \) when \( x = -3 \), find \( x \) when \( y = -6 \). \( xy = 72; -12 \)
12. If \( y = 15 \) when \( x = 1 \), find \( x \) when \( y = -3 \). \( xy = 15; -5 \)
13. If \( y = 48 \) when \( x = -4 \), find \( y \) when \( x = 6 \). \( xy = -192; -32 \)
14. If \( y = -4 \) when \( x = \frac{1}{2} \), find \( x \) when \( y = 2 \). \( xy = -2; -1 \)
Inverse Variation

1. Determine whether each table or equation represents an inverse or a direct variation. Explain.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>40</td>
</tr>
<tr>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>1.25</td>
</tr>
</tbody>
</table>

inverse; $xy = k$

direct; $y = kx$

Assume that $y$ varies inversely as $x$. Write an inverse variation equation that relates $x$ and $y$. Then graph the equation.

5. $y = -2$ when $x = 12$

6. $y = -6$ when $x = -5$

7. $y = 2.5$ when $x = 2$

Write an inverse variation equation that relates $x$ and $y$. Assume that $y$ varies inversely as $x$. Then solve.

8. If $y = 124$ when $x = 12$, find $y$ when $x = 24$. $xy = 1488; -62$

9. If $y = -8.5$ when $x = 6$, find $y$ when $x = -2.5$. $xy = -51; 20.4$

10. If $y = 3.2$ when $x = -5.5$, find $y$ when $x = 6.4$. $xy = -17.6; -2.75$

11. If $y = 0.6$ when $x = 7.5$, find $y$ when $x = -1.25$. $xy = 4.5; -3.6$

12. EMPLOYMENT The manager of a lumber store schedules 6 employees to take inventory in an 8-hour work period. The manager assumes all employees work at the same rate.

a. Suppose 2 employees call in sick. How many hours will 4 employees need to take inventory? 12 h

b. If the district supervisor calls in and says she needs the inventory finished in 6 hours, how many employees should the manager assign to take inventory? 8

13. TRAVEL Jesse and Joaquin can drive to their grandparents’ home in 3 hours if they average 50 miles per hour. Since the road between the homes is winding and mountainous, their parents prefer they average between 40 and 45 miles per hour. How long will it take to drive to the grandparents’ home at the reduced speed? between 3 h 20 min and 3 h 45 min

Inverse Variation

1. PHYSICAL SCIENCE The illumination $I$ produced by a light source varies inversely as the square of the distance $d$ from the source. The illumination produced 5 feet from the light source is 80 foot-candles.

$Id^2 = k$

$80(5)^2 = k$

$2000 = k$

Find the illumination produced 8 feet from the same source.

31.25 foot-candles

2. MONEY A formula called the Rule of 72 approximates how fast money will double in a savings account. It is based on the relation that the number of years it takes for money to double varies inversely as the annual interest rate. Use the information in the table to write the Rule of 72 formula. $yr = 72$

<table>
<thead>
<tr>
<th>Years to Double Money</th>
<th>Annual Interest Rate (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>14.4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>10.29</td>
<td>7</td>
</tr>
</tbody>
</table>

5. SOUND The sound produced by a string inside a piano depends on its length. The frequency of a vibrating string varies inversely as its length.

a. Write an equation that represents the relationship between frequency $f$ and length $ℓ$. Use $k$ for the constant of variation. $f \times ℓ = k$ or $f = \frac{k}{ℓ}$

b. If you have two different length strings, which one vibrates more quickly (that is, which string has a greater frequency)? The shorter string vibrates more quickly than the longer string.

c. Suppose a piano string 2 feet long vibrates 300 cycles per second. What would be the frequency of a string 4 feet long? 150 cycles per second

Lesson 11-1
**11-1 Enrichment**

**Direct or Indirect Variation**

Fill in each table below. Then write inversely, or directly to complete each conclusion.

1. $\begin{array}{c|cccc}
\hline
x & 2 & 4 & 8 & 16 & 32 \\
\hline
W & 4 & 4 & 4 & 4 & 4 \\
A & 8 & 16 & 32 & 64 & 128 \\
\hline
\end{array}$

For a set of rectangles with a width of 4, the area varies directly as the length.

2. $\begin{array}{c|cccc}
\hline
Hours & 2 & 4 & 5 & 6 \\
\hline
Speed & 55 & 55 & 55 & 55 \\
Distance & 110 & 220 & 275 & 330 \\
\hline
\end{array}$

For a car traveling at 55 m/h, the distance covered varies directly as the hours driven.

3. Oat Bran $\frac{1}{3}$ cup $\frac{2}{3}$ cup 1 cup

Water 1 cup 2 cup 3 cup

The number of servings of oat bran varies directly as the number of cups of oat bran.

4. $\begin{array}{c|cccc}
\hline
Hours of Work & 128 & 128 & 128 \\
People Working & 2 & 4 & 8 \\
Hours per Person & 64 & 32 & 16 \\
\hline
\end{array}$

A job requires 128 hours of work. The number of hours each person works varies inversely as the number of people working.

5. $\begin{array}{c|cccc}
\hline
Miles & 100 & 100 & 100 & 100 \\
Rate & 20 & 25 & 50 & 100 \\
Hours & 5 & 4 & 2 & 1 \\
\hline
\end{array}$

For a 100-mile car trip, the time the trip takes varies inversely as the average rate of speed the car travels.

6. $\begin{array}{c|cccc}
\hline
6 & 3 & 4 & 5 & 6 \\
\hline
h & 10 & 10 & 10 & 10 \\
A & 15 & 20 & 25 & 30 \\
\hline
\end{array}$

For a set of right triangles with a height of 10, the area varies directly as the base.

Use the table at the right.

7. $x$ varies directly as $y$.

8. $z$ varies inversely as $y$.

9. $x$ varies inversely as $z$.

**11-2 Study Guide and Intervention**

**Rational Functions**

**Identify Excluded Values** The function $y = \frac{10}{x}$ is an example of a rational function. Because division by zero is undefined, any value of a variable that results in a denominator of zero must be excluded from the domain of that variable. These are called excluded values of the rational function.

**Example** State the excluded value for each function.

a. $y = \frac{3}{x}$

The denominator cannot equal zero.

The excluded value is $x = 0$.

b. $y = \frac{4}{x - 5}$

Set the denominator equal to 0.

$x - 5 = 0$

$x = 5$

The excluded value is $x = 5$.

**Exercises**

State the excluded value for each function.

1. $y = \frac{2}{x}$ $x = 0$

2. $y = \frac{1}{x - 4}$ $x = 4$

3. $y = \frac{x - 3}{x + 1}$ $x = -1$

4. $y = \frac{4}{x - 2}$ $x = 2$

5. $y = \frac{x}{2x - 6}$ $x = 3$

6. $y = \frac{5}{x}$ $x = 0$

7. $y = \frac{3x - 4}{x + 3}$ $x = -3$

8. $y = \frac{x - 1}{5x + 10}$ $x = -2$

9. $y = \frac{x + 1}{x}$ $x = 0$

10. $y = \frac{x - 7}{2x + 8}$ $x = -4$

11. $y = \frac{x - 5}{6x}$ $x = 0$

12. $y = \frac{x - 2}{x + 11}$ $x = -11$

13. $y = \frac{7}{3x + 21}$ $x = -7$

14. $y = \frac{2x - 4}{x + 4}$ $x = -4$

15. $y = \frac{3x - 4}{x}$ $x = 5$

16. **DINING** Mya and her friends are eating at a restaurant. The total bill of $36 is split among $x$ friends. The amount each person pays is given by $y = \frac{36}{x}$, where $x$ is the number of people. Graph the function.
### 11-2 Study Guide and Intervention (continued)

**Rational Functions**

**Identify and Use Asymptotes**

Because excluded values are undefined, they affect the graph of the function. An asymptote is a line that the graph of a function approaches. A rational function in the form \( y = \frac{a}{x - b} + c \) has a vertical asymptote at the \( x \)-value that makes the denominator equal zero, \( x = b \). It has a horizontal asymptote at \( y = c \).

**Example**

Identify the asymptotes of \( y = \frac{1}{x - 1} + 2 \). Then graph the function.

**Step 1** Identify and graph the asymptotes using dashed lines.
- Vertical asymptote: \( x = 1 \)
- Horizontal asymptote: \( y = 2 \)

**Step 2** Make a table of values and plot the points. Then connect them.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>15</td>
<td>1</td>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>

**Exercises**

Identify the asymptotes of each function. Then graph the function.

1. \( y = \frac{3}{x} \) \( x = 0 \); \( y = 0 \)
2. \( y = -\frac{5}{x} \) \( x = 0 \); \( y = 0 \)
3. \( y = \frac{2}{x} + 1 \) \( x = 0 \); \( y = 1 \)
4. \( y = \frac{2}{x} - 3 \) \( x = 0 \); \( y = 3 \)
5. \( y = \frac{2}{x + 1} \) \( x = 1 \); \( y = 0 \)
6. \( y = -\frac{2}{x - 2} \) \( x = 3 \); \( y = 0 \)

**11-2 Skills Practice**

**Rational Functions**

State the excluded value for each function.

1. \( y = \frac{6}{x} \) \( x = 0 \)
2. \( y = -\frac{2}{x - 2} \) \( x = 2 \)
3. \( y = \frac{x}{x + 6} \) \( x = -6 \)

4. \( y = \frac{x - 3}{x + 4} \) \( x = -4 \)
5. \( y = \frac{3x - 5}{x + 8} \) \( x = -8 \)
6. \( y = -\frac{5}{2x + 14} \) \( x = 7 \)

7. \( y = \frac{x}{3x + 21} \) \( x = -7 \)
8. \( y = \frac{x - 1}{9x - 36} \) \( x = 4 \)
9. \( y = \frac{9}{5x + 40} \) \( x = -8 \)

Identify the asymptotes of each function. Then graph the function.

10. \( y = \frac{1}{x} \) \( x = 0, y = 0 \)
11. \( y = \frac{3}{x^2} \) \( x = 0, y = 0 \)
12. \( y = -\frac{2}{x + 1} \) \( x = -1, y = 0 \)

13. \( y = \frac{3}{x - 2} \) \( x = 2, y = 0 \)
14. \( y = \frac{2}{x + 1} - 1 \) \( x = -1, y = -1 \)
15. \( y = \frac{1}{x - 2} + 3 \) \( x = 2, y = 3 \)
11-2 Practice

Rational Functions

State the excluded value for each function.

1. \( y = \frac{1}{x} \) \( x = 0 \)
2. \( y = \frac{3}{x + 5} \) \( x = -5 \)
3. \( y = \frac{2x}{x - 5} \) \( x = 5 \)
4. \( y = \frac{x - 1}{12x + 36} \) \( x = -3 \)
5. \( y = \frac{4x + 1}{2x + 3} \) \( x = -\frac{3}{2} \)
6. \( y = \frac{1 - 5x}{5x - 2} \) \( x = \frac{2}{5} \)

Identify the asymptotes of each function. Then graph the function.

7. \( y = \frac{1}{x} \) \( x = 0, y = 0 \)
8. \( y = \frac{3}{x} \) \( x = 0, y = 0 \)
9. \( y = \frac{2}{x - 1} \) \( x = 1, y = 0 \)

10. \( y = \frac{2x - 4}{x - 2} \) \( x = 2, y = 0 \)
11. \( y = \frac{1}{x - 3} + 2 \) \( x = 3, y = 2 \)
12. \( y = \frac{2}{x + 1} - 1 \) \( x = -1, y = -1 \)

13. AIR TRAVEL Denver, Colorado, is located approximately 1000 miles from Indianapolis, Indiana. The average speed of a plane traveling between the two cities is given by \( y = \frac{1000}{x} \) where \( x \) is the total flight time. Graph the function.

11-2 Word Problem Practice

Rational Functions

1. BULLET TRAINS The Shinkansen, or Japanese bullet train network, provides high-speed transportation throughout Japan. Trains regularly operate at speeds in excess of 200 kilometers per hour. The average speed of a bullet train traveling between Tokyo and Kyoto is given by \( y = \frac{51.5}{x} \) where \( x \) is the total travel time in hours. Graph the function.

2. DRIVING Peter is driving to his grandparents’ house 40 miles away. During the trip, Peter makes a 30-minute stop for lunch. The average speed of Peter’s trip is given by \( y = \frac{800 + 30}{x + 0.5} \) where \( x \) is the total time spent in the car. What are the asymptotes of the function? \( x = -0.5, y = 0 \)

3. ERROR ANALYSIS Nicolas is graphing the equation \( y = \frac{40}{x + 3} - 6 \) and draws a graph with asymptotes at \( y = 3 \) and \( x = -6 \). Explain the error that Nicolas made in his graph. The asymptotes should be \( x = -3, y = -6 \).

4. USED CARS While researching cars to purchase online, Ms. Jacobs found that the value of a used car is inversely proportional to the age of the car. The average price of a used car is given by \( y = \frac{17,900}{x + 1.2} + 100 \), where \( x \) is the age of the car. What are the asymptotes of the function? Explain why \( x = 0 \) cannot be an asymptote.

5. FAMILY REUNION The Gaudet family is holding their annual reunion at Watkins Park. It costs $50 to get a permit to hold the reunion at the park, and the family is spending $8 per person on food. The Gaudets have agreed to split the cost of the event evenly among all those attending.

a. Write an equation showing the cost per person if \( x \) people attend the reunion. \( y = 8 + \frac{50}{x} \)

b. What are the asymptotes of the equation? \( x = 0, y = 8 \)

c. Now assume that the family wants to let a long-lost cousin attend for free. Rewrite the equation to find the new cost per paying person \( y \).

\( y = 8 + \frac{58}{x - 1} \)

d. What are the asymptotes for the new equation? \( x = 1, y = 8 \)
11-2 Enrichment

Inequalities involving Rational Functions

Inequalities involving rational functions can be graphed much like those involving linear functions.

Example

Graph \( y \geq \frac{1}{x} \).

Step 1 Plot points and draw a smooth solid curve. Because the inequality involves a greater than or equal to sign, solutions that satisfy \( y = \frac{1}{x} \) will be a part of the graph.

Step 2 Plot the asymptotes, \( x = 0 \) and \( y = 0 \), as dashed lines.

Step 3 Begin testing values. A value must be tested between each set of lines, including asymptotes.

Region 1 Test \((-1, 1)\). This returns a true value for the inequality.

Region 2 Test \((-1, -0.5)\). This returns a true value for the inequality.

Region 3 Test \((-1, -2)\). This returns a false value for the inequality.

Region 4 Test \((1, 0.5)\). This returns a false value for the inequality.

Region 5 Test \((1, 0.5)\). This returns a false value for the inequality.

Region 6 Test \((1, -1)\). This returns a false value for the inequality.

Step 4 Shade the regions where the inequality is true.

Exercises

Graph each inequality.

1. \( y \leq \frac{1}{x + 1} \)

2. \( y > \frac{2}{x} \)

3. \( y \leq \frac{1}{x + 1} - 1 \)


11-3 Study Guide and Intervention

Simplifying Rational Expressions

Identify Excluded Values

Because a rational expression involves division, the denominator cannot equal zero. Any value of the denominator that results in division by zero is called an excluded value of the denominator.

Example 1

State the excluded value of \( \frac{4m - 8}{m + 2} \).

Exclude the values for which \( m + 2 = 0 \).

\( m + 2 - 2 = 0 - 2 \)

Subtract 2 from each side.

\( m = -2 \)

Simplify.

Therefore, \( m \) cannot equal \(-2\).

Example 2

State the excluded values of \( \frac{x^2 + 1}{x^2 - 9} \).

Exclude the values for which \( x^2 - 9 = 0 \).

\( x^2 - 9 = 0 \)

The denominator cannot equal 0.

\( x + 3x - 3 = 0 \)

Factor.

\( x + 3 = 0 \) or \( x - 3 = 0 \)

Zero Product Property

\( x = -3 \) or \( x = 3 \)

Therefore, \( x \) cannot equal \(-3 \) or \( 3 \).

Exercises

State the excluded values for each rational expression.

1. \( \frac{2b}{b^2 - 8} \)

2. \( \frac{12 - a}{32 + a} \)

3. \( \frac{x^2 - 2}{x^2 + 4} \)

2, \(-2\)

4. \( \frac{m^2 - 4}{2m^2 - 8} \)

2, \(-2\)

5. \( \frac{2n - 12}{n^2 - 4} \)

\(-2, 2\)

6. \( \frac{2x + 18}{x^2 - 16} \)

\(-4, 4\)

7. \( \frac{x^2 - 4}{x^2 + 4x + 4} \)

\(-2\)

8. \( \frac{x^2 - 4}{x^2 + 4x + 4} \)

\(-2, 2\)

9. \( \frac{m^2 + 1}{2m^2 - m - 1} \)

\(-\frac{1}{2}, 1\)

10. \( \frac{m^2 - 1}{2m^2 - m - 1} \)

\(-\frac{1}{2}, 1\)

11. \( \frac{2x + 5x + 1}{x^2 - 10x + 36} \)

\(2, 8\)

12. \( \frac{3x^2 + 5x + 1}{3x^2 - 10x + 16} \)

\(2, 8\)

13. \( \frac{x^2 - 1}{x^2 - 10x + 16} \)

\(-5, 1\)

14. \( \frac{x^2 - y - 2}{3y^2 - 12} \)

\(-2, 2\)

15. \( \frac{x^2 - 1}{x^2 - 10x + 36} \)

\(2, 8\)

16. \( \frac{x^2 - 1}{x^2 - 10x + 36} \)

\(-3, \frac{1}{4}\)
### 11-3 Skills Practice

#### Simplifying Rational Expressions

State the excluded values for each rational expression.

1. \(-\frac{4p}{p - 7}\)

2. \(-\frac{m + 1}{n + 4}\)

3. \(-\frac{h + 2}{h - 4}\)

4. \(-\frac{2x + 15}{x^2 - 25}\)

5. \(-\frac{y^2 - 9}{y^6 + 3y - 18}\)

6. \(-\frac{a^2 - 3b - 8}{b^2 + 7b + 10}\)

#### Exercises

Simplify each expression. State the excluded values of the variables.

7. \(-\frac{21bc}{20bc^2}; \quad \frac{3}{4}; \quad 0, 0\)

8. \(-\frac{12m^3x^2}{24m^3x^2}; \quad m; \quad 0, 0\)

9. \(-\frac{16y^2z^3}{20y^2z^3}; \quad \frac{4}{5}; \quad 0, 0\)

10. \(-\frac{a^2}{b^2}; \quad 0, 0\)

11. \(-\frac{n + 6}{3n + 18}; \quad \frac{1}{3}; \quad 6\)

12. \(-\frac{4x - 4}{x + 4}; \quad \frac{x = 1}{x + 1}; \quad -1\)

13. \(-\frac{y^2 - 64}{y + 8}; \quad y - 8; \quad -8\)

14. \(-\frac{x^2 - 7y - 18}{y - 9}; \quad y + 2; \quad 9\)

15. \(-\frac{z + 1}{z - 1}; \quad \frac{1}{z}; \quad -1, 1\)

16. \(-\frac{x + 6}{x^2 + 2x - 24}; \quad \frac{x = 4}{x - 4}; \quad -6, 4\)

17. \(-\frac{2d + 10}{d^2 - 2d - 35}; \quad \frac{-2}{d - 7}; \quad -5, 7\)

18. \(-\frac{3b - 9}{b^2 - 7b + 12}; \quad \frac{3}{b - 3}; \quad 3, 4\)

19. \(-\frac{p^2 + 5t}{p^2 + 6t + 8}; \quad \frac{t + 3}{t + 4}; \quad -4, -2\)

20. \(-\frac{n^2 + 3n - 4}{n^2 - 3n + 2}; \quad \frac{a - 1}{a - 2}; \quad -4, 2\)

21. \(-\frac{x^2 + 10x + 24}{x + 6}; \quad \frac{x + 6}{x - 6}; \quad 4, 6\)

22. \(-\frac{b - 6b + 9}{b^2 - 9b + 18}; \quad \frac{b - 3}{b - 6}; \quad 3, 6\)
11-3 Practice

Simplifying Rational Expressions

State the excluded values for each rational expression.

1. \( \frac{4x - 28}{p^2 - 49} \); \(-7, 7\)
2. \( \frac{p^2 - 16}{p^2 - 13p + 36} \); \(4, 9\)
3. \( \frac{3a^2 - 3a - 15}{a^2 + 8a + 15} \); \(-5, -3\)

Simplify each expression. State the excluded values of the variables.

4. \( \frac{b^2 - 4b}{b^{2} - 9} \); \(0, -2, d; 0\)
5. \( \frac{6x - 1}{3x^2 - y} \); \(x, y; 0, 0, 0\)
6. \( \frac{2bc^2 + 9c}{2b^2c^2} \); \(0, 0, 0\)
7. \( \frac{c^2d^2}{a^2} \); \(0, -2, d; 0\)
8. \( \frac{p^2 - 9p + 12}{p - 3} \); \(p - 6, 2\)
9. \( \frac{m + 3}{m - 9} \); \(m - 3; -3, 3\)
10. \( \frac{2}{b^2 - 9} \); \(b; 2, 6\)
11. \( \frac{2}{b^2 - 9} \); \(b; 2\)
12. \( \frac{2x^2 - 7x + 10}{x^2 - 2x - 15} \); \(x - 2, 5; x + 3\)
13. \( \frac{2x^2 - 7x + 6}{p^2 - 6x} \); \(x - 2, 5; x + 3\)
14. \( \frac{a^2 - 2a + 1}{a^2 - 4} \); \(a; 0, 1\)
15. \( \frac{t^2 - 81}{t^2 - 12t + 27} \); \(t - 3, 3\)
16. \( \frac{r^2 + 3}{t - 3} \); \(t; 3, 9\)
17. \( \frac{2x^2 + 18x + 36}{3x^2 - 36} \); \(2(x + 6); 3(x - 4)\)
18. \( \frac{2y^2 + 9y + 4}{3y^2 - 4y - 3} \); \(2y - 3, 2; 2\)

19. ENTERTAINMENT

Fairfield High spent \(d\) dollars for refreshments, decorations, and advertising for a dance. In addition, they hired a band for $550.

a. Write an expression that represents the cost of the band as a fraction of the total amount spent for the school dance.

\( \frac{550}{d + 550} \)

b. If \(d\) is $1650, what percent of the budget did the band account for? 25%

20. PHYSICAL SCIENCE

Mr. Kaminski plans to dislodge a tree stump in his yard by using a 6-foot bar as a lever. He places the bar so that 0.5 foot extends from the fulcrum to the end of the bar under the tree stump. In the diagram, \(b\) represents the total length of the bar and \(t\) represents the portion of the bar beyond the fulcrum.

a. Write an equation that can be used to calculate the mechanical advantage. \( MA = \frac{b - t}{t} \)

b. What is the mechanical advantage? 11

c. If a force of 200 pounds is applied to the end of the lever, what is the force placed on the tree stump? 2200 lb

Chapter 11

NAME ____________________________ DATE __________ PERIOD __________

11-3 Word Problem Practice

Simplifying Rational Expressions

1. PHYSICAL SCIENCE

Pressure is equal to the magnitude of a force divided by the area over which the force acts.

\( P = \frac{F}{A} \)

Gabe and Shelby each push open a door with one hand. In order to open, the door requires 20 pounds of force. The surface area of Gabe’s hand is 10 square inches, and the surface area of Shelby’s hand is 8 square inches. Whose hand feels the greater pressure?

Shelby’s: \( 2.5 \text{ lb/in}^2 \) (vs Gabe’s \( 2 \text{ lb/in}^2 \))

2. GRAPHING

Recall that the slope of a line is a ratio of the vertical change to the horizontal change in coordinates for two given points. Write a rational expression that represents the slope of the line containing the points at \( (p, r) \) and \((7, -3)\).

\( \frac{r + 3}{p - 7} \) or \( \frac{3 - r}{7 - p} \)

3. AUTOMOBILES

The force needed to keep a car from skidding out of a turn on a particular road is given by the formula below. What force is required to keep a 2000-pound car traveling at 50 miles per hour on a curve with radius of 750 feet on the road? What value of \( r \) is excluded?

\( f = \frac{0.0072v^2}{w} \)

\( f = \text{force in pounds} \)

\( w = \text{weight in pounds} \)

\( v = \text{speed in mph} \)

\( r = \text{radius in feet} \)

\( 448 \text{ lb, } r \neq 0 \)

4. PACKAGING

In order to safely ship a small electronic device, the distribution manager at Data Products Company determines that the package must contain a certain amount of cushioning on each side of the device. The device is shaped like a cube with side length \( x \), and some sides need more cushioning than others because of the device’s design. The volume of a shipping container is represented by the expression \( (x^2 + 6x + 8)x + 6) \). Find the polynomial that represents the area of the top of the box if the height of the box is \( x + 2 \).

\( x^2 + 10x + 24 \)

5. SCHOOL CHOICE

During a recent school year, the ratio of public school students to private school students in the United States was approximately 7 to 1. The students attending public school outnumbered those attending private schools by 42,240,000.

a. Write a rational expression to express the ratio of public school students to private school students.

\( \frac{x + 42,240,000}{x} \)

b. How many students attended private school? 6,400,000
Shannon’s Juggling Theorem

Mathematicians look at various mathematical ways to represent juggling. One way they have found to represent juggling is Shannon’s Juggling Theorem. Shannon’s Juggling Theorem uses the rational equation

\[ \frac{f + d}{v + d} = \frac{b}{h} \]

where \( f \) is the flight time, or how long a ball is in the air, \( d \) is the dwell time, or how long a ball is in a hand, \( v \) is the vacant time, or how long a hand is empty, \( b \) is the number of balls, and \( h \) is the number of hands (either 1 or 2 for a real-life situation, possibly more for a computer simulation).

So, given the values for \( f, d, v, \) and \( h \), it is possible to determine the number of balls being juggled. If the flight time is 9 seconds, the dwell time is 3 seconds, the vacant time is 1 second, and the number of hands is 2, how many balls are being juggled?

\[
\begin{align*}
\frac{f + d}{v + d} &= \frac{b}{h} \\
9 + 3 &= b \\
2 &= b \\
\frac{12}{4} &= \frac{b}{2} \\
3 &= \frac{b}{2} \\
24 &= 4b \\
6 &= b
\end{align*}
\]

So, the number of balls being juggled is 6.

Given the following information, determine the number of balls being juggled.

1. Flight time = 6 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2
2. Flight time = 13 seconds, vacant time = 1 second, dwell time = 5 seconds, number of hands = 1
3. Flight time = 4 seconds, vacant time = 1 second, dwell time = 1 second, number of hands = 2
4. Flight time = 16 seconds, vacant time = 1 second, dwell time = 2 seconds, number of hands = 2
5. Flight time = 18 seconds, vacant time = 3 seconds, dwell time = 2 seconds, number of hands = 1

Exercises

Find each product.

1. \( \frac{6b + c}{a - b} \cdot \frac{5a}{c} \)
2. \( \frac{4p}{3} \cdot \frac{mp}{3} \)
3. \( \frac{x + 2}{x - 4} \cdot \frac{x + 2}{x - 1} \)
4. \( \frac{x}{x^2 - 4} \cdot \frac{1}{x - 1} \)
5. \( \frac{2n - 8}{n + 2} \cdot \frac{2n + 4}{n - 4} \)
6. \( \frac{x - 4}{x - 1} \cdot \frac{x + 4}{x + 1} \)
7. \( \frac{1}{x - 4} \cdot \frac{x + 4}{x - 1} \)
8. \( \frac{x^2 - 4}{x + 2} \cdot \frac{x^2 + 2x + 1}{x + 1} \)
9. \( \frac{x^2 + 2x + 1}{x - 4} \cdot \frac{1}{x - 1} \)
10. \( \frac{x^2 + 1}{x - 1} \cdot \frac{x - 1}{x^2 - 4} \)
11. \( \frac{n + 1}{n - 2} \cdot \frac{n - 1}{n - 2} \)
12. \( \frac{n + 2}{n - 2} \cdot \frac{n + 1}{n - 2} \)
13. \( \frac{a^2 + 7a + 12}{a^2 + 3a - 10} \cdot \frac{a^2 + 2a - 8}{a^2 + 2a - 8} \)
14. \( \frac{(a + 3)(a + 5)}{(a - 2)(a + 4)} \)

Multiply Rational Expressions

To multiply rational expressions, you multiply the numerators and multiply the denominators. Then simplify.

Example 1

Find \( \frac{2x^2 + 4x}{5x^2 + 10x} \cdot \frac{9b}{3a} \)

\[
\begin{align*}
\frac{2x^2 + 4x}{5x^2 + 10x} &= \frac{2x(x + 2)}{5x(x + 2)} \\
&= \frac{2x}{5x} \\
&= \frac{2}{5} \\
\frac{9b}{3a} &= \frac{3b}{a} \\
\frac{2}{5} \cdot \frac{3b}{a} &= \frac{6b}{5a}
\end{align*}
\]

Example 2

Find \( \frac{x^2 + 2}{x^2 + 8x + 16} \cdot \frac{x + 4}{x - 4} \)

\[
\begin{align*}
\frac{x^2 + 2}{x^2 + 8x + 16} &= \frac{(x + 4)(x - 4)}{(x + 4)(x + 4)} \\
&= \frac{x - 4}{x + 4} \\
\frac{x + 4}{x - 4} &= \frac{x + 4}{x + 4} \\
\frac{x - 4}{x + 4} \cdot \frac{x + 4}{x - 4} &= \frac{1}{1} \\
&= 1
\end{align*}
\]
11-4 Study Guide and Intervention (continued)

Multiply and Dividing Rational Expressions

Example 1
Find \( \frac{12y^2}{5a^3b^2} \cdot \frac{-c^2}{10ab} \).

Evaluate each product.

Example 2
Find \( \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x + 2} \).

Example 2
Find \( \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x + 2} \).

Exercises
Find each quotient.

Find each product.

\( \frac{12y^2}{5a^3b^2} \cdot \frac{-c^2}{10ab} \).

\( \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x + 2} \).

Answers

1. Evaluate each product.

2. Find each quotient.

\( \frac{12y^2}{5a^3b^2} \cdot \frac{-c^2}{10ab} \).

\( \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x + 2} \).

NAME

Period

DATE

NAME

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Chapter 11

Glencoe Algebra 1

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Chapter 11

11-4 Practice

Multiplying and Dividing Rational Expressions

Find each product or quotient.

1. \( \frac{16x^2 \cdot 15y^3 \cdot 8y}{10x^3 \cdot 24x} \)

2. \( \frac{24r^2 \cdot 12r^2 \cdot 1}{8r^3 \cdot 36r^2} \)

3. \( \frac{(x + 2)(x + 2)}{8(x + 2)(x - 2)} \)

4. \( \frac{m + 7}{m - 6} \cdot \frac{m + 4}{m + 7} \)

5. \( \frac{a - 6}{a - 12} \cdot \frac{a + 3}{a - 6} \)

6. \( \frac{4x + 8}{x^2 - x^2 - 3x - 14} \cdot \frac{4}{x(x - 7)} \)

7. \( \frac{n^2 + 10n + 16}{5n - 10} \cdot \frac{n - 2}{n^2 + 9n + 8} \)

8. \( \frac{x^3 - 9}{y^2 - y + 20} \cdot \frac{y - 3}{x^2 - 4} \)

9. \( \frac{b^2 + 5b + 6}{b^2 - 36} \cdot \frac{b^2 + 5b - 6}{b^2 - 28 - b} \)

10. \( \frac{b^2 + 5b + 6}{b^2 - 36} \cdot \frac{b^2 + 5b - 6}{b^2 - 28 - b} \)

11. \( \frac{28x^2 + 21a^3}{12x^2 - 35ab} \)

12. \( \frac{2a}{a - 1} \div \frac{2a}{(a + 1)(a - 1)} \)

13. \( \frac{4x + 20}{5} \div \frac{x + 5}{2y - 6} \)

14. \( \frac{x^2 - 16}{3x} \div \frac{z + 4}{3z} \)

15. \( \frac{x^2 + 12x + 36}{b^2 + 12x + 36} \div \frac{b^2 - 12x + 36}{b^2 - 18x + 2b - 18} \)

16. \( \frac{a^2 + 8a + 12}{a^2 - 3a + 10} \div \frac{a^2 + 8a + 12}{a - 6}(a + 5) \)

17. \( \frac{(a + 5)(a + 5)}{(a - 6)(a - 5)} \)

21. **BIOLOGY** The heart of an average person pumps about 9000 liters of blood per day. How many quarts of blood does the heart pump per hour? (Hint: One quart is equal to 0.946 liters.) Round to the nearest whole number. \( 396 \) q/h

22. **TRAFFIC** On Saturday, it took Ms. Torres 24 minutes to drive 20 miles from her home to her office. During Friday’s rush hour, it took 75 minutes to drive the same distance.

a. What was Ms. Torres’s average speed in miles per hour on Saturday? \( 50 \) mph

b. What was her average speed in miles per hour on Friday? \( 16 \) mph

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Chapter 11

11-4 Word Problem Practice

Multiplying and Dividing Rational Expressions

1. **JOBS** Rosa earned $26.25 for babysitting for 3.5 hours. At this rate, how much will she earn babysitting for 5 hours? $37.50

2. **HOMEWORK** Alejandro and Ander were working on the following homework problem. Find \( \frac{n - 10}{n + 3} \div \frac{2n + 6}{n + 3} \).

3. **GEOMETRY** Suppose the rational expression \( \frac{5m}{3m - 2b} \) represents the area of a section in a tiled floor and \( \frac{2b}{8m} \) represents the section’s length. Write a rational expression to represent the section’s width. \( \frac{8m}{5m - 2b} \)

4. **TRAVEL** Helene travels 800 miles from Amarillo to Brownsville at an average speed of 80 miles per hour. She makes the return trip driving an average of 60 miles per hour. What is the average rate for the entire trip? (Hint: Recall that \( t = \frac{d}{r} ). \) \( 48 \) mph

5. **MANUFACTURING** India works in a metal shop and needs to drill equally spaced holes along a strip of metal. The centers of the holes on the ends of the strip must be exactly 1 inch from each end. The remaining holes will be equally spaced.

a. If there are \( x \) equally spaced holes, write an expression for the number of equal spaces there are between holes. \( x - 1 \)

b. Write an expression for the distance between the end screws if the length is \( t \). \( t - 2 \)

c. Write a rational equation that represents the distance between the holes on a piece of metal that is \( f \) inches long and must have \( x \) equally spaced holes. \( d = \frac{f - 2}{x - 1} \)

d. How many holes will be drilled in a metal strip that is 6 feet long with a distance of 7 inches between the centers of each screw? \( 11 \)
Geometric Series

A geometric series is a sum of the terms in a geometric sequence. Each term of a geometric sequence is formed by multiplying the previous term by a constant term called the common ratio.

\[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots\] geometric sequence where the common ratio is \(\frac{1}{2}\)

The sum of a geometric series can be represented by the rational expression \(s_n = r - 1\), where \(r\) is the first term of the series, \(r\) is the common ratio, and \(n\) is the number of terms.

In the example above, \(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 \cdot \left(\frac{1}{2}\right) - \frac{1}{2} = 1\) or \(\frac{15}{8}\)

You can check this by entering \(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\) into a calculator. The result is the same.

Rewrite each sum as a rational expression and simplify.

1. \(9 + 3 + 1 = \frac{121}{9}\)

2. \(500 + 250 + 125 + 62 = \frac{1875}{2}\)

3. \(6 + 1 + \frac{1}{6} = \frac{259}{36}\)

4. \(100 + 20 + 4 = \frac{624}{5}\)

5. \(1000 + 100 + 10 + 1 = \frac{1,111,111}{1000}\)

6. \(55 + 5 + \frac{5}{11} = \frac{7320}{121}\)

One of the characteristics that makes a spreadsheet powerful is the ability to recalculate values in formulas automatically. You can use this ability to investigate real-world situations.

**Example**

Use a spreadsheet to investigate the effect of doubling the diameter of a tire on the number of revolutions the tire makes at a given speed.

Use dimensional analysis to find the formula for the revolutions per minute of a tire with diameter of \(x\) inches traveling at \(y\) miles per hour.

\[
\text{RPM} = \left(\frac{\pi x}{1} \cdot \frac{1}{60 \text{ minutes}} \cdot \frac{63,360}{1} \right) = 1056y \text{ revolutions per minute}
\]

**Step 1** Use Column A of the spreadsheet for diameter of the tire in inches. Use Column B for the speed in miles per hour.

**Step 2** Column C contains the formula for the number of rotations per minute.

Notice that in Excel, \(\pi\) is entered as PI().

**Step 3** Choose values for the diameter and speed and study the results shown in the spreadsheet. To compare the revolutions per minute for doubled diameters, keep the speed constant and change the diameters. It appears that when the diameter is doubled, the number of revolutions per minute is halved.

**Exercises**

Use the spreadsheet of revolutions per minute.

1. How is the number of revolutions affected if the speed of a wheel of a given diameter is doubled? **RPM is cut in half.**

2. Name two ways that you can double the RPM of a bicycle wheel. Double the speed or halve the diameter of the wheel, keeping the same speed.
11-5 Study Guide and Intervention

Dividing Polynomials

Divide Polynomials by Monomials To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

**Example 1** Find \((4x^3 - 12x) \div 2x\).

\[
\begin{align*}
(4x^3 - 12x) \div 2x & = \frac{4x^3}{2x} - \frac{12x}{2x} \\
& = 2x^2 - 6 \\
& \text{Simplify}
\end{align*}
\]

**Example 2** Find \((3x^3 - 8x + 4) \div (4x)\).

\[
\begin{align*}
(3x^3 - 8x + 4) \div 4x & = \frac{3x^3}{4x} - \frac{8x}{4x} + \frac{4}{4x} \\
& = \frac{3x^2}{4} - \frac{8x}{4} + \frac{4}{4x} \\
& = \frac{3x^2}{4} - 2 + \frac{1}{x} \\
& \text{Simplify}
\end{align*}
\]

**Exercises**

Find each quotient.

1. \((x^3 + 2x^2 - x) \div x = x^2 + 2 - 1\)

2. \((2x^3 + 12x^2 - 8x) \div (2x) = x^2 + 6x - 4\)

3. \((x^2 + 3x - 4) \div x = x + 3 - \frac{4}{x}\)

4. \((4m^2 + 6m - 8) \div (2m^2) = 2 + \frac{3}{m} - \frac{4}{m^2}\)

5. \((3x^3 + 15x^2 - 21x) \div (3x) = x^2 + 5x - 7\)

6. \((8m^2p^3 + 4mp - 8p) \div p = 8mp^3 + 4m - 8\)

7. \((8y^4 + 16y^3 - 4) \div (4y^2) = 2y^2 + 4 - \frac{1}{y}\)

8. \((16x^4y^2 + 24xy + 5) \div (xy) = 16x^3y + 24 + \frac{5}{xy}\)

9. \(\frac{15x^3 - 25x^2 + 30x}{5} = 3x^2 - 5x + 6\)

10. \(\frac{6m^3 + 9m^2 + 9m}{3m} = 2m^2 + 3x + \frac{3}{x}\)

11. \(\frac{mp^2 - 5mp + 6}{mp^2} = 1 - \frac{5}{mp} + \frac{6}{mp^2}\)

12. \(\frac{m^3 - 12m^2 + 41}{3m^2} = 4 - \frac{144}{m^2}\)

13. \(\frac{3y^3 - 4y^2 + 6y}{2y} = 3y^2 - 2y + 3\)

14. \(\frac{2xy^3 - 4xy^2 - 8y}{2xy} = xy^2 - 2xy - 4\)

15. \(\frac{9y^3z - 2y + 12}{xyz} = 9y^2 - 2z + 12\)

16. \(\frac{3a^2b^2 + 8ab - 10b + 12}{2a^2b^2} = \frac{3a^2b^2}{2a^2b^2} + \frac{8ab}{2a^2b^2} - \frac{10b}{2a^2b^2} + \frac{12}{2a^2b^2}\)

17. \(\frac{2a^3b + 5a - 45}{2a^3b + \frac{5}{2a^3b} - \frac{45}{2a^3b}} = ab + 4 - \frac{5}{ab} + \frac{6}{a^2b^2}\)

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11-5 Study Guide and Intervention (continued)

Dividing Polynomials

Divide Polynomials by Binomials To divide a polynomial by a binomial, factor the divisor if possible and divide both dividend and divisor by the GCF. If the polynomial cannot be factored, use long division.

**Example** Find \((x^2 + 7x + 10) \div (x + 3)\).

**Step 1** Divide the first term of the dividend, \(x^2\), by the first term of the divisor, \(x\).

\[
\begin{align*}
x & + 3 \frac{x^2}{x} + \frac{7x}{x} + \frac{10}{x} \\
& = x + 3 + \frac{3}{x} + \frac{10}{x} \\
& \text{Simplify}
\end{align*}
\]

**Step 2** Bring down the next term, 10. Divide the first term of 4x + 10 by \(x + 3\).

\[
\begin{align*}
x & + 3 \frac{4x}{x} + \frac{10}{x + 3} \\
& = 4x + \frac{10}{x + 3} \\
& \text{Simplify}
\end{align*}
\]

The quotient is \(x + 4\) with remainder \(-2\). The quotient can be written as \(x + 4 + \frac{-2}{x + 3}\).

**Exercises**

Find each quotient.

1. \((7y - 5b + 6) \div (b - 2) = b - 3\)

2. \((x^2 - x - 6) \div (x - 3) = x + 2\)

3. \((x^2 + 3x - 4) \div (x - 1) = x + 4\)

4. \((m^2 + 2m - 8) \div (m + 4) = m - 2\)

5. \((x^2 + 5x + 6) \div (x + 2) = x + 3\)

6. \((m^2 + 4m + 4) \div (m + 2) = m + 2\)

7. \((2y^2 + 5y + 2) \div (y + 2) = 2y + 1\)

8. \((8y^2 - 15y - 2) \div (y - 2) = 8y + 1\)

9. \(\frac{8x^2 - 6x - 9}{4x + 3} = 2x - 3\)

10. \(\frac{m^2 - 5m - 6}{m - 2} = m + 1\)

11. \(\frac{x^2 + 1}{x - 2} = x^2 + 2x + 4 + \frac{9}{x - 2}\)

12. \(\frac{3a^2 - 4ab + 12}{2a + 3} = 3a - 4 + \frac{25}{2a + 3}\)

13. \(\frac{b^3 - 2a^2b}{2a + 5} = 4p^2 - 6p + 9\)

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11-5 *Skills Practice*

Dividing Polynomials

Find each quotient.

1. \((20x^2 + 12x) ÷ 4x\)  \(5x + 3\)
2. \((18n^2 + 6n) ÷ 3n\)  \(6n + 2\)
3. \((h^2 - 12b + 5) ÷ 2b\)  \(b - 6 + \frac{5}{2b}\)
4. \((8r^2 + 5r - 20) ÷ 4r\)  \(2r + \frac{5}{4} - \frac{5}{r}\)
5. \(\frac{12p^3 + 18p^2 - 6p}{6p^2} ÷ \frac{2pr + 3 - \frac{1}{p}}{p}\)
6. \(\frac{15k^n - 10k^m + 25k^n}{5k^m} ÷ \frac{3k - 2 + \frac{5u}{k}}{k}\)
7. \((x^2 - 5x - 6) ÷ (x - 6)\)  \(x + 1\)
8. \((a^2 - 10a + 16) ÷ (a - 2)\)  \(a - 8\)
9. \((n^2 - n - 20) ÷ (n + 4)\)  \(n - 5\)
10. \((y^2 + 4y - 21) ÷ (y - 3)\)  \(y + 7\)
11. \((h^2 - 6h + 9) ÷ (h - 2)\)  \(h - 4 + \frac{1}{h - 2}\)
12. \((b^2 + 5b - 2) ÷ (b + 6)\)  \(b - 1 + \frac{4}{b + 6}\)
13. \((y^4 + 6y + 1) ÷ (y + 2)\)  \(y + 4 - \frac{7}{y + 2}\)
14. \((m^2 - 2m - 5) ÷ (m - 3)\)  \(m + 1 - \frac{2}{m - 3}\)
15. \(\frac{2k^3 - 5k - 3}{2k + 1} ÷ c - 3\)
16. \(\frac{2r^2 + 6r - 20}{2r - 4} ÷ 5 + \frac{r}{5}\)
17. \(\frac{x^3 - 3x^2 - 6x - 20}{x - 5} ÷ x^2 + 2x + 4\)
18. \(\frac{r^2 - 4p^2 + p + 6}{p - 2} ÷ p^2 - 2p - 3\)
19. \(\frac{n^2 - 6n - 2}{n + 1} ÷ n^2 - n - 5 + \frac{3}{n + 1}\)
20. \(\frac{y^2 + 4y - 40}{y - 4} ÷ y^2 + 3y + 12 + \frac{8}{y - 4}\)

LANDSCAPING  Jocelyn is designing a bed for cactus specimens at a botanical garden. The total area can be modeled by the expression \(2x^2 + 7x + 3\), where \(x\) is in feet.

a. Suppose in one design the length of the cactus bed is \(4x\) and in another, the length is \(2x + 1\). What are the widths of the two designs?

b. If \(x = 3\) feet, what will be the dimensions of the cactus bed in each of the designs?

12 ft by 3.5 ft; 7 ft by 6 ft

20. FURNITURE  Teri is upholstering the seats of four chairs and a bench. She needs \(\frac{1}{2}\) square yard of fabric for each chair, and \(\frac{1}{3}\) square yard for the bench. If the fabric at the store is 45 inches wide, how many yards of fabric will Teri need to cover the chairs and the bench if there is no waste? 1 \(\frac{1}{5}\) yd
1. TECHNOLOGY The surface area in square millimeters of a rectangular computer microchip is represented by the expression \( x^2 - 12x + 35 \), where \( x \) is the number of circuits. If the width of the chip is \( x = 5 \) millimeters, write a polynomial that represents the length. \( x = 7 \) mm

2. HOMEWORK Your classmate Ava writes her answer to a homework problem on the chalkboard. She has simplified \( 6x - 12x \) as \( x = 12x \). Is this correct? If not, what is the correct simplification? This is not correct. She forgot to factor 6 from the \(-12x\) term. The correct answer should be \( \frac{6(x^2 - 2x)}{6} = x^2 - 2x \).

3. CIVIL ENGINEERING Suppose 5400 tons of concrete costs \((500 + d)\) dollars. Write a formula that gives the cost \( C \) of \( t \) tons of concrete. \( \frac{500t + dt}{5400} \)

4. SHIPPING The Overseas Shipping Company loads cargo into a container to be shipped around the world. The volume of their shipping containers is determined by the following equation. \( x^3 + 21x^2 + 99x + 135 \) The container's height is \( x + 3 \). Write an expression that represents the area of the base of the shipping container. \( x^2 + 18x + 45 \)

5. CIVIL ENGINEERING Greenshield's formula can be used to determine the amount of time a traffic light at an intersection should remain green. \( G = 2 \ln n + 3.7 \)

6. SOLID GEOMETRY The surface area of a right cylinder is given by the formula \( S = 2\pi r^2 + 2\pi rh \).

   a. Write a simplified rational expression that represents the ratio of the surface area to the circumference of the cylinder. \( \frac{r + h}{r} \)

   b. Write a simplified rational expression that represents the ratio of the surface area to the area of the base. \( 2 + 2h \)

---

**Synthetic Division**

You can divide a polynomial such as \( 3x^3 - 4x^2 - 3x + 2 \) by a binomial such as \( x - 3 \) by a process called synthetic division. Compare the process with long division in the following explanation.

Example: Divide \( (3x^3 - 4x^2 - 3x + 2) \) by \( (x - 3) \) using synthetic division.

1. Show the coefficients of the terms in descending order: \( \frac{3 | 3 -4 -3 2}{9 15 -36} \)
2. The divisor is \( x - 3 \). Since 3 is to be subtracted, write 3 in the corner. 
3. Bring down the first coefficient, 3.
4. Multiply. \( 3 \cdot 3 = 9 \)
5. Add. \( -4 + 9 = 5 \)
6. Multiply. \( 3 \cdot 5 = 15 \)
7. Add. \( -3 + 15 = 12 \)
8. Multiply. \( 3 \cdot 12 = 36 \)
9. Add. \( -2 + 36 = 34 \)

Check Use long division.

\[
\begin{array}{c|cccc}
 & 3x^2 & +5x & +12 \hline \\
 x - 3 | 3x^3 & -4x^2 & -3x & +2 \\
 & 3x^3 & -9x^2 & \\
 \hline \\
 & & 5x^2 & -3x & \\
 & & 5x^2 & -15x & \\
 \hline \\
 & & & -12x & +2 & \\
 & & & -12x & +36 & \\
 \hline \\
 & & & & 34 & The result is \( 3x^3 + 5x + 12 + \frac{34}{x - 3} \)
\end{array}
\]

Divide by using synthetic division. Check your result using long division.

1. \( x^3 + 6x^2 + 3x + 1 \div (x - 2) \)
2. \( (x^3 - 3x^2 - 6x - 20) \div (x - 5) \)
3. \( 2x^2 - 2x - 3 \div (x + 1) \)
4. \( x^4 + 7x^2 + 4 \div (x - 2) \)
5. \( (x^4 + 2x^3 + 4x + 4) \div (x + 3) \)
6. \( (x^3 + 4x^2 - 3x - 11) \div (x - 4) \)
7. \( x^2 - 2x - 2 \div (x + 3) \)
8. \( x^2 + 8x + 29 \div (x - 4) \)
11-5 TI-Nspire® Activity

Dividing Polynomials

You can use a graphing calculator with a computer algebra system (CAS) to divide polynomials with any divisor. One function is used to find the quotient, while another function is used to find the remainder.

**Example**

Use CAS to find \((x^4 + x^3 - 3x^2 + x) ÷ (x^2 + 4x + 5)\).

**Step 1** Add a new Calculator page on the TI-Nspire.

**Step 2** From the menu, select Algebra, Polynomial Tools and Quotient of Polynomial.

**Step 3** Type the dividend, a comma, and the divisor.

The CAS indicates that \((x^4 + x^3 - 3x^2 + x) ÷ (x^2 + 4x + 5)\) is \(x^2 - 3x + 4\).

We need to determine whether there is a remainder.

**Step 4** Use the Remainder of a Polynomial option from the Algebra, Polynomial Tools menu to determine the remainder. Then type the dividend, a comma, and the divisor.

The remainder is \(-20\). Therefore, \((x^4 + x^3 - 3x^2 + x) ÷ (x^2 + 4x + 5)\) is \(x^2 - 3x + 4 - \frac{20}{x^2 + 4x + 5}\).

**Check** Use the Expand option from the Algebra menu to confirm your answer. Type the quotient multiplied by the sum of the divisor and the remainder over the divisor. The CAS confirms that the division was correct.

**Exercises**

Find each quotient.

1. \((x^4 + 2x^3 - 19x^2 + 22x - 12) ÷ (x^2 + 5x - 6)\)
2. \((2x^5 + 3x^3 + x + 9) ÷ (x^2 - 2x + 3)\)
3. \((3x^4 - 20x^2 + 58x^2 - 30x + 8) ÷ (3x^2 - 2x + 1)\)
4. \((4x^4 - 7x^2 + 29x^2 - 3x - 40) ÷ (4x^2 + x + 5)\)
5. \((2x^5 + 15x^4 + 10x^3 + 4x^2 + 15x + 16) ÷ (x^2 + 3x + 5)\)
6. \((2x^4 + 9x^3 - 27x + 40) ÷ (x^2 + 3x + 5)\)
7. \((x^4 + 4x^3 - 5x^2 + 12x + 40x + 60) ÷ (x^2 + x + 8)\)
8. \((x^2 + 3x + 5) ÷ (x^2 + x + 8)\)
9. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
10. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
11. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
12. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
13. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
14. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
15. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
16. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
17. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
18. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
19. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)
20. \((x^2 + 2x + 3x + 2) ÷ (x + 2)\)

**Chapter 11**

Glencoe Algebra 1

**11-6 Study Guide and Intervention**

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions with Like Denominators

To add rational expressions with like denominators, add the numerators and then write the sum over the common denominator. To subtract fractions with like denominators, subtract the numerators. If possible, simplify the resulting rational expression.

**Example 1**

Find \(\frac{5n + 7n}{15} + \frac{12m}{15}\).

1. Add the numerators.
2. Simplify.
3. Divide by 3.
4. Subtract.
5. Add \(-1\).

**Example 2**

Find \(\frac{3x + 2}{x - 2} - \frac{4x}{x - 2}\).

1. The common denominator is \(x - 2\).
2. Subtract.

**Exercises**

Find each sum or difference.

1. \(\frac{3}{a} + \frac{4}{a}\)
2. \(\frac{x^2 + x}{8} - \frac{x}{8}\)
3. \(\frac{5x + x}{9} - \frac{4x}{9}\)
4. \(\frac{16}{x^2} - \frac{16}{y^2}\)
5. \(\frac{2a - 4}{a - 4} - \frac{a}{a - 4}\)
6. \(\frac{m + 1}{2m - 1} + \frac{3m - 3}{2m - 1}\)
7. \(\frac{x + 2}{x + 2} = \frac{y + 6}{y + 6}\)
8. \(\frac{3y + 5}{5} - \frac{2y}{5}\)
9. \(\frac{x + 1}{x - 2} - \frac{x + 5}{x - 2}\)
10. \(\frac{5a + 10b}{3b^2}\)
11. \(\frac{3a + 2}{a} - \frac{a + 1}{a}\)
12. \(\frac{a + 2}{a + 1} ÷ \frac{a + 2}{a + 1}\)
13. \(\frac{a + 2}{a + 1} ÷ \frac{a + 2}{a + 1}\)
14. \(\frac{a + 2}{a + 1} ÷ \frac{a + 2}{a + 1}\)
11-6 Study Guide and Intervention (continued)

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions with Unlike Denominators. Adding or subtracting rational expressions with unlike denominators is similar to adding and subtracting fractions with unlike denominators.

Adding and Subtracting Rational Expressions

Step 1 Find the LCD of the expressions.
Step 2 Change each expression into an equivalent expression with the LCD as the denominator.
Step 3 Add or subtract just as with expressions with like denominators.
Step 4 Simplify if necessary.

Example 1

Find \( \frac{5}{n} + \frac{3}{n} + \frac{8n - 4}{4n} \).

Factor each denominator.

\( n = n \)

\( \frac{4n}{4} \)

\( 4n = 4n \)

Since the denominator of \( \frac{8n - 4}{4n} \) is already \( 4n \), only \( \frac{5}{n} + \frac{3}{n} \) needs to be renamed.

\( \frac{5}{n} + \frac{3}{n} = \frac{8n - 4}{4n} \)

\( \frac{4n}{4} \)

\( = \frac{30n + 2}{n} \)

Exercises

Find each sum or difference.

1. \( \frac{4}{a} + \frac{2}{3a} \)

2. \( \frac{1}{2a} + \frac{3}{4a} + \frac{9x}{4} \)

3. \( \frac{5}{x} - \frac{1}{2} \)

4. \( \frac{6}{x^2} - \frac{3}{2} \)

5. \( \frac{a}{a^2} - \frac{2}{a^2} \)

6. \( \frac{x^2 - 4x - 5}{x^2} \)

7. \( \frac{y}{y - 3} - \frac{1}{3} \)

8. \( \frac{a - 1}{a + 1} + \frac{2}{a - 1} \)

9. \( \frac{4}{x^2} + \frac{2}{x} + \frac{1}{2} \)

10. \( \frac{3}{x^2 - 4} + \frac{2}{x^2 + 2} \)

11. \( \frac{3}{x^2 - 4} + \frac{2}{x^2 + 2} \)

12. \( \frac{2}{x^2 - 4} + \frac{1}{x^2 + 2} \)

13. \( \frac{2}{x^2 - 4} + \frac{1}{x^2 + 2} \)

Answers

11-6 Skills Practice

Adding and Subtracting Rational Expressions

Find each sum or difference.

1. \( \frac{2}{3y} + \frac{3}{4} \)

2. \( \frac{5}{3} + \frac{2}{5} \)

3. \( \frac{3}{7} + \frac{3}{7} \)

4. \( \frac{c + 8}{4} - \frac{c + 6}{2} \)

5. \( \frac{2 + 2}{4} + \frac{x + 5}{3} \)

6. \( \frac{k + 2}{4} - \frac{g - 8}{4} \)

7. \( \frac{5}{x - 1} - \frac{1}{x - 1} \)

8. \( \frac{3r}{r + 3} - \frac{r}{r + 3} \)

Find the LCM of each pair of polynomials.

9. \( 4x^2y, 12x^2y^2 \)

10. \( n + 2, n - 3 \)

11. \( 2r - 1, r + 4 \)

12. \( t + 4, 4t + 16 \)

Find each sum or difference.

13. \( \frac{5}{4} - \frac{2}{5} \)

14. \( \frac{5x}{3y} - \frac{2x}{9y} \)

15. \( \frac{x}{x + 2} - \frac{4}{x - 1} \)

16. \( \frac{d - 1}{d - 2} - \frac{3}{d + 5} \)

17. \( \frac{b}{b - 1} - \frac{2}{b - 1} \)

18. \( \frac{h - 1}{h - 5} - \frac{1}{h - 5} \)

19. \( \frac{3x + 15}{x^2 - 25} + \frac{x}{x + 5} \)

20. \( \frac{x - 3}{x^2 - 4x + 4} + \frac{x + 2}{x - 2} \)
11-6 Practice
Adding and Subtracting Rational Expressions

Find each sum or difference.
1. \( \frac{n}{2} + \frac{3m}{5} \)
2. \( \frac{7m}{10} + \frac{5m}{16} \)
3. \( \frac{w + 9}{9} + \frac{w + 4}{9} \)
4. \( \frac{x - 6}{2} - \frac{x - 7}{2} \)
5. \( \frac{n + 14}{5} - \frac{n - 14}{5} \)
6. \( \frac{6}{c - 1} - \frac{-2}{c - 1} \)
7. \( \frac{x - 5}{2} + 2 \)
8. \( \frac{x + z}{2} - \frac{x + 2}{5} \)
9. \( \frac{4p + 14}{p + 4} + \frac{3p + 10}{p + 4} \)

Find the LCM of each pair of polynomials.
10. \( 3a^2b^2, 18ab^2 \)
11. \( w - 4, w + 2 \)
12. \( 5d - 20, d - 4 \)
13. \( 6p + 1, p - 1 \)
14. \( x^2 + 5x + 4, (x + 1)^2 \)
15. \( m^2 + 3m - 10, m^2 - 4 \)

Find each sum or difference.
16. \( \frac{6p}{5x} - \frac{2p}{3x} \)
17. \( \frac{18x - 10xp}{15x^2} \)
18. \( \frac{y + 3}{y^2 - 16} - \frac{3y - 2}{y^2 - 8y + 16} \)
19. \( \frac{y^2 - y^2}{(y + 4)(y - 4)} \)
20. \( \frac{t + 3}{t^3 - 10} + \frac{-4t + 8}{t^3 - 10(t - 5)^2} \)

22. Service Members of the ninth grade class at Pine Ridge High School are organizing into service groups. What is the minimum number of students who must participate for all students to be divided into groups of 4, 6, or 9 students with no one left out? 36

23. Geometry Find an expression for the perimeter of rectangle \( ABCD \). Use the formula \( P = 2l + 2w \).

\( \frac{4(4a + 3b)}{2a + b} \)

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11-6 Enrichment

**Sum and Difference of Any Two Like Powers**

The sum of any two like powers can be written $a^n + b^n$, where $n$ is a positive integer. The difference of like powers is $a^n - b^n$. Under what conditions are these expressions exactly divisible by $a + b$ or $(a - b)$? The answer depends on whether $n$ is an odd or even number.

Use long division to find the following quotients. (Hint: Write $a^n + b^n$ as $a^n + b^n$.) Is the numerator exactly divisible by the denominator? Write yes or no.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Divisible by $a + b$?</th>
<th>Divisible by $a - b$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^n + b^n$</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>$a^n - b^n$</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

13. Use the words odd and even to complete these two statements.

a. $a^n + b^n$ is divisible by $a + b$ if $n$ is **odd** and by neither $a + b$ nor $a - b$ if $n$ is **even**.

b. $a^n - b^n$ is divisible by $a - b$ if $n$ is **odd**, and by both $a + b$ and $a - b$ if $n$ is **even**.

14. Describe the signs of the terms of the quotient when the divisor is $a - b$.

   The terms are all positive.

15. Describe the signs of the terms of the quotient when the divisor is $a + b$.

   The terms are alternately positive and negative.

---

11-7 Study Guide and Intervention

**Mixed Expressions and Complex Fractions**

Simplify Mixed Expressions

Algebraic expressions such as $\frac{a}{b}$ and $5 + \frac{x + y}{x + 3}$ are called **mixed expressions**. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

**Example 1**

Simplify $5 + \frac{2}{n}$.

$$5 + \frac{2}{n} = \frac{5n + 2}{n}$$

Therefore, $5 + \frac{2}{n} = \frac{5n + 2}{n}$.

**Example 2**

Simplify $2 + \frac{3}{n + 3}$.

$$2 + \frac{3}{n + 3} = \frac{2n + 3}{n + 3} + \frac{3}{n + 3}$$

$$= \frac{2n + 6}{n + 3} + \frac{3}{n + 3}$$

$$= \frac{(2n + 6) + 3}{n + 3}$$

Therefore, $2 + \frac{3}{n + 3} = \frac{2n + 9}{n + 3}$.

**Exercises**

Write each mixed expression as a rational expression.

1. $4 + \frac{6}{a} + \frac{6a}{a}$
2. $\frac{1}{3} - 3 - \frac{27x}{9x}$
3. $3x - \frac{1}{x} - \frac{3x^2 - 1}{x^2}$
4. $\frac{3}{b^2} - \frac{2x^2}{b^2}$
5. $\frac{10}{x + 5} + \frac{60}{x + 5}$
6. $\frac{h + 2}{h + 4} + \frac{3h + 8}{h + 4}$
7. $\frac{y}{y - 2} + \frac{y^2 - 2y + y}{y - 2}$
8. $\frac{4}{2x + 1} - \frac{8x}{2x + 1}$
9. $1 + \frac{1}{x} + \frac{x + 1}{x}$
10. $\frac{4m - 2m}{m - 2} - \frac{4 - 2m^2 + 4m}{m - 2}$
11. $x^2 + \frac{x + 2}{x - 3} - \frac{x^2 + 3x^2 + x + 2}{x - 3}$
12. $a - 3 + \frac{a - 2}{a + 3}$
13. $8m + \frac{3p}{2x} + \frac{8m + 3p}{2x}$
14. $2q^2 + \frac{q}{r + q}$
15. $\frac{2}{y^2 + 1} - \frac{4y^2 + 4y^2}{y^2 - 1}$
16. $p^2 + \frac{p + t}{p + t}$

---

**Answers** (Lesson 11-6 and Lesson 11-7)
11-7 Study Guide and Intervention (continued)

Mixed Expressions and Complex Fractions

Simplify Complex Fractions If a fraction has one or more fractions in the numerator or denominator, it is called a complex fraction.

Example

Simplify \( \frac{2 + \frac{4}{a}}{a + \frac{2}{3}} \).

\[
\frac{2 + \frac{4}{a}}{a + \frac{2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}}
\]

Find the LCD for the numerator and rewrite as like fractions.

\[
= \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}}
\]

Simplify the numerator.

\[
= \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}}
\]

Rewrite as the product of the numerator and the reciprocal of the denominator.

\[
= \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}}
\]

Factor.

\[
= \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}} = \frac{\frac{2a + 4}{a}}{\frac{3a + 2}{3}}
\]

Divide and simplify.

Exercises

Simplify each expression.

1. \( \frac{2 + \frac{4}{a}}{a + \frac{2}{3}} \)

2. \( \frac{\frac{3}{x} + \frac{3}{y}}{4x} \)

3. \( \frac{x^2 + \frac{x}{y}}{x^2 + \frac{1}{y}} \)

4. \( \frac{1 - \frac{1}{x}}{x + 1} \)

5. \( \frac{1 - \frac{2}{x}}{x + 1} \)

6. \( \frac{x + 3}{\frac{2}{x^2 - 9}} \)

7. \( \frac{x^2 - 5x}{x^2 - 25} \)

8. \( \frac{x + \frac{2}{x - 2}}{x - \frac{2}{x - 2}} \)

9. \( \frac{y - 10}{x} \)

10. \( \frac{y - 6}{x} \)

11. \( \frac{1}{x} \)

12. \( \frac{3}{x} \)

13. \( \frac{1}{m} \)

14. \( \frac{a^2}{b^2} \)

15. \( \frac{x^2}{y^2} \)

16. \( \frac{3}{r + 3} \)

17. \( \frac{w + \frac{4}{w}}{w^2 - 16} \)

18. \( \frac{x^2 - 1}{x + 1} \)

19. \( \frac{\frac{b^2 - 4}{b - 2}}{\frac{b + 5}{b + 6}} \)

20. \( \frac{w + \frac{4}{w}}{w^2 - 16} \)

21. \( \frac{g + \frac{2}{g}}{g + 8} \)

22. \( \frac{w + \frac{4}{w}}{w^2 - 16} \)
11-7 Practice

Mixed Expressions and Complex Fractions

Write each mixed expression as a rational expression.

1. \( \frac{1.14 - \frac{b}{a}}{u} \)

2. \( \frac{7d + \frac{4c}{c}}{2c} \)

3. \( \frac{3n + \frac{6-n}{n}}{n} \)

4. \( \frac{4.56 - \frac{b+3}{2b}}{a} \)

5. \( \frac{3 + \frac{f+5}{p-1}}{p} \)

6. \( \frac{2a + \frac{a-1}{a+1}}{2a + \frac{3a-1}{a+1}} \)

7. \( \frac{2p + \frac{b+1}{p-3}}{2b} \)

8. \( \frac{2p^2 - \frac{3p}{p-1}}{n^2 - 3-n} \)

9. \( \frac{t+1 + \frac{4}{t+5}}{t+6 + \frac{9}{t+5}} \)

Simplify each expression.

10. \( \frac{3 \frac{5}{6}}{2 \frac{5}{6}} \)

11. \( \frac{\frac{m^2}{3m}}{\frac{mp}{18}} \)

12. \( \frac{\frac{x+y}{x-y}}{\frac{3(x-y)}{x}} \)

13. \( \frac{\frac{a}{a^2 - 16}}{\frac{a^2 + a}{a + 4}} \)

14. \( \frac{\frac{q-3}{q}}{\frac{1}{q+4}} \)

15. \( \frac{\frac{k^2 + 6k}{k}}{\frac{k^2 + 4k - 5}{k + 5}} \)

16. \( \frac{\frac{b^2 + 12}{b^2 + 3b + 4}}{\frac{b^2 - 3b - 12}{b^2 - b}} \)

17. \( \frac{\frac{g}{g + 9} + \frac{5}{g - 4}}{\frac{g + 9}{g} + \frac{5}{g - 4}} \)

18. \( \frac{\frac{x + 6}{x - y}}{\frac{2y - 5}{y + 7}} \)

19. TRAVEL Ray and Jan are on a 12 1/2-hour drive from Springfield, Missouri, to Chicago, Illinois. They stop for a break every 3 1/4 hours.
   a. Write an expression to model this situation.
   b. How many stops will Ray and Jan make before arriving in Chicago?
   c. CARPENTERY Tai needs several 2 1/4-inch wooden rods to reinforce the frame on a futon. She can cut the rods from a 24 1/2-inch dowel purchased from a hardware store. How many wooden rods can she cut from the dowel?

11-7 Word Problem Practice

Mixed Expressions and Complex Fractions

1. CYCLING Natalie rode in a bicycle event for charity on Saturday. It took her 2 hours to complete the 18-mile race. What was her average speed in miles per hour?

2. QUILTING Mrs. Tantora sews and sells Amish baby quilts. She bought 42 3/4 yards of backing fabric, and 2 1/2 yards are needed for each quilt she sews. How many quilts can she make with the backing fabric she bought?

3. TRAVEL The Franz family traveled from Galveston to Waco for a family reunion. Driving their minivan, they averaged 30 miles per hour on the way to Waco and 45 miles per hour on the return trip home to Galveston. What is their average rate for the entire trip? (Hint: Remember that average rate equals total distance divided by total time and that time can be represented as a ratio of distance x to rate.)

4. PHYSICAL SCIENCE The volume of a gas varies directly as the Kelvin temperature \( T \) and inversely as the pressure \( P \), where \( k \) is the constant of variation.

   \( V = \frac{kT}{P} \)

   If \( k = \frac{12}{T} \), find the volume in liters of helium gas at 273 degrees Kelvin and 3 atmospheres of pressure. Round your answer to the nearest hundredth.

   \( 5.22 \text{ L} \)

5. SAFETY The Occupational Safety and Health Administration provides safety standards in the workplace to keep workers free from dangerous working conditions. OSHA recommends that for general construction there be 5 foot-candles of illumination in which to work. A foreman using a light meter finds that the illumination of a construction light on a surface 8 feet from the source is 11 foot-candles. The illumination produced by a light source varies inversely as the square of the distance from the source.

   \( I = \frac{k}{d^2} \)

   \( k \) is a constant.

   a. Find the illumination of the same light at a distance of 15 2/3 feet. Round your answer to the nearest hundredth.
   b. Is there enough illumination at this distance to meet OSHA requirements for lighting? no
   c. In order to comply with OSHA, what is the maximum allowable working distance from this light source? Round your decimal answer to nearest tenth.

   \( 11.9 \text{ ft} \)
11-7 Enrichment

Continued Fractions

Continued fractions are a special type of complex fraction. Each fraction in a continued fraction has a numerator of 1.

Example 1 Evaluate the continued fraction above. Start at the bottom and work your way up.

Step 1 \[3 + \frac{1}{4} = \frac{13}{4}\]
Step 2 \[\frac{1}{4} = \frac{1}{4}\]
Step 3 \[\frac{1}{2 + \frac{1}{2}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}\]
Step 4 \[\frac{1}{\frac{3}{5}} = \frac{5}{3}\]
Step 5 \[\frac{1}{\frac{5}{3}} = \frac{3}{5}\]

Evaluate each continued fraction.

1. \[0 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{6 + \frac{1}{3}}}}} = \frac{204}{63}\]
2. \[0 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3}}} = \frac{13}{19}\]
3. \[3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}} = \frac{5}{3}\]
4. \[36 + \frac{1}{115}\]

Example 2 Change \(\frac{13}{7}\) into a continued fraction.

Step 1 \[\frac{13}{7} = \frac{6}{7} = 1 + \frac{6}{7}\]
Step 2 \[\frac{2}{1} = \frac{6}{7}\]
Step 3 \[\frac{2}{1} = \frac{6}{7}\]

Thus, \(\frac{13}{7}\) can be written as \(1 + \frac{1}{1 + \frac{1}{1}}\).

11-8 Study Guide and Intervention

Rational Equations

Solve Rational Equations Rational equations are equations that contain rational expressions. To solve equations containing rational expressions, multiply each side of the equation by the least common denominator.

Rational equations can be used to solve work problems and rate problems.

Example 1 Solve \(\frac{x - 3}{3} + \frac{3}{2} = 4\).

Answer: \(x = 2\)

Example 2 Solve \(\frac{15}{x^2 - 1} = \frac{5}{2(x - 1)}\). State any extraneous solutions.

Answer: \(x = 3, x = 2\)

Exercises

Solve each equation. State any extraneous solutions.

1. \(\frac{x - 5}{5} = \frac{x}{4}\)
2. \(\frac{3}{4} = \frac{6}{x + 1}\)
3. \(\frac{x - 1}{3} = \frac{2x - 2}{5}\)
4. \(\frac{8}{n - 1} = \frac{10}{n + 1}\)
5. \(t - \frac{4}{t + 3} = t + 3 - \frac{4}{3}\)
6. \(\frac{m + 4}{m} = \frac{m + 3}{3} - 4; 0\)
7. \(\frac{q + 4}{q} = \frac{q}{q} + 1\)
8. \(\frac{5 - 2x}{2} = \frac{4x + 3}{6}\)
9. \(\frac{m}{m - 1}, \frac{m}{1 - m} = 1 - \frac{2}{2}\)
10. \(\frac{4}{x - 3} + x = 9 - 3\) or 2
11. \(\frac{9}{x - 6} = 0; 6\)
12. \(\frac{4}{x^2 + 4} = \frac{6}{x} - 1\)
13. \(\frac{4}{x - 4} - \frac{3}{p - 4} = 4 - 6, 2\)
14. \(\frac{x - 16}{x + 4} + x^2 = 16 - 4, 3; 4\)

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11-8 Study Guide and Intervention (continued)

Use Rational Equations to Solve Problems Rational equations can be used to solve work problems and rate problems.

Example WORK PROBLEM Marla can paint Percy’s kitchen in 3 hours. Percy can paint it in 2 hours. Working together, how long will it take Marla and Percy to paint the kitchen?

In \( t \) hours, Marla completes \( t \cdot \frac{1}{3} \) of the job and Percy completes \( t \cdot \frac{1}{2} \) of the job. So an equation for completing the whole job is \( \frac{t}{3} + \frac{t}{2} = 1 \).

\[
\frac{2t}{6} + \frac{3t}{6} = 1
\]

Multiply each term by 6.

\[
5t = 6
\]

Add like terms.

\[
t = \frac{6}{5}
\]

Solve.

So it will take Marla and Percy \( \frac{6}{5} \) hours to paint the room if they work together.

Exercises

1. **GREETING CARDS** It takes Kenesha 45 minutes to prepare 20 greeting cards. It takes Paula 30 minutes to prepare the same number of cards. Working together at this rate, how long will it take them to prepare the cards? 18 min

2. **BOATING** A motorboat went upstream at 15 miles per hour and returned downstream at 20 miles per hour. How far did the boat travel one way if the round trip took 3.5 hours? 30 mi

3. **FLOORING** Maya and Reginald are installing hardwood flooring. Maya can install flooring in a room in 4 hours. Reginald can install flooring in a room in 3 hours. How long would it take them if they worked together? \( \frac{24}{7} \) hr or about 3.43 hours

4. **BICYCLING** Stefan is bicycling on a bike trail at an average of 10 miles per hour. Erik starts bicycling on the same trail 30 minutes later. If Erik averages 16 miles per hour, how long will it take him to pass Stefan? 50 min

11-8 Skills Practice

Rational Equations

Solve each equation. State any extraneous solutions.

1. \( \frac{5}{x} = \frac{-2}{x + 3} - 5 \)

2. \( \frac{a}{b} = \frac{-5}{a + 4} \)

3. \( \frac{7}{m + 1} = \frac{12}{m + 2} \)

4. \( \frac{3}{x + 2} = \frac{5}{x + 8} \)

5. \( \frac{y}{y - 2} = \frac{y + 1}{y - 5} \)

6. \( \frac{b - 2}{b} = \frac{b + 4}{b + 2} - 1 \)

7. \( \frac{3m - 1}{4} = \frac{10m + 1}{8} \)

8. \( \frac{7x + 1}{3} = \frac{5x}{6} \)

9. \( \frac{9x + 5}{6} = \frac{2x}{3} - \frac{1}{2} \)

10. \( \frac{n - 3}{10} + \frac{n - 5}{5} = \frac{3}{2} \)

11. \( \frac{c + 2}{c} + \frac{c + 3}{c} = 7 \)

12. \( \frac{3b - 4}{b} = \frac{b - 7}{b} - 1 \)

13. \( \frac{m - 4}{m} = \frac{m - 11}{m + 4} \)

14. \( \frac{f + 2}{f} - \frac{f + 1}{f + 5} = \frac{1}{f} - 1 \)

15. \( \frac{f + 3}{r - 1} - \frac{f}{r - 3} = 0 \)

16. \( \frac{a + 1}{a - 2} - \frac{a}{a + 1} = 0 \)

17. \( \frac{-2}{k + 1} + \frac{2}{k} = 1 - 2, 1 \)

18. \( \frac{5}{m - 4} - \frac{m}{2m - 8} = 1 \)

19. **ACTIVISM** Maury and Tyra are making phone calls to state representatives’ offices to lobby for an issue. Maury can call all 120 state representatives in 10 hours. Tyra can call all 120 state representatives in 8 hours. How long would it take them to call all 120 state representatives together? 4 \( \frac{2}{9} \) hr
11-8 Practice

Rational Equations

Solve each equation. State any extraneous solutions.

1. \(\frac{5}{x + 2} = \frac{4}{x + 4}\)
2. \(\frac{2x}{x - 5} = \frac{4}{x - 6}\)
3. \(\frac{h + 5}{h} = \frac{h - 1}{h + 9}\)
4. \(\frac{2x}{x - 1} = \frac{2x + 1}{x + 2}\)
5. \(\frac{4y}{3} + \frac{2}{y} = \frac{5y}{6}\)
6. \(x - 2 + \frac{x + 2}{5} = -1\)
7. \(\frac{2x - 1}{x} - \frac{q}{x} = \frac{g + 4}{18}\)
8. \(\frac{5}{p - 1} - \frac{3}{p + 2} = \frac{1}{2}\)
9. \(x - 3 + \frac{1}{y} = \frac{1}{x + 3}\)
10. \(\frac{4x}{2x + 1} - \frac{2x}{2x + 3} = 1\)
11. \(\frac{d - 3}{d - 2} - \frac{d - 4}{d} = 1\)
12. \(\frac{2x}{y - 2} - \frac{x}{y + 3} = 3\)
13. \(\frac{2m}{m + 2} - \frac{m + 2}{m - 2} = \frac{7}{3}\)
14. \(\frac{n + 2}{n} + \frac{n + 5}{n + 3} = \frac{6}{n}\)
15. \(\frac{1}{x + 1} - \frac{6 - x}{6x} = \frac{1}{2}\)
16. \(\frac{n + 2}{n} - \frac{n + 6}{n^2 - 4} = 1\)
17. \(\frac{x^2 - 9}{x - 3} = \frac{x}{x + 3}\)
18. \(-\frac{2a}{n - 4} - \frac{n + 6}{n^2 - 4} = 1\)

3; extraneous: \(-2\)

19. PUBLISHING Tracey and Alan publish an 10-page independent newspaper one a month. At production, Alan usually spends 6 hours on the layout of the paper. When Tracey helps, layout takes 3 hours and 20 minutes.

a. Write an equation that could be used to determine how long it would take Tracey to do the layout by herself. Sample answer: \(\frac{3}{10} + \frac{1}{6} = 1\)
   b. How long would it take Tracey to do the job alone? 7 h 30 min

20. TRAVEL Emilio made arrangements to have Lynda pick him up from an auto repair shop after he dropped his car off. He called Lynda to tell her he would start walking and to look for him on the way. Emilio and Lynda live 10 miles from the auto-shop. It takes Emilio 1/2 hours to walk the distance and Lynda 15 minutes to drive the distance.

a. If Emilio and Lynda leave at the same time, when should Lynda expect to spot Emilio on the road? in \(13\frac{1}{2}\) min
   b. How far will Emilio have walked when Lynda picks him up? 1 mi

11-8 Word Problem Practice

Rational Equations

1. ELECTRICITY The current in a simple electric circuit varies inversely as the resistance. If the current is 20 amps when the resistance is 5 ohms, find the current when the resistance is 8 ohms. 12.5 amps

2. MASONRY Sam and Belai are masons who are working to build a stone wall that will be 120 feet long. Sam works from one end and is able to build a ten-foot section in 5 hours. Belai works from the other end and is able to finish a ten-foot section in 4 hours. How long will it take Sam and Belai to finish building the wall?
   26 hours and 40 minutes

3. NUMBERS The formula to find the sum of the first \(n\) whole numbers is \(\sum_{k=1}^{n} k = \frac{n(n + 1)}{2}\) In order to encourage students to show up early to a school dance, the dance committee decides to charge less for those who come to the dance early. Their plan is to charge the first student to arrive 1 penny. The second student through the door is charged 2 pennies; the third student through the door is charged 3 pennies, and so on. How much money, in total, would be paid by the first 150 students? 11,325 pennies or $113.25

4. NAUTICAL A ferry captain keeps track of the progress of his ship in the ship’s log. One day, he records the following entry.

   With the recent spring snow melt, the current is running strong today. The six-mile trip downstream to Whyte’s Landing was very quick. However, we only covered two miles in the same amount of time we headed back upstream.

Write a rational equation using \(b\) for the speed of the boat and \(c\) for the speed of the stream and solve for \(b\) in terms of \(c\).

   \(b = \frac{2}{c}\)

5. HEALTH CARE The total number of Americans waiting for kidney and heart transplants is approximately 66,500. The ratio of those awaiting a kidney transplant to those awaiting a heart transplant is about 20 to 1.

   a. How many people are on each of the waiting lists? Round your answers to the nearest hundred.
      Kidney: 63,300; Heart: 3200

   b. These two groups make up about \(\frac{3}{4}\) of the transplant candidates for all organs. About how many organ transplant candidates are there altogether? Round your answer to the nearest thousand.
      89,000
Winning Distances

In 1999, Hicham El Guerrouj set a world record for the mile run with a time of 3:43.13 (3 min 43.13 s). In 1954, Roger Bannister ran the first mile under 4 minutes at 3:59.4. Had they run those times in the same race, how far in front of Bannister would El Guerrouj have been at the finish?

Use \( \frac{d}{t} = r \). Since 3 min 43.13 s = 223.13 s, and 3 min 59.4 s = 239.4 s, El Guerrouj’s rate was \( \frac{5280}{223.13} \) and Bannister’s rate was \( \frac{5280}{239.4} \).

\[
\begin{array}{ccc}
\text{r} & \text{t} & \text{d} \\
223.13 & 5280 & 223.13 \\
239.4 & 5280 & 223.13 or 4921.2 \\
\end{array}
\]

Therefore, when El Guerrouj hit the tape, he would be 4921.2 feet, or 358.8 feet, ahead of Bannister. Let’s see whether we can develop a formula for this type of problem.

Let \( D \) = the distance raced,

\( W \) = the winner’s time,

and \( L \) = the loser’s time.

Following the same pattern, you obtain the results shown in the table on the right.

The winning distance will be \( D - \frac{DW}{L} \).

1. Show that the expression for the winning distance is equivalent to \( \frac{DL - W}{L} \).

\[
D - \frac{DW}{L} = \frac{DL}{L} - \frac{DW}{L} = \frac{DL - DW}{L} = \frac{DL - W}{L}
\]

Use the formula winning distance = \( \frac{DL - W}{L} \) to find the winning distance to the nearest tenth for each of the following Olympic races.

2. women’s 400 meter relay: East Germany 41.6 s (1980); Canada 48.4 s (1928) \( 56.2 \) meters

3. men’s 200 meter freestyle swimming: Michael Phelps 1 min 42.96 s (2008); Yevgeny Sudovyi 1 min 46.70 s (1992) \( 71 \) meters

4. men’s 50,000 meter walk: Alex Schwazer 3 h 37 min 9 s (2008); Hartwig Gauter 3 h 49 min 24 s (1980) \( 2670.0 \) meters

5. women’s 400 meter freestyle swimming relay: Netherlands 3 min 33.76 s (2008); United States 3 min 39.29 s (1996) \( 10.1 \) meters
Chapter 11 Assessment Answer Key

Quiz 1 (Lessons 11-1 through 11-2) Page 57

1. **D**
2. **-2**
3. **xy = -18; y = 2**
4. **x = -1, y = 2**

Quiz 2 (Lessons 11-3 and 11-4) Page 57

1. **A**
2. **16/5c**
3. **3q/2r**
4. **1**
5. **w²y/x³z³**
6. **20j**
7. **(m+1)/(m(m-1))**
8. **-4**
9. **7**

Quiz 3 (Lessons 11-5 and 11-6) Page 58

1. **x/2 + 3 - 5/2x**
2. **5x - 2**
3. **3x + 4**
4. **24t**
5. **(x + 7)(x + 3)**
6. **3n/n² - 16**
7. **(4x + 14)/(x - 5)**
8. **2a/3**
9. **(4x + 7)/(x - 3)**
10. **D**

Quiz 4 (Lessons 11-7 and 11-8) Page 58

1. **m² + m + 1/m**
2. **3/xy^4n²**
3. **(x² + 3x - 1)/(x² + 3x + 2)**
4. **x = -1**
5. **B**
6. **H**
7. **xy = 21**
8. **9**
9. **x = 0, y = 2**
10. **-3**
11. **x + 1/x - 5**
12. **x + 2/2(x + 3)**
### Chapter 11 Assessment Answer Key

**Vocabulary Test**

**Form 1**

<table>
<thead>
<tr>
<th>Page 61</th>
<th>Page 62</th>
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</thead>
<tbody>
<tr>
<td>1. F</td>
<td>11. A</td>
</tr>
<tr>
<td>2. A</td>
<td>12. H</td>
</tr>
<tr>
<td>5. F</td>
<td>15. A</td>
</tr>
<tr>
<td>6. H</td>
<td>16. F</td>
</tr>
<tr>
<td>7. C</td>
<td>17. C</td>
</tr>
<tr>
<td>10. F</td>
<td>20. G</td>
</tr>
</tbody>
</table>

| Sample answer: The equation \( x/y_1 = x/y_2 \) is the product rule for inverse variations. Use it to solve problems involving inverse variations. |
| Sample answer: A rational equation is an equation that is made up of rational expressions. |

| 1. least common denominator |
| 2. rational expression |
| 3. work problem |
| 4. extraneous solutions |
| 5. inverse variation |
| 6. complex fraction |
| 7. excluded values |
| 8. mixed expression |
| 9. least common multiple |
| 10. rate problem |

---

**Chapter 11**

**Glencoe Algebra 1**

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**Answers**
Chapter 11 Assessment Answer Key

Form 2A
Page 63

1. B
2. G
3. D
4. H
5. C
6. F
7. A
8. G
9. B
10. G

B: 6

Form 2B
Page 65

1. A
2. G
3. D
4. J
5. A
6. H
7. A
8. H
9. D
10. J

B: x + 2, x + 3
Chapter 11 Assessment Answer Key

Form 2C
Page 67

1. \[ xy = 126.4; 4 \]

2. \[ \frac{m + 5}{5m} \]

3. \[ \frac{a - 7}{a - 1}; -4, 1 \]

4. \[ \frac{2(y - 3)}{1} \]

5. \[ \frac{6(x + 7)}{7(x - 5)} \]

6. \[ \frac{3b - 5 + \frac{2}{b - 4}}{2t + \frac{4rt}{3} - 3r} \]

7. \[ \frac{8r + 4}{r - 5} \]

8. \[ \frac{3}{a + 3} \]

9. \[ \frac{2n + 25}{n^2 - 36} \]

10. \[ \frac{20}{40} \]

11. \[ \frac{3}{a + 3} \]

12. \[ \frac{20}{40} \]

13. \[ \frac{20}{40} \]

14. \[ \frac{-3}{4n^2 + 9} \]

15. \[ \frac{9}{2n - 3} \]

16. \[ \frac{-9y + 20}{5y(y - 7)} \]

17. \[ \frac{a^2 + 3a + 7}{a + 2} \]

18. \[ \frac{r^2 - 4}{3r} \]

19. \[ \frac{(x - 3)(x + 6)}{(x + 2)(x + 1)} \]

20. \[ \frac{2}{2} \]

21. \[ -7; \text{extraneous 4} \]

22. \[ 4.06 \text{ yd/yr} \]

23. \[ 52 \text{ min} \]

24. \[ 2\frac{8}{11} \text{ hr} \]

25. \[ 1 \text{ ohm} \]

B: \[ 66 \text{ ft/s} \]
Chapter 11 Assessment Answer Key

Form 2D
Page 69

1. \[ xy = 110.4; 8 \]

2. \[ \frac{k + 5}{k - 1}; 1, 3 \]

3. \[ \frac{1}{2(m - 2)} \]

4. \[ \frac{1}{r + 1} \]

5. \[ \frac{x + 1}{4x} \]

6. \[ \frac{5(x + 3)}{2(x - 9)} \]

7. \[ \frac{4a^2b - \frac{5b^2}{3} - 5ab}{5r + 2 + \frac{8}{r - 3}} \]

8. \[ \frac{9t - 2}{t + 6} \]

9. \[ \frac{4}{v - 2} \]

10. \[ \frac{3p + 32}{p^2 - 49} \]

11. \[ \frac{25n^2 + 16}{5n - 4} \]

12. \[ \frac{-8x + 3}{3x(x - 5)} \]

13. \[ n^2 + 4n + 7 \]

14. \[ n + 3 \]

15. \[ \frac{(x - 2)^2}{(x - 3)(x + 2)} \]

16. \[ \frac{(x + 1)(x - 2)}{9} \]

17. \[ -4 \]

18. \[ -5; \text{ extraneous 3} \]

19. \[ 108 \text{ qt/day} \]

20. \[ 67.5 \text{ min} \]

21. \[ 1\frac{1}{5} \text{ hr} \]

22. \[ 9 \text{ ohms} \]

B: \[ 44 \text{ ft/s} \]
Chapter 11 Assessment Answer Key

Form 3
Page 71

1. 
\[ xy = \frac{12}{5}, \frac{3}{5} \]

2. 
\[ x = 0.5, y = 0 \]

3. 
\[ -3, -1, 0 \]

4. 
\[ \frac{2x - 3}{2x - 1} = \frac{1}{3}, \frac{1}{2} \]

5. 
\[ \frac{5a(a + 2)}{4(a + 3)} \]

6. 
\[ \frac{2b + 5}{2b + 3} \]

7. 
\[ \frac{1}{w + 2} \]

8. 
\[ \frac{x^3}{4y^2(x - 3)} \]

9. 
\[ (x + 4)(x - 4) \]

10. 
\[ \frac{2w + 4}{5w^2 - 1}, \frac{6x - 7}{(2x + 7)^2(2x - 7)} \]

11. 
\[ 5x - 2 \]

12. 
\[ 4x - 1, -2y^2 + 12y + 30 \]

13. 
\[ y^2 - 36 \]

14. 
\[ 7w + 12 \]

15. 
\[ w + 5 \]

16. 
\[ 2y + 2 \]

17. 
\[ y - 7 \]

18. 
\[ p^2 - p - 7 \]

19. 
\[ p - 4 \]

20. 
\[ \frac{1}{x + 4} \]

21. 
\[ 10 \]

22. 
\[ 2 \frac{1}{7} \text{ hr} \]

23. 
\[ 3 \frac{3}{4} \text{ hr} \]

24. 
\[ 29 \frac{1}{3} \text{ ft/s} \]

25. 
\[ 2x + 3, x + 3 \]

B: 
\[ c = 5; k = 2 \]
# Chapter 11 Assessment Answer Key

## Page 73, Extended-Response Test

### Scoring Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
| 4     | Superior            | • Shows thorough understanding of the concepts of inverse variations, rational expressions, division of a polynomial, simplifying complex fractions, and solving rational equations.  
       |                     | • Uses appropriate strategies to solve problems.  
       |                     | • Computations are correct.  
       |                     | • Written explanations are exemplary.  
       |                     | • Graphs are accurate and appropriate.  
       |                     | • Goes beyond requirements of some or all problems. |
| 3     | Satisfactory        | • Shows an understanding of the concepts of inverse variations, rational expressions, division of a polynomial, simplifying complex fractions, and solving rational equations.  
       |                     | • Uses appropriate strategies to solve problems.  
       |                     | • Computations are mostly correct.  
       |                     | • Written explanations are effective.  
       |                     | • Graphs are mostly accurate and appropriate.  
       |                     | • Satisfies all requirements of problems. |
| 2     | Nearly Satisfactory | • Shows an understanding of most of the concepts of inverse variations, rational expressions, division of a polynomial, simplifying complex fractions, and solving rational equations.  
       |                     | • May not use appropriate strategies to solve problems.  
       |                     | • Computations are mostly correct.  
       |                     | • Written explanations are satisfactory.  
       |                     | • Graphs are mostly accurate.  
       |                     | • Satisfies the requirements of most of the problems. |
| 1     | Nearly Unsatisfactory | • Final computation is correct.  
       |                    | • No written explanations or work is shown to substantiate the final computation.  
       |                    | • Graphs may be accurate but lack detail or explanation.  
       |                    | • Satisfies minimal requirements of some of the problems. |
| 0     | Unsatisfactory      | • Shows little or no understanding of most of the concepts of inverse variations, rational expressions, division of a polynomial, simplifying complex fractions, and solving rational equations.  
       |                    | • Does not use appropriate strategies to solve problems.  
       |                    | • Computations are incorrect.  
       |                    | • Written explanations are unsatisfactory.  
       |                    | • Graphs are inaccurate or inappropriate.  
       |                    | • Does not satisfy requirements of problems.  
       |                    | • No answer may be given. |
1. Sample answer: \(rt = 300; r = 100, t = 3\);
   \(r = 50, t = 6; r = 30, t = 10; r = 10, t = 30; r = 4, t = 75\);

2a. Sample answer: 
   \(\frac{(x - 9)(2x + 3)}{x - 9}; 2x + 3; 9\)

2b. Sample answer: 
   \(\frac{x - 2}{x}, \frac{x}{x + 3}; \frac{x^2}{x - 3}\)

2c. Sample answer: 
   \(\frac{8x + 5}{4x - 7}, \frac{5x + 4}{4x - 7}; \frac{13x + 9}{4x - 7}\)

3a. Sample answer: 
   \(\frac{(x - 3)(2x + 5)}{x - 3}; \frac{2x^2 - x - 15}{x - 3}; 2x + 5\)

   (Students should have used long division to simplify the expression.)

3b. The student should recognize that the two methods will sometimes yield the same result, and will sometimes yield different results. If the denominator is a factor of the numerator, then simplifying a rational expression by dividing out the greatest common factor will yield a polynomial. In this case using long division will also yield the same polynomial. If the denominator is not a factor of the numerator, then simplifying a rational expression by dividing out the greatest common factor will yield a rational expression. However, in this case using long division will yield the sum of a polynomial and a rational expression.

4. The student should explain that the number of flies Roshanda can make is equivalent to \(18 \frac{1}{2}\) yards per fly. The answer can be found by converting yards to inches in the numerator of this complex fraction (666 in.), then performing the division that is implied. Roshanda can make 46 flies with \(12 \frac{4}{5}\) inches of thread left over.

5a. Working together Pat and Harold will irrigate the fields faster than Pat will irrigate them working alone. The students should recognize that \(y\) will never equal \(x\) in this situation.

5b. Sample answer: 6; 2.4; 2.4 h

5c. Sample answer: 4; \(5 + \sqrt{17}\); 9.1 h
## Chapter 11 Assessment Answer Key

### Standardized Test Practice

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>○ ○ ○ ○</td>
</tr>
<tr>
<td>2.</td>
<td>○ ○ ○ ○</td>
</tr>
<tr>
<td>3.</td>
<td>○ ○ ○ ●</td>
</tr>
<tr>
<td>4.</td>
<td>○ ○ ○ ●</td>
</tr>
<tr>
<td>5.</td>
<td>○ ○ ● ○</td>
</tr>
<tr>
<td>6.</td>
<td>● ○ ○ ○</td>
</tr>
<tr>
<td>7.</td>
<td>● ○ ○ ○</td>
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<tr>
<td>8.</td>
<td>● ○ ○ ○</td>
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<td>9.</td>
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<tr>
<td>17.</td>
<td></td>
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<tr>
<td>18.</td>
<td></td>
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</tbody>
</table>

### Notes
- The answer for question 17 is a grid with options B, 4, and 6.
- The answer for question 18 is another grid with options 3 and 6.
Chapter 11 Assessment Answer Key

Standardized Test Practice
Page 76

19. yes; $4x - 9y = 60$

Sample answer:

$$y - 1500 = \frac{-670}{59}(x - 0)$$

20. $\emptyset$

21. $-2$

22. $0$ or $6$

23. $(x - 3)(x + 8)$

24. $x = 1; (1, -3)$

25. $4$

26. $22.8$ cm

27. no, $19^2 \neq 5^2 + 17^2$

28. $11$ in.

29. $15.6$ in.