**CONSUMABLE WORKBOOKS** Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th>Workbook</th>
<th>MHID</th>
<th>ISBN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-660292-3</td>
<td>978-0-07-660292-6</td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660291-5</td>
<td>978-0-07-660291-9</td>
</tr>
<tr>
<td>Spanish Version</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660294-X</td>
<td>978-0-07-660294-0</td>
</tr>
</tbody>
</table>

**Answers For Workbooks** The answers for Chapter 2 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

**ConnectED** All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

**Spanish Assessment Masters** (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 2 Test Form 2A and Form 2C.
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Teacher’s Guide to Using the
Chapter 2 Resource Masters

The Chapter 2 Resource Masters includes the core materials needed for Chapter 2. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give to students before beginning Lesson 2-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master presented in both English and Spanish is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the Chapter 2 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 9 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- Form 1 contains multiple-choice questions and is intended for use with below grade level students.
- Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 2. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>dimensional analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>duh·MEHNCH·NUHL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>equivalent equations</td>
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<td>ih·KWIHV·luhnt</td>
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<tr>
<td>formula</td>
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<td></td>
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<tr>
<td>identity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>multi-step equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition/Description/Example</td>
</tr>
<tr>
<td>--------------------------</td>
<td>---------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>percent of change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>proportion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pruh-POHR-shun</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rate</td>
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<td></td>
</tr>
<tr>
<td>ratio</td>
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<tr>
<td>scale model</td>
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<tr>
<td>solve an equation</td>
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<td></td>
</tr>
<tr>
<td>unit rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>weighted average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## 2 Anticipation Guide
### Linear Equations

**Step 1** *Before you begin Chapter 2*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>Statement</th>
<th>STEP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, or NS</td>
<td></td>
<td>A or D</td>
</tr>
<tr>
<td>1. When writing equations the phrases <em>as much as, is, and is identical to</em>, all suggest the equals sign.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. The solving an equation strategy cannot be used if an equation is not given in the problem.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Given a true equation, any value can be added or subtracted to both sides resulting in a true equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Since the equation ( t - 23 = 54 ) involves subtraction, subtraction would be used to solve for ( t ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. To solve ( 21 = -7x ), you could divide by ( -7 ) or multiply by ( -\frac{1}{7} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. To solve equations with more than one operation, undo operations in the same order as the Order of Operations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Equations with the variable on both sides have no solution.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. 3 to 5, 3:5, and ( \frac{3}{5} ) are all examples of ratios.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Because ( 5 \cdot 12 = 15 \cdot 4 ), ( \frac{12}{15} ) is in proportion to ( \frac{4}{5} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. A percent of change is found by dividing the amount of change by the original amount.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Equations containing two variables cannot be solved since variables cannot be added or subtracted from each side of the equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. To find a weighted average, extremely high or low values are not included.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** *After you complete Chapter 2*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
# Ejercicios preparatorios
## Ecuaciones lineales

### Paso 1
**Antes de comenzar el Capítulo 2**
- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Al escribir ecuaciones, los enunciados: <em>tanto como, es y es idéntico a</em>, sugieren el signo de igualdad.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Al resolver un problema, no se puede usar una estrategia de ecuación si el problema no incluye una ecuación.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Dada una ecuación verdadera, se puede sumar y restar cualquier valor de ambos lados de la ecuación, lo cual resulta en una ecuación verdadera.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Puesto que la ecuación $t - 23 = 54$ implica sustracción, ésta se usaría para despejar $t$.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Para resolver $21 = -7x$, podrías dividir entre $-7$ o multiplicar por $\frac{1}{-7}$.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Para resolver ecuaciones con más de una operación, anula las operaciones en el mismo orden en que lo establece el orden de las operaciones.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Ecuaciones con la variable en ambos lados no tienen solución.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>3 a 5, 3:5 y $\frac{3}{5}$ son todos ejemplos de razones.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Dado que $5 \cdot 12 = 15 \cdot 4$, $\frac{12}{15}$ está en proporción a $\frac{4}{5}$.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Un porcentaje de cambio se encuentra dividiendo la cantidad de cambio entre la cantidad original.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Ecuaciones que contienen dos variables no pueden resolverse porque las variables no se pueden sumar o restar de cada lado de la ecuación.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>Para calcular el promedio ponderado, no se incluyen los valores extremadamente altos o bajos.</td>
<td></td>
</tr>
</tbody>
</table>

### Paso 2
**Después de completar el Capítulo 2**
- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
Write Equations Writing equations is one strategy for solving problems. You can use a variable to represent an unspecified number or measure referred to in a problem. Then you can write a verbal expression as an algebraic expression.

Example 1 Translate each sentence into an equation or a formula.

a. Ten times a number \( x \) is equal to 2.8 times the difference \( y \) minus \( z \).
\[ 10x = 2.8(y - z) \]
The equation is \( 10x = 2.8(y - z) \).

b. A number \( m \) minus 8 is the same as a number \( n \) divided by 2.
\[ m - 8 = n \div 2 \]
The equation is \( m - 8 = \frac{n}{2} \).

c. The area of a rectangle equals the length times the width. Translate this sentence into a formula.
Let \( A = \text{area} \), \( \ell = \text{length} \), and \( w = \text{width} \).
Formula: \( \text{Area equals length times width} \).
\[ A = \ell \times w \]
The formula for the area of a rectangle is \( A = \ell w \).

Example 2 Use the Four-Step Problem-Solving Plan.

POPULATION The population of the United States in July 2007 was about 301,000,000, and the land area of the United States is about 3,500,000 square miles. Find the average number of people per square mile in the United States.

Step 1 Read You know that there are 301,000,000 people. You want to know the number of people per square mile.

Step 2 Plan Write an equation to represent the situation. Let \( p \) represent the number of people per square mile.
\[ 3,500,000 \times p = 301,000,000 \]

Step 3 Solve \( 3,500,000 \times p = 301,000,000 \) Divide each side by \( 3,500,000 \).
\[ p = \frac{301,000,000}{3,500,000} = 86 \]
There are 86 people per square mile.

Step 4 Check If there are 86 people per square mile and there are 3,500,000 square miles, \( 86 \times 3,500,000 = 301,000,000 \). The answer makes sense.

Exercises
Translate each sentence into an equation or formula.

1. Three times a number \( t \) minus twelve equals forty.

2. One-half of the difference of \( a \) and \( b \) is 54.

3. Three times the sum of \( d \) and 4 is 32.

4. The area \( A \) of a circle is the product of \( \pi \) and the radius \( r \) squared.

5. WEIGHT LOSS Lou wants to lose weight to audition for a part in a play. He weighs 160 pounds now. He wants to weigh 150 pounds.

   a. If \( p \) represents the number of pounds he wants to lose, write an equation to represent this situation.

   b. How many pounds does he need to lose to reach his goal?
Write Verbal Sentences You can translate equations into verbal sentences.

Example

Translate each equation into a sentence.

a. \(4n - 8 = 12\).

\[
4n - 8 = 12
\]

Four times \(n\) minus eight equals twelve.

b. \(a^2 + b^2 = c^2\)

\[
a^2 + b^2 = c^2
\]

The sum of the squares of \(a\) and \(b\) is equal to the square of \(c\).

Exercises

Translate each equation into a sentence.

1. \(4a - 5 = 23\)

2. \(10 + k = 4k\)

3. \(6xy = 24\)

4. \(x^2 + y^2 = 8\)

5. \(p + 3 = 2p\)

6. \(b = \frac{1}{3}(h - 1)\)

7. \(100 - 2x = 80\)

8. \(3(g + h) = 12\)

9. \(p^2 - 2p = 9\)

10. \(C = \frac{5}{9}(F - 32)\)

11. \(V = \frac{1}{3}Bh\)

12. \(A = \frac{1}{2}hb\)
Skills Practice

Writing Equations

Translate each sentence into an equation.

1. Two added to three times a number $m$ is the same as 18.
2. Twice $a$ increased by the cube of $a$ equals $b$.
3. Seven less than the sum of $p$ and $t$ is as much as 6.
4. The sum of $x$ and its square is equal to $y$ times $z$.
5. Four times the sum of $f$ and $g$ is identical to six times $g$.

Translate each sentence into a formula.

6. The perimeter $P$ of a square equals four times the length of a side $\ell$.
7. The area $A$ of a square is the length of a side $\ell$ squared.
8. The perimeter $P$ of a triangle is equal to the sum of the lengths of sides $a$, $b$, and $c$.

9. The area $A$ of a circle is pi times the radius $r$ squared.
10. The volume $V$ of a rectangular prism equals the product of the length $\ell$, the width $w$, and the height $h$.

Translate each equation into a sentence.

11. $g + 10 = 3g$
12. $2p + 4t = 20$
13. $4(a + b) = 9a$
14. $8 - 6x = 4 + 2x$
15. $\frac{1}{2}(f + y) = f - 5$
16. $k^2 - n^2 = 2b$

Write a problem based on the given information.

17. $c =$ cost per pound of plain coffee beans
   $c + 3 =$ cost per pound of flavored coffee beans
   $2c + (c + 3) = 21$

18. $p =$ cost of dinner
   $0.15p =$ cost of a 15% tip
   $p + 0.15p = 23$
2-1 Practice

Writing Equations

Translate each sentence into an equation.

1. Fifty-three plus four times \(b\) is as much as 21.
2. The sum of five times \(h\) and twice \(g\) is equal to 23.
3. One fourth the sum of \(r\) and ten is identical to \(r\) minus 4.
4. Three plus the sum of the squares of \(w\) and \(x\) is 32.

Translate each sentence into a formula.

5. Degrees Kelvin \(K\) equals 273 plus degrees Celsius \(C\).
6. The total cost \(C\) of gas is the price \(p\) per gallon times the number of gallons \(g\).
7. The sum \(S\) of the measures of the angles of a polygon is equal to 180 times the difference of the number of sides \(n\) and 2.

Translate each equation into a sentence.

8. \(r - (4 + p) = \frac{1}{3}r\)  
9. \(\frac{3}{5}t + 2 = t\)

10. \(9(y^2 + x) = 18\)  
11. \(2(m - n) = x + 7\)

Write a problem based on the given information.

12. \(a = \text{cost of one adult's ticket to zoo}\)  
\(a - 4 = \text{cost of one children's ticket to zoo}\)  
\(2a + 4(a - 4) = 38\)

13. \(c = \text{regular cost of one airline ticket}\)  
\(0.20c = \text{amount of 20\% promotional discount}\)  
\(3(c - 0.20c) = 330\)

14. GEOGRAPHY About 15\% of all federally-owned land in the 48 contiguous states of the United States is in Nevada. If \(F\) represents the area of federally-owned land in these states, and \(N\) represents the portion in Nevada, write an equation for this situation.

15. FITNESS Deanna and Pietra each go for walks around a lake a few times per week. Last week, Deanna walked 7 miles more than Pietra.

a. If \(p\) represents the number of miles Pietra walked, write an equation that represents the total number of miles \(T\) the two girls walked.

b. If Pietra walked 9 miles during the week, how many miles did Deanna walk?

c. If Pietra walked 11 miles during the week, how many miles did the two girls walk together?
2-1 Word Problem Practice

Writing Equations

1. HOUSES The area of the Hartstein’s kitchen is 182 square feet. This is 20% of the area of the first floor of their house. Let $F$ represent the area of the first floor. Write an equation to represent the situation.

2. FAMILY Katie is twice as old as her sister Mara. The sum of their ages is 24. Write a one-variable equation to represent the situation.

3. GEOMETRY The formula $F + V = E + 2$ shows the relationship between the number of faces $F$, edges $E$, and vertices $V$ of a polyhedron, such as a pyramid. Write the formula in words.

4. WIRELESS PHONE Spinfrog wireless phone company bills on a monthly basis. Each bill includes a $29.95 service fee for 1000 minutes plus a $2.95 federal communication tax. Additionally, there is a charge of $0.05 for each minute used over 1000. Let $m$ represent the number of minutes over 1000 used during the month. Write an equation to describe the cost $p$ of the wireless phone service per month.

5. TEMPERATURE The table below shows the temperature in Fahrenheit for some corresponding temperatures in Celsius.

<table>
<thead>
<tr>
<th>Celsius</th>
<th>Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20°</td>
<td>-4°</td>
</tr>
<tr>
<td>-10°</td>
<td>14°</td>
</tr>
<tr>
<td>0°</td>
<td>32°</td>
</tr>
<tr>
<td>10°</td>
<td>50°</td>
</tr>
<tr>
<td>20°</td>
<td>68°</td>
</tr>
<tr>
<td>30°</td>
<td>86°</td>
</tr>
</tbody>
</table>

a. Write a formula for converting Celsius temperatures to Fahrenheit temperatures.

b. Find the Fahrenheit equivalents for 25°C and 35°C.
Guess the Number

Think of a number. Add five to your number. Now, double your result. Double your result again. Divide you answer by four. Finally, subtract your original number. Your result is five.

How is it possible to know what the answer is without knowing the original number? Write the steps listed above as an expression in equation form. Then use algebra to show why this trick works.

Think of a number: \( x \)
Add five to your number: \( x + 5 \)
Double your result: \( 2(x + 5) \)
Double your result again: \( 2(2(x + 5)) \)
Divide you answer by four: \( \frac{2(2(x + 5))}{4} \)
Subtract your original number: \( \frac{2(2(x + 5))}{4} - x \)
Simplify the final expression: \( \frac{4(x + 5)}{4} - x \)  
Multiply. 
\( x + 5 - x \)  
Divide. 
\( 5 \)  
Simplify. 

So, the result will always be five, no matter what the starting number is.

Write variable expressions to determine why each number trick works.

1. Think of a number. Add eight. Double your result. Next, subtract 16. Finally, divide your result by 2. You get your original number back.

2. Think of a number. Multiply by 10. Add 5 to your result. Next, subtract 3. Then add 2. Next, subtract 4. Divide your result by 5. Finally, subtract your original number. Your result is your original number.

3. Think of a number. Add 1. Multiply your result by 6. Now, double your result. Next, divide your result by 12. Finally, subtract your original number. Your result is 1.

4. Think of a number. Multiply by 5. Add five to your result. Now, divide by 5. Subtract 1 from your result. Finally, subtract your original number. Your final result is 0.

Lesson 2-2

Study Guide and Intervention

Solving One-Step Equations

Solve Equations Using Addition and Subtraction If the same number is added to each side of an equation, the resulting equation is equivalent to the original one. In general if the original equation involves subtraction, this property will help you solve the equation. Similarly, if the same number is subtracted from each side of an equation, the resulting equation is equivalent to the original one. This property will help you solve equations involving addition.

<table>
<thead>
<tr>
<th>Addition Property of Equality</th>
<th>Subtraction Property of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>For any numbers a, b, and c, if a = b, then a + c = b + c.</td>
<td>For any numbers a, b, and c, if a = b, then a − c = b − c.</td>
</tr>
</tbody>
</table>

Example 1 Solve \( m - 32 = 18 \).

\[
\begin{align*}
  m - 32 &= 18 & \text{Original equation} \\
  m - 32 + 32 &= 18 + 32 & \text{Add 32 to each side.} \\
  m &= 50 & \text{Simplify.}
\end{align*}
\]

The solution is 50.

Example 2 Solve \( 22 + p = -12 \).

\[
\begin{align*}
  22 + p &= -12 & \text{Original equation} \\
  22 + p - 22 &= -12 - 22 & \text{Subtract 22 from each side.} \\
  p &= -34 & \text{Simplify.}
\end{align*}
\]

The solution is -34.

Exercises

Solve each equation. Check your solution.

1. \( h - 3 = -2 \)  
2. \( m - 8 = -12 \)  
3. \( p - 5 = 15 \)  
4. \( 20 = y - 8 \)  
5. \( k - 0.5 = 2.3 \)  
6. \( w - \frac{1}{2} = \frac{5}{8} \)  
7. \( h - 18 = -17 \)  
8. \( -12 = -24 + k \)  
9. \( j - 0.2 = 1.8 \)  
10. \( b - 40 = -40 \)  
11. \( m - (-12) = 10 \)  
12. \( w - \frac{3}{2} = \frac{1}{4} \)  
13. \( x + 12 = 6 \)  
14. \( w + 2 = -13 \)  
15. \( -17 = b + 4 \)  
16. \( k + (-9) = 7 \)  
17. \( -3.2 = \ell + (-0.2) \)  
18. \( -\frac{3}{8} + x = \frac{5}{8} \)  
19. \( 19 + h = -4 \)  
20. \( -12 = k + 24 \)  
21. \( j + 1.2 = 2.8 \)  
22. \( b + 80 = -80 \)  
23. \( m + (-8) = 2 \)  
24. \( w + \frac{3}{2} = \frac{5}{8} \)
Solving One-Step Equations

Solve Equations Using Multiplication and Division If each side of an equation is multiplied by the same number, the resulting equation is equivalent to the given one. You can use the property to solve equations involving multiplication and division. To solve equations with multiplication and division, you can also use the Division Property of Equality. If each side of an equation is divided by the same number, the resulting equation is true.

### Multiplication Property of Equality
For any numbers $a$, $b$, and $c$, if $a = b$, then $ac = bc$.

### Division Property of Equality
For any numbers $a$, $b$, and $c$, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

#### Example 1
Solve $3\frac{1}{2}p = 1\frac{1}{2}$.

Original equation

$$3\frac{1}{2}p = 1\frac{1}{2}$$

Rewrite each mixed number as an improper fraction.

$$\frac{7}{2}p = \frac{3}{2}$$

Multiply each side by $\frac{7}{2}$.

$$\frac{2(7p)}{7} = \frac{2(3)}{7(2)}$$

Simplify.

$$p = \frac{3}{7}$$

The solution is $\frac{3}{7}$.

#### Example 2
Solve $-5n = 60$.

Original equation

$$-5n = 60$$

Divide each side by $-5$.

$$\frac{-5n}{-5} = \frac{60}{-5}$$

Simplify.

$$n = -12$$

The solution is $-12$.

### Exercises
Solve each equation. Check your solution.

1. $\frac{h}{3} = -2$
2. $\frac{1}{8}m = 6$
3. $\frac{1}{5}p = \frac{3}{5}$

4. $5 = \frac{y}{12}$
5. $-\frac{1}{4}k = -2.5$
6. $-\frac{m}{8} = \frac{5}{8}$

7. $-1\frac{1}{2}h = 4$
8. $-12 = -\frac{3}{2}k$
9. $\frac{j}{3} = \frac{2}{5}$

10. $-3\frac{1}{3}b = 5$
11. $\frac{7}{10}m = 10$
12. $\frac{p}{5} = -\frac{1}{4}$

13. $3h = -42$
14. $8m = 16$
15. $-3t = 51$

16. $-3r = -24$
17. $8k = -64$
18. $-2m = 16$

19. $12h = 4$
20. $-2.4p = 7.2$
21. $0.5j = 5$

22. $-25 = 5m$
23. $6m = 15$
24. $-1.5p = -75$
Solve each equation. Check your solution.

1. \( y - 7 = 8 \)
2. \( w + 14 = -8 \)
3. \( p - 4 = 6 \)
4. \(-13 = 5 + x \)
5. \( 98 = b + 34 \)
6. \( y - 32 = -1 \)
7. \( n + (-28) = 0 \)
8. \( y + (-10) = 6 \)
9. \( -1 = t + (-19) \)
10. \( j - (-17) = 36 \)
11. \( 14 = d + (-10) \)
12. \( u + (-5) = -15 \)
13. \( 11 = -16 + y \)
14. \( c - (-3) = 100 \)
15. \( 47 = w - (-8) \)
16. \( x - (-74) = -22 \)
17. \( 4 - (-h) = 68 \)
18. \( -56 = 20 - (-j) \)
19. \( 12z = 108 \)
20. \( -7t = 49 \)
21. \( 18f = -216 \)
22. \( -22 = 11v \)
23. \( -6d = -42 \)
24. \( 96 = -24a \)
25. \( \frac{c}{4} = 16 \)
26. \( \frac{a}{16} = 9 \)
27. \( -84 = \frac{d}{3} \)
28. \( -\frac{d}{7} = -13 \)
29. \( \frac{t}{4} = -13 \)
30. \( 31 = \frac{1}{6}n \)
31. \( -6 = \frac{2}{3}z \)
32. \( \frac{2}{7}q = -4 \)
33. \( \frac{5}{9}p = -10 \)
34. \( \frac{a}{10} = \frac{2}{5} \)
2-2 Practice

Solving One-Step Equations

Solve each equation. Check your solution.

1. \( d - 8 = 17 \)  
2. \( v + 12 = -5 \)  
3. \( b - 2 = -11 \)
4. \( -16 = m + 71 \)  
5. \( 29 = a - 76 \)  
6. \( -14 + y = -2 \)
7. \( 8 - (-n) = 1 \)  
8. \( 78 + r = -15 \)  
9. \( f + (-3) = -9 \)
10. \( 8j = 96 \)  
11. \( -13z = -39 \)  
12. \( -180 = 15m \)
13. \( 243 = 27r \)  
14. \( \frac{y}{9} = -8 \)  
15. \( \frac{-j}{12} = -8 \)
16. \( \frac{a}{15} = \frac{4}{5} \)  
17. \( \frac{g}{27} = \frac{2}{9} \)  
18. \( \frac{q}{24} = \frac{1}{6} \)

Write an equation for each sentence. Then solve the equation.

19. Negative nine times a number equals \(-117\).
20. Negative one eighth of a number is \(-3\) \(-\frac{3}{4}\).
21. Five sixths of a number is \(-\frac{5}{9}\).
22. 2.7 times a number equals 8.37.

23. HURRICANES The day after a hurricane, the barometric pressure in a coastal town has risen to 29.7 inches of mercury, which is 2.9 inches of mercury higher than the pressure when the eye of the hurricane passed over.
   a. Write an addition equation to represent the situation.
   b. What was the barometric pressure when the eye passed over?

24. ROLLER COASTERS Kingda Ka in New Jersey is the tallest and fastest roller coaster in the world. Riders travel at an average speed of 61 feet per second for 3118 feet. They reach a maximum speed of 187 feet per second.
   a. If \( x \) represents the total time that the roller coaster is in motion for each ride, write an expression to represent the situation. (Hint: Use the distance formula \( d = rt \).)
   b. How long is the roller coaster in motion?
1. **SUPREME COURT** Chief Justice William Rehnquist served on the Supreme Court for 33 years until his death in 2005. Write and solve an equation to determine the year he was confirmed as a justice on the Supreme Court.

2. **SALARY** In a recent year, the annual salary of the Governor of New York was $179,000. During the same year, the annual salary of the Governor of Tennessee was $94,000 less. Write and solve an equation it to find the annual salary of the Governor of Tennessee in that year.

3. **WEATHER** On a cold January day, Mavis noticed that the temperature dropped 21 degrees over the course of the day to −9°C. Write and solve an equation to determine what the temperature was at the beginning of the day.

4. **FARMING** Mr. Hill’s farm is 126 acres. Mr. Hill’s farm is \(\frac{1}{4}\) the size of Mr. Miller’s farm. How many acres is Mr. Miller’s farm?

5. **NAUTICAL** On the sea, distances are measured in nautical miles rather than miles.

   a. If a boat travels 16 knots in 1 hour, how far will it have traveled in feet? Write and solve an equation.

   b. About how fast was the boat traveling in miles per hour? Round your answer to the nearest hundredth.
Elevator Puzzle
José gets on the elevator and rides without pushing any buttons. First, the elevator goes up 4 floors where Bob gets on. Bob goes down 6 floors and gets off. At that same floor Florence gets on and goes up one floor before getting off. The elevator then moves down 8 floors to pick up the Hartt family who ride down 3 floors and get off. Then the elevator goes up one floor, picks up Kris, and goes down 6 floors to the street level where José exits the elevator.

1. Suppose \( x \) is your starting point. Write an equation that represents José’s elevator ride.

2. At what floor did José get on the elevator?

Now that you know the starting point of José, the starting point of every other person who rode the elevator can be determined.

3. At what floor did Bob get on the elevator? At what floor did Bob get off?

4. At what floor did Florence get on the elevator? At what floor did Florence get off?

5. At what floor did the Hartt family get on the elevator? At what floor did the Hartt family get off?

6. At what floor did Kris get on the elevator? At what floor did Kris get off?
Lesson 2-3

Study Guide and Intervention

Solving Multi-Step Equations

Work Backward  Working backward is one of many problem-solving strategies that you can use to solve problems. To work backward, start with the result given at the end of a problem and undo each step to arrive at the beginning number.

Example 1  A number is divided by 2, and then 8 is subtracted from the quotient. The result is 16. What is the number?

Solve the problem by working backward.

The final number is 16. Undo subtracting 8 by adding 8 to get 24. To undo dividing 24 by 2, multiply 24 by 2 to get 48.

The original number is 48.

Example 2  A bacteria culture doubles each half hour. After 3 hours, there are 6400 bacteria. How many bacteria were there to begin with?

Solve the problem by working backward.

The bacteria have grown for 3 hours. Since there are 2 one-half hour periods in one hour, in 3 hours there are 6 one-half hour periods. Since the bacteria culture has grown for 6 time periods, it has doubled 6 times. Undo the doubling by halving the number of bacteria 6 times.

\[
6400 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 6400 \times \frac{1}{64} = 100
\]

There were 100 bacteria to begin with.

Exercises

Solve each problem by working backward.

1. A number is divided by 3, and then 4 is added to the quotient. The result is 8. Find the number.

2. A number is multiplied by 5, and then 3 is subtracted from the product. The result is 12. Find the number.

3. Eight is subtracted from a number, and then the difference is multiplied by 2. The result is 24. Find the number.

4. Three times a number plus 3 is 24. Find the number.

5. CAR RENTAL  Angela rented a car for $29.99 a day plus a one-time insurance cost of $5.00. Her bill was $124.96. For how many days did she rent the car?

6. MONEY  Mike withdrew an amount of money from his bank account. He spent one fourth for gasoline and had $90 left. How much money did he withdraw?
Solve Multi-Step Equations  To solve equations with more than one operation, often called multi-step equations, undo operations by working backward. Reverse the usual order of operations as you work.

Example  Solve $5x + 3 = 23$.

\[
\begin{align*}
5x + 3 &= 23 & \text{Original equation} \\
5x + 3 - 3 &= 23 - 3 & \text{Subtract 3 from each side.} \\
5x &= 20 & \text{Simplify.} \\
\frac{5x}{5} &= \frac{20}{5} & \text{Divide each side by 5.} \\
x &= 4 & \text{Simplify.}
\end{align*}
\]

Exercises
Solve each equation. Check your solution.

1. $5x + 2 = 27$
2. $6x + 9 = 27$
3. $5x + 16 = 51$
4. $14n - 8 = 34$
5. $0.6x - 1.5 = 1.8$
6. $\frac{7}{8}p - 4 = 10$
7. $16 = \frac{d - 12}{14}$
8. $8 + \frac{3n}{12} = 13$
9. $\frac{g}{5} + 3 = -13$
10. $\frac{4b + 8}{-2} = 10$
11. $0.2x - 8 = -2$
12. $3.2y - 1.8 = 3$
13. $-4 = \frac{7x - (-1)}{-8}$
14. $8 = -12 + \frac{k}{-4}$
15. $0 = 10y - 40$

Write an equation and solve each problem.

16. Find three consecutive integers whose sum is 96.
17. Find two consecutive odd integers whose sum is 176.
18. Find three consecutive integers whose sum is $-93$. 
Skills Practice

2-3

Solving Multi-Step Equations

Solve each problem by working backward.

1. A number is divided by 2, and then the quotient is added to 8. The result is 33.
   Find the number.

2. Two is subtracted from a number, and then the difference is divided by 3.
   The result is 30. Find the number.

3. A number is multiplied by 2, and then the product is added to 9. The result is 49.
   What is the number?

4. ALLOWANCE After Ricardo received his allowance for the week, he went to the mall
   with some friends. He spent half of his allowance on a new paperback book. Then he
   bought himself a snack for $1.25. When he arrived home, he had $5.00 left. How much
   was his allowance?

Solve each equation. Check your solution.

5. \(5x + 3 = 23\)  
6. \(4 = 3a - 14\)  
7. \(2y + 5 = 19\)

8. \(6 + 5c = -29\)  
9. \(8 - 5w = -37\)  
10. \(18 - 4v = 42\)

11. \(\frac{n}{3} - 8 = -2\)  
12. \(5 + \frac{x}{4} = 1\)  
13. \(-\frac{h}{3} - 4 = 13\)

14. \(-\frac{d}{6} + 12 = -7\)  
15. \(\frac{a}{5} - 2 = 9\)  
16. \(\frac{w}{7} + 3 = -1\)

17. \(\frac{3}{4}q - 7 = 8\)  
18. \(\frac{2}{3}g + 6 = -12\)  
19. \(\frac{5}{2}z - 8 = -3\)

20. \(\frac{4}{5}m + 2 = 6\)  
21. \(c - \frac{5}{4} = 3\)  
22. \(\frac{b + 1}{3} = 2\)

Write an equation and solve each problem.

23. Twice a number plus four equals 6. What is the number?

24. Sixteen is seven plus three times a number. Find the number.

25. Find two consecutive integers whose sum is 35.

26. Find three consecutive integers whose sum is 36.
2-3 Practice

Solving Multi-Step Equations

Solve each problem by working backward.

1. Three is added to a number, and then the sum is multiplied by 4. The result is 16. Find the number.

2. A number is divided by 4, and the quotient is added to 3. The result is 24. What is the number?

3. Two is subtracted from a number, and then the difference is multiplied by 5. The result is 30. Find the number.

4. BIRD WATCHING While Michelle sat observing birds at a bird feeder, one fourth of the birds flew away when they were startled by a noise. Two birds left the feeder to go to another station a few feet away. Three more birds flew into the branches of a nearby tree. Four birds remained at the feeder. How many birds were at the feeder initially?

Solve each equation. Check your solution.

5. \(-12n - 19 = 77\)  
6. \(17 + 3f = 14\)  
7. \(15t + 4 = 49\)

8. \(\frac{u}{5} + 6 = 2\)  
9. \(\frac{d}{-4} + 3 = 15\)  
10. \(\frac{b}{3} - 6 = -2\)

11. \(\frac{1}{2}y - \frac{1}{8} = \frac{7}{8}\)  
12. \(-32 - \frac{3}{5}f = -17\)  
13. \(8 - \frac{3}{8}k = -4\)

14. \(\frac{r + 13}{12} = 1\)  
15. \(\frac{15 - a}{3} = -9\)  
16. \(\frac{3k - 7}{5} = 16\)

17. \(\frac{x}{7} - 0.5 = 2.5\)  
18. \(2.5g + 0.45 = 0.95\)  
19. \(0.4m - 0.7 = 0.22\)

Write an equation and solve each problem.

20. Seven less than four times a number equals 13. What is the number?

21. Find two consecutive odd integers whose sum is 116.

22. Find two consecutive even integers whose sum is 126.

23. Find three consecutive odd integers whose sum is 117.

24. COIN COLLECTING Jung has a total of 92 coins in his coin collection. This is 8 more than three times the number of quarters in the collection. How many quarters does Jung have in his collection?
2-3 Word Problem Practice

Solving Multi-Step Equations

1. TEMPERATURE The formula for converting a Fahrenheit temperature to a Celsius temperature is \( C = \frac{F - 32}{1.8} \).
   Find the equivalent Celsius temperature for 68°F.

2. HUMAN HEIGHT It is a commonly used guideline that for the average American child, their maximum adult height will be about twice their height at age 2. Suppose that Micah's adult height fits the following equation \( a = 2c - 1 \), where \( a \) represents his adult height and \( c \) represents his height at age 2. At age 2, Micah was 35 inches tall. What is Micah’s adult height? Write and solve an equation.

3. CHEMISTRY The half-life of a radioactive substance is the time required for half of a sample to undergo radioactive decay, or for the quantity to fall to half its original amount. Carbon 14 has a half-life of 5730 years. Suppose given samples of carbon 14 weigh \( \frac{5}{8} \) of a pound and \( \frac{7}{8} \) of a pound. What was the total weight of the samples 11,460 years ago?

4. NUMBER THEORY Write and solve an equation to find three consecutive odd integers whose sum is 3.

5. GEOMETRY A rectangular swimming pool is surrounded by a concrete sidewalk that is 3 feet wide. The dimensions of the rectangle created by the sidewalk are 21 feet by 31 feet.

   a. Find the length and width of the pool.

   b. Find the area of the pool.

   c. Write and solve an equation to find the area of the sidewalk in square feet.
Consecutive Integer Problems

Many types of problems and puzzles involve the idea of consecutive integers. Knowing how to represent these integers algebraically can help to solve the problem.

Example

Find four consecutive odd integers whose sum is \(-80\).

An odd integer can be written as \(2n + 1\), where \(n\) is any integer.

If \(2n + 1\) is the first odd integer, then add 2 to get the next largest odd integer, and so on.

Now write an equation to solve this problem.

\[(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) = -80\]

Exercises

Write an equation for each problem. Then solve.

1. Complete the solution to the problem in the example.

2. Find three consecutive even integers whose sum is 132.

3. Find two consecutive integers whose sum is 19.

4. Find two consecutive integers whose sum is 100.

5. The lesser of two consecutive even integers is 10 more than one-half the greater. Find the integers.

6. The greater of two consecutive even integers is 6 less than three times the lesser. Find the integers.

7. Find four consecutive integers such that twice the sum of the two greater integers exceeds three times the first by 91.

8. Find a set of four consecutive positive integers such that the greatest integer in the set is twice the least integer in the set.
Lesson 2-4
Study Guide and Intervention
Solving Equations with the Variable on Each Side

Variables on Each Side To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to write an equivalent equation that has the variable on just one side of the equation. Then solve the equation.

Example 1 Solve $5y - 8 = 3y + 12$.

\[
5y - 8 = 3y + 12 \\
5y - 8 - 3y = 3y + 12 - 3y \\
2y - 8 = 12 \\
2y - 8 + 8 = 12 + 8 \\
2y = 20 \\
\frac{2y}{2} = \frac{20}{2} \\
y = 10
\]

The solution is 10.

Example 2 Solve $-11 - 3y = 8y + 1$.

\[
-11 - 3y = 8y + 1 \\
-11 - 3y + 3y = 8y + 1 + 3y \\
-11 = 11y + 1 \\
-11 - 1 = 11y + 1 - 1 \\
-12 = 11y \\
\frac{-12}{11} = \frac{11y}{11} \\
-1\frac{1}{11} = y
\]

The solution is $-1\frac{1}{11}$.

Exercises
Solve each equation. Check your solution.

1. $6 - b = 5b + 30$
2. $5y - 2y = 3y + 2$
3. $5x + 2 = 2x - 10$

4. $4n - 8 = 3n + 2$
5. $1.2x + 4.3 = 2.1 - x$
6. $4.4m + 6.2 = 8.8m - 1.8$

7. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$
8. $\frac{3}{4}k - 5 = \frac{1}{4}k - 1$
9. $8 - 5p = 4p - 1$

10. $4b - 8 = 10 - 2b$
11. $0.2x - 8 = -2 - x$
12. $3y - 1.8 = 3y - 1.8$

13. $-4 - 3x = 7x - 6$
14. $8 + 4k = -10 + k$
15. $20 - a = 10a - 2$

16. $\frac{2}{3}n + 8 = \frac{1}{2}n + 2$
17. $\frac{2}{5}y - 8 = 9 - \frac{3}{5}y$
18. $-4r + 5 = 5 - 4r$

19. $-4 - 3x = 6x - 6$
20. $18 - 4k = -10 - 4k$
21. $12 + 2y = 10y - 12$
# 2-4 Study Guide and Intervention (continued)

## Solving Equations with the Variable on Each Side

### Grouping Symbols
When solving equations that contain grouping symbols, first use the Distributive Property to eliminate grouping symbols. Then solve.

### Example
Solve $4(2a - 1) = -10(a - 5)$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Step</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4(2a - 1) = -10(a - 5)$</td>
<td>Original equation</td>
<td></td>
</tr>
<tr>
<td>$8a - 4 = -10a + 50$</td>
<td>Distributive Property</td>
<td></td>
</tr>
<tr>
<td>$8a - 4 + 10a = -10a + 50 + 10a$</td>
<td>Add $10a$ to each side.</td>
<td></td>
</tr>
<tr>
<td>$18a - 4 = 50$</td>
<td>Simplify.</td>
<td></td>
</tr>
<tr>
<td>$18a - 4 + 4 = 50 + 4$</td>
<td>Add $4$ to each side.</td>
<td></td>
</tr>
<tr>
<td>$18a = 54$</td>
<td>Simplify.</td>
<td></td>
</tr>
<tr>
<td>$\frac{18a}{18} = \frac{54}{18}$</td>
<td>Divide each side by $18$.</td>
<td></td>
</tr>
<tr>
<td>$a = 3$</td>
<td>Simplify.</td>
<td></td>
</tr>
</tbody>
</table>

The solution is $3$.

### Exercises
Solve each equation. Check your solution.

1. $-3(x + 5) = 3(x - 1)$
2. $2(7 + 3t) = -t$
3. $3(a + 1) - 5 = 3a - 2$

4. $75 - 9g = 5(-4 + 2g)$
5. $5(f + 2) = 2(3 - f)$
6. $4(p + 3) = 36$

7. $18 = 3(2t + 2)$
8. $3(d - 8) = 3d$
9. $5(p + 3) + 9 = 3(p - 2) + 6$

10. $4(b - 2) = 2(5 - b)$
11. $1.2(x - 2) = 2 - x$
12. $\frac{3 + y}{4} = \frac{-y}{8}$

13. $\frac{a - 8}{12} = \frac{2a + 5}{3}$
14. $2(4 + 2k) + 10 = k$
15. $2(w - 1) + 4 = 4(w + 1)$

16. $6(n - 1) = 2(2n + 4)$
17. $2[2 + 3(y - 1)] = 22$
18. $-4(r + 2) = 4(2 - 4r)$

19. $-3(x - 8) = 24$
20. $4(4 - 4k) = -10 - 16k$
21. $6(2 - 2y) = 5(2y - 2)$
Lesson 2-4
Skills Practice

Solving Equations with the Variable on Each Side

Justify each step.

1. \(4k - 3 = 2k + 5\)
   \[4k - 3 - 2k = 2k + 5 - 2k\]
   \[2k - 3 = 5\]
   \[2k - 3 + 3 = 5 + 3\]
   \[2k = 8\]
   \[\frac{2k}{2} = \frac{8}{2}\]
   \[k = 4\]

   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

2. \(2(8u + 2) = 3(2u - 7)\)
   \[16u + 4 = 6u - 21\]
   \[16u + 4 - 6u = 6u - 21 - 6u\]
   \[10u + 4 = -21\]
   \[10u + 4 - 4 = -21 - 4\]
   \[10u = -25\]
   \[\frac{10u}{10} = \frac{-25}{10}\]
   \[u = -2.5\]

   a. 
   b. 
   c. 
   d. 
   e. 
   f. 
   g. 

Solve each equation. Check your solution.

3. \(2m + 12 = 3m - 31\)
4. \(2h - 8 = h + 17\)
5. \(7a - 3 = 3 - 2a\)
6. \(4n - 12 = 12 - 4n\)
7. \(4x - 9 = 7x + 12\)
8. \(-6y - 3 = 3 - 6y\)
9. \(5 + 3r = 5r - 19\)
10. \(-9 + 8k = 7 + 4k\)
11. \(8q + 12 = 4(3 + 2q)\)
12. \(3(5j + 2) = 2(3j - 6)\)
13. \(6(-3v + 1) = 5(-2v - 2)\)
14. \(-7(2b - 4) = 5(-2b + 6)\)
15. \(3(8 - 3t) = 5(2 + t)\)
16. \(2(3u + 7) = -4(3 - 2u)\)
17. \(8(2f - 2) = 7(3f + 2)\)
18. \(5(-6 - 3d) = 3(8 + 7d)\)
19. \(6(w - 1) = 3(3w + 5)\)
20. \(7(-3y + 2) = 8(3y - 2)\)
21. \(\frac{2}{3}v - 6 = 6 - \frac{2}{3}v\)
22. \(\frac{1}{2} - \frac{5}{8}x = \frac{7}{8}x + \frac{7}{2}\)
2-4 Practice

Solving Equations with the Variable on Each Side

Solve each equation. Check your solution.

1. \(5x - 3 = 13 - 3x\)  
   2. \(-4r - 11 = 4r + 21\)

3. \(1 - m = 6 - 6m\)  
   4. \(14 + 5n = -4n + 17\)

5. \(\frac{1}{2}k - 3 = 2 - \frac{3}{4}k\)  
   6. \(\frac{1}{2}(6 - y) = y\)

7. \(3(-2 - 3x) = -9x - 4\)  
   8. \(4(4 - w) = 3(2w + 2)\)

9. \(9(4b - 1) = 2(9b + 3)\)  
   10. \(3(6 + 5y) = 2(-5 + 4y)\)

11. \(-5x - 10 = 2 - (x + 4)\)  
    12. \(6 + 2(3j - 2) = 4(1 + j)\)

13. \(\frac{5}{2}t - t = 3 + \frac{3}{2}t\)  
    14. \(1.4f + 1.1 = 8.3 - f\)

15. \(\frac{2}{3}x - \frac{1}{6} = \frac{1}{2}x + \frac{5}{6}\)  
    16. \(2 - \frac{3}{4}k = \frac{1}{8}k + 9\)

17. \(\frac{1}{2}(3g - 2) = \frac{g}{2}\)  
    18. \(\frac{1}{3}(n + 1) = \frac{1}{6}(3n - 5)\)

19. \(\frac{1}{2}(5 - 2h) = \frac{h}{2}\)  
    20. \(\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)\)

21. \(3(d - 8) - 5 = 9(d + 2) + 1\)  
    22. \(2(a - 8) + 7 = 5(a + 2) - 3a - 19\)

23. **NUMBERS** Two thirds of a number reduced by 11 is equal to 4 more than the number. Find the number.

24. **NUMBERS** Five times the sum of a number and 3 is the same as 3 multiplied by 1 less than twice the number. What is the number?

25. **NUMBER THEORY** Tripling the greater of two consecutive even integers gives the same result as subtracting 10 from the lesser even integer. What are the integers?

26. **GEOMETRY** The formula for the perimeter of a rectangle is \(P = 2\ell + 2w\), where \(\ell\) is the length and \(w\) is the width. A rectangle has a perimeter of 24 inches. Find its dimensions if its length is 3 inches greater than its width.
2-4 Word Problem Practice

Solving Equations with the Variable on Each Side

1. **OLYMPICS** In the 2010 Winter Olympic Games in Vancouver, Canada, the United States athletes won 1 more than 4 times the number of gold metals won by the French athletes. The United States won 7 more gold medals than the French. Solve the equation \(7 + F = 4F + 1\) to find the number of gold metals won by the French athletes.

2. **AGE** Diego’s mother is twice as old as he is. She is also as old as the sum of the ages of Diego and both of his younger twin brothers. The twins are 11 years old. Solve the equation \(2d = d + 11 + 11\) to find the age of Diego.

3. **GEOMETRY** Supplementary angles are angles whose measures have a sum of 180°. Complementary angles are angles whose measures have a sum of 90°. Find the measure of an angle whose supplement is 10° more than twice its complement. Let \(90 - x\) equal the degree measure of its complement and \(180 - x\) equal the degree measure of its supplement. Write and solve an equation.

4. **NATURE** The table shows the current heights and average growth rates of two different species of trees. How long will it take for the two trees to be the same height?

<table>
<thead>
<tr>
<th>Tree Species</th>
<th>Current Height</th>
<th>Annual growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>38 inches</td>
<td>4 inches</td>
</tr>
<tr>
<td>B</td>
<td>45.5 inches</td>
<td>2.5 inches</td>
</tr>
</tbody>
</table>

5. **NUMBER THEORY** Mrs. Simms told her class to find two consecutive even integers such that twice the lesser of two integers is 4 less than two times the greater integer.

   a. Write and solve an equation to find the integers.

   b. Does the equation have one solution, no solutions, or is it an identity? Explain.
Identities

An equation that is true for every value of the variable is called an identity. When you try to solve an identity, you end up with a statement that is always true. Here is an example.

Example

Solve \( 8 - (5 - 6x) = 3(1 + 2x) \).

\[
8 - (5 - 6x) = 3(1 + 2x) \quad \text{Original equation}
\]
\[
8 - 5 - (-6x) = 3(1 + 2x) \quad \text{Distributive Property}
\]
\[
8 - 5 + 6x = 3 + 6x \quad \text{Distributive Property}
\]
\[
3 + 6x = 3 + 6x \quad \text{Add.}
\]

Exercises

State whether each equation is an identity. If it is not, find its solution.

1. \( 2(2 - 3x) = 3(3 + x) + 4 \)
2. \( 5(m + 1) + 6 = 3(4 + m) + (2m - 1) \)

3. \( (5t + 9) - (3t - 13) = 2(11 + t) \)
4. \( 14 - (6 - 3c) = 4c - c \)

5. \( 3y - 2(y + 19) = 9y - 3(9 - y) \)
6. \( 3(3h - 1) = 4(h + 3) \)

7. Use the true equation \( 3x - 2 = 3x - 2 \) to create an identity of your own.

8. Use the false equation \( 1 = 2 \) to create an equation with no solution.

9. Create an equation whose solution is \( x = 3 \).
Graphing Calculator Activity

Solving Linear Equations

One way to solve a linear equation in one variable is to graph each side of the equation as a separate entity. The intersection of these two graphs identifies the \( x \) value for which the equation is true.

**Example 1** Solve \( 5x - 9 = -3x + 7 \).

Enter the left side of the equation into \( Y_1 \) and the right side into the \( Y_2 \).

Keystrokes: \( Y= 5 \text{X,T,\theta,n} - 9 \text{ ENTER} \ (-) 3 \text{X,T,\theta,n} + 7 \)

Graph the equations.

Keystrokes: \( \text{ZOOM} 0 \text{ ZOOM} 8 \text{ ENTER} \)

Use the **TRACE** key to find the intersection.

Notice the coordinates at the bottom of the screen do not change as you toggle up and down. This indicates a common solution, 2. Substitute 2 into the equation to check the answer.

**Example 2** Solve \( \frac{3}{4} a + 16 = 2 - \frac{1}{8} a \).

Enter each side of the equation into \( Y= \) list.

Keystrokes: \( Y= (3 \div 4) \text{X,T,\theta,n} + 16 \text{ ENTER} 2 - (1 \div 8) \text{X,T,\theta,n} \text{GRAPH} \).

**TRACE** to locate the point of intersection at \( x = -16 \). So, the solution to the equation is \( a = -16 \).

**Exercises**

Solve each equation.

1. \( 7(x - 3) = 7 \)  
2. \( 5 - \frac{1}{2}(x - 6) = 4 \)  
3. \( \frac{1}{4}(7 + 3x) = -\frac{x}{8} \)

4. \( 28 - 2.2x = 11.6x + 262.6 \)  
5. \( 1.03x - 4 = -2.15x + 8.72 \)  
6. \( \frac{c + 1}{8} - \frac{2 - c}{3} = \frac{5}{6} \)

7. \( 6x - 2(x - 3) = 4(x + 1) + 4 \)  
8. \( 5(x - 2) + 2x = 7(x + 4) - 38 \)  
9. \( \frac{8}{x} = 2x \)

Not all equations have integer solutions. Investigate using **[CALC]** 5 to solve these equations.

10. \( 2.4(2x + 3) = -0.1(2x + 3) \)  
11. \( 9 - 2(5d + 4) = -3(2d - 7) - 10 \)


**2-5 Study Guide and Intervention**

**Solving Equations Involving Absolute Value**

**Absolute Value Expressions** Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable.

**Example** Evaluate $|t - 5| - 7$ if $t = 3$.

$$
|t - 5| - 7 = |3 - 5| - 7 \quad \text{Replace } t \text{ with } 3.
$$

$$
= |-2| - 7 \quad 3 - 5 = -2
$$

$$
= 2 - 7 \quad |-2| = 2
$$

$$
= -5 \quad \text{Simplify.}
$$

**Exercises**

Evaluate each expression if $r = -2$, $n = -3$, and $t = 3$.

1. $|8 - t| + 3$
2. $|t - 3| - 7$
3. $5 + |3 - n|$

4. $|r + n| - 7$
5. $|n - t| + 4$
6. $-|r + n + t|$

Evaluate each expression if $n = 2$, $q = -1.5$, $r = -3$, $v = -8$, $w = 4.5$, and $x = 4$.

7. $|2q + r|$
8. $10 - |2n + v|$
9. $|3x - 2w| - q$

10. $v - |3n + x|$
11. $1 + |5q - w|$
12. $2 |3r - v|$

13. $| -2x + 5n| + (n - x)$
14. $4w - |2r + v|$
15. $3 |w - n| - 5 |q - r|$
Absolute Value Equations  When solving equations that involve absolute value, there are two cases to consider.

Case 1: The value inside the absolute value symbols is positive.

Case 2: The value inside the absolute value symbols is negative.

**Example 1**  Solve \(|x + 4| = 1\). Then graph the solution set.

Write \(|x + 4| = 1\) as \(x + 4 = 1\) or \(x + 4 = -1\).

\[x + 4 = 1 \quad \text{or} \quad x + 4 = -1\]

\[x = -3 \quad \text{or} \quad x = -5\]

The solution set is \((-5, -3)\).

The graph is shown below.

**Example 2**  Write an equation involving absolute value for the graph.

Find the point that is the same distance from \(-2\) as it is from \(4\).

The distance from 1 to \(-2\) is 3 units. The distance from 1 to 4 is 3 units.

So, \(|x - 1| = 3\).

---

**Exercises**

Solve each equation. Then graph the solution set.

1. \(|y| = 3\)

2. \(|x - 4| = 4\)

3. \(|y + 3| = 2\)

4. \(|b + 2| = 3\)

5. \(|w - 2| = 5\)

6. \(|t + 2| = 4\)

7. \(|2x| = 8\)

8. \(|5y - 2| = 7\)

9. \(|p - 0.2| = 0.5\)

10. \(|d - 100| = 50\)

11. \(|2x - 1| = 11\)

12. \(|3x + \frac{1}{2}| = 6\)

Write an equation involving absolute value for each graph.

13.

14.

15.
2-5 Skills Practice

Solving Equations Involving Absolute Value

Evaluate each expression if \( a = 2 \), \( b = -3 \), and \( c = -4 \).

1. \( |a - 5| - 1 \) 
2. \( |b + 1| + 8 \)
3. \( 5 - |c + 1| \) 
4. \( |a + b| - c \)

Solve each equation. Then graph the solution set.

5. \( |w + 1| = 5 \) 
6. \( |c - 3| = 1 \)
7. \( |n + 2| = 1 \) 
8. \( |t + 6| = 4 \)
9. \( |w - 2| = 2 \) 
10. \( |k - 5| = 4 \)

Write an equation involving absolute value for each graph.

11. 
12. 
13. 
14.
2-5 Practice

Solving Equations Involving Absolute Value

Evaluate each expression if \( x = -1, \ y = 3, \) and \( z = -4. \)

1. \( 16 - |2z + 1| \)
2. \( |x - y| + 4 \)
3. \( | -3y + z| - x \)
4. \( 3|z - x| + |2 - y| \)

Solve each equation. Then graph the solution set.

5. \( |2z - 9| = 1 \)
6. \( |3 - 2r| = 7 \)

7. \( |3t + 6| = 9 \)
8. \( |2g - 5| = 9 \)

Write an equation involving absolute value for each graph.

9. \( \)
10. \( \)

11. \( \)
12. \( \)

13. **FITNESS** Taisha uses the elliptical cross-trainer at the gym. Her general goal is to burn 280 Calories per workout, but she varies by as much as 25 Calories from this amount on any given day. Write and solve an equation to find the maximum and minimum number of Calories Taisha burns on the cross-trainer.

14. **TEMPERATURE** A thermometer is guaranteed to give a temperature no more than 1.2°F from the actual temperature. If the thermometer reads 28°F, write and solve an equation to find the maximum and minimum temperatures it could be.
2-5 Word Problem Practice

Solving Open Sentences Involving Absolute Value

1. ENGINEERING Tolerance in engineering is an allowance made for imperfections in a manufactured object. The manufacturer of an oven specifies a temperature tolerance of \( \pm 15^\circ F \). This means that the temperature inside the oven will be within \( 15^\circ F \) of the temperature that it is set on. Write an absolute value expression to represent the maximum and minimum temperatures inside the oven when the thermostat is set on \( 400^\circ F \).

2. AVIATION The circle graph shows the results of a survey that asked 4300 students ages 7 to 18 what they thought would be the most important benefit of air travel in the future. There are about 40 million students in the United States. If the margin of error is \( \pm 3\% \), what is the range of the number of students ages 7 to 18 who would likely say that “finding new resources for Earth” is the most important benefit of future flight?

3. COLLEGE A certain scholarship and student loan fund uses a formula to determine whether or not a student qualifies for college funding. The formula is \(|3k + 6| = 15\), where \( k \) is a need score determined by an interview. What are the possible need scores?

4. STATISTICS The most familiar statistical measure is the arithmetic mean, or average. A second important statistical measure is the standard deviation, which is a measure of how far the individual scores are from the mean. For example, the mean score on the Wechsler IQ test is 100 and the standard deviation is 15. This means that people within one deviation of the mean have IQ scores that are 15 points higher or lower than the mean.

   a. Write an absolute value equation to find the maximum and minimum test scores within one standard deviation if the mean was 80 and the standard deviation was 12.

   b. What is the range of Wechsler IQ test scores \( \pm 3 \) standard deviations from the mean?
**Precision of Measurement**

The precision of a measurement depends both on your accuracy in measuring and the number of divisions on the ruler you use. Suppose you measured a length of wood to the nearest one-eighth of an inch and got a length of $6\frac{5}{8}$ in.

The drawing shows that the actual measurement lies somewhere between $6\frac{9}{16}$ in. and $6\frac{11}{16}$ in. This measurement can be written using the symbol $\pm$, which is read *plus or minus*.

$$6\frac{5}{8} \pm \frac{1}{16} \text{ in.}$$

In this example, $\frac{1}{16}$ in. is the absolute error. The absolute error is one-half the smallest unit used in a measurement.

**Find the maximum and minimum lengths for each measurement.**

1. $3\frac{1}{2} \pm \frac{1}{4}$ in.
2. $9.78 \pm 0.005$ cm
3. $2.4 \pm 0.05$ g
4. $28 \pm \frac{1}{2}$ ft
5. $15 \pm 0.5$ cm
6. $\frac{11}{16} \pm \frac{1}{64}$ in.

**For each measurement, give the smallest unit used and the absolute error.**

7. The actual measurement is between 12.5 cm and 13.5 cm.

8. The actual measurement is between $12\frac{1}{8}$ in. and $12\frac{3}{8}$ in.

9. The actual measurement is between $56\frac{1}{2}$ in. and $57\frac{1}{2}$ in.

10. The actual measurement is between 23.05 mm and 23.15 mm.
Ratios and Proportions

A ratio is a comparison of two numbers by division. The ratio of \( x \) to \( y \) can be expressed as \( x \) to \( y \), \( x:y \) or \( \frac{x}{y} \). Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a proportion. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1

Determine whether the ratios \( \frac{24}{36} \) and \( \frac{12}{18} \) are equivalent ratios.

Write yes or no. Justify your answer.

\[
\frac{24}{36} = \frac{2}{3} \quad \text{when expressed in simplest form.}
\]

\[
\frac{12}{18} = \frac{2}{3} \quad \text{when expressed in simplest form.}
\]

The ratios \( \frac{24}{36} \) and \( \frac{12}{18} \) form a proportion because they are equal when expressed in simplest form.

Example 2

Use cross products to determine whether \( \frac{10}{18} \) and \( \frac{25}{45} \) form a proportion.

Write the proportion.

\[
\frac{10}{18} = \frac{25}{45}
\]

Cross products

\[
10(45) \neq 18(25)
\]

Simplify.

\[
450 = 450
\]

The cross products are equal, so \( \frac{10}{18} = \frac{25}{45} \).

Since the ratios are equal, they form a proportion.

Exercises

Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. \( \frac{1}{2}, \frac{16}{32} \)
2. \( \frac{5}{8}, \frac{10}{15} \)
3. \( \frac{10}{20}, \frac{25}{49} \)

4. \( \frac{25}{36}, \frac{15}{20} \)
5. \( \frac{12}{32}, \frac{3}{16} \)
6. \( \frac{4}{9}, \frac{12}{27} \)

7. \( \frac{0.1}{2}, \frac{5}{100} \)
8. \( \frac{15}{20}, \frac{9}{12} \)
9. \( \frac{14}{12}, \frac{20}{30} \)

10. \( \frac{2}{3}, \frac{20}{30} \)
11. \( \frac{5}{9}, \frac{25}{45} \)
12. \( \frac{72}{64}, \frac{9}{8} \)

13. \( \frac{5}{5}, \frac{30}{20} \)
14. \( \frac{18}{24}, \frac{50}{75} \)
15. \( \frac{100}{75}, \frac{44}{33} \)

16. \( \frac{0.05}{1}, \frac{1}{20} \)
17. \( \frac{1.5}{2}, \frac{6}{8} \)
18. \( \frac{0.1}{0.2}, \frac{0.45}{0.9} \)
Study Guide and Intervention (continued)

Ratios and Proportions

Solve Proportions  If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion \( \frac{x}{5} = \frac{10}{13} \), \( x \) and 13 are called extremes. They are the first and last terms of the proportion. 5 and 10 are called means. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions  For any numbers \( a, b, c, \) and \( d \), if \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).

Example  Solve \( \frac{x}{5} = \frac{10}{13} \).

\[
\frac{x}{5} = \frac{10}{13} \\
13(x) = 5(10) \\
13x = 50 \\
\frac{13x}{13} = \frac{50}{13} \\
x = 3 \frac{11}{13}
\]

Exercises

Solve each proportion. If necessary, round to the nearest hundredth.

1. \( \frac{-3}{x} = \frac{2}{8} \)  
2. \( \frac{1}{t} = \frac{5}{3} \)  
3. \( \frac{0.1}{2} = \frac{0.5}{x} \)
4. \( \frac{x + 1}{4} = \frac{3}{4} \)  
5. \( \frac{4}{6} = \frac{8}{x} \)  
6. \( \frac{x}{21} = \frac{3}{63} \)
7. \( \frac{9}{y + 1} = \frac{18}{54} \)  
8. \( \frac{3}{d} = \frac{18}{3} \)  
9. \( \frac{5}{8} = \frac{p}{24} \)
10. \( \frac{4}{b - 2} = \frac{4}{12} \)  
11. \( \frac{1.5}{x} = \frac{12}{x} \)  
12. \( \frac{3 + y}{4} = \frac{-y}{8} \)
13. \( \frac{a - 18}{12} = \frac{15}{3} \)  
14. \( \frac{12}{k} = \frac{24}{k} \)  
15. \( \frac{2 + w}{6} = \frac{12}{9} \)

Use a proportion to solve each problem.

16. MODELS  To make a model of the Guadeloupe River bed, Hermie used 1 inch of clay for 5 miles of the river’s actual length. His model river was 50 inches long. How long is the Guadeloupe River?

17. EDUCATION  Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems?
2-6 Skills Practice

Ratios and Proportions

Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. \( \frac{4}{5} = \frac{20}{25} \)

2. \( \frac{5}{9} = \frac{7}{11} \)

3. \( \frac{6}{7} = \frac{24}{28} \)

4. \( \frac{8}{9} = \frac{72}{81} \)

5. \( \frac{7}{16} = \frac{42}{90} \)

6. \( \frac{13}{19} = \frac{26}{38} \)

7. \( \frac{3}{14} = \frac{21}{98} \)

8. \( \frac{12}{17} = \frac{50}{85} \)

Solve each proportion. If necessary, round to the nearest hundredth.

9. \( \frac{1}{a} = \frac{2}{14} \)

10. \( \frac{5}{b} = \frac{3}{9} \)

11. \( \frac{9}{g} = \frac{15}{10} \)

12. \( \frac{3}{a} = \frac{1}{6} \)

13. \( \frac{6}{z} = \frac{3}{5} \)

14. \( \frac{5}{f} = \frac{35}{21} \)

15. \( \frac{12}{7} = \frac{36}{m} \)

16. \( \frac{6}{23} = \frac{y}{69} \)

17. \( \frac{42}{56} = \frac{6}{f} \)

18. \( \frac{7}{b} = \frac{1}{9} \)

19. \( \frac{10}{14} = \frac{30}{m} \)

20. \( \frac{11}{15} = \frac{n}{60} \)

21. \( \frac{9}{c} = \frac{27}{39} \)

22. \( \frac{5}{12} = \frac{20}{g} \)

23. \( \frac{4}{21} = \frac{y}{84} \)

24. \( \frac{22}{x} = \frac{11}{30} \)

25. BOATING Hue’s boat used 5 gallons of gasoline in 4 hours. At this rate, how many gallons of gasoline will the boat use in 10 hours?
2-6 Practice

Ratios and Proportions

Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. \(\frac{7}{6}\) \(\frac{52}{48}\)

2. \(\frac{3}{11}\) \(\frac{15}{66}\)

3. \(\frac{18}{24}\) \(\frac{36}{48}\)

4. \(\frac{12}{11}\) \(\frac{108}{99}\)

5. \(\frac{8}{9}\) \(\frac{72}{81}\)

6. \(\frac{1}{9}\) \(\frac{1}{6}\)

7. \(\frac{3.4}{5.2}\) \(\frac{7.14}{10.92}\)

8. \(\frac{1.5}{9}\) \(\frac{1}{6}\)

Solve each proportion. If necessary, round to the nearest hundredth.

10. \(\frac{5}{a} = \frac{30}{54}\)

11. \(\frac{v}{46} = \frac{34}{23}\)

12. \(\frac{40}{56} = \frac{k}{7}\)

13. \(\frac{28}{49} = \frac{4}{w}\)

14. \(\frac{3}{u} = \frac{27}{162}\)

15. \(\frac{y}{3} = \frac{48}{9}\)

16. \(\frac{2}{y} = \frac{10}{60}\)

17. \(\frac{5}{11} = \frac{35}{x}\)

18. \(\frac{3}{51} = \frac{z}{17}\)

19. \(\frac{6}{61} = \frac{12}{h}\)

20. \(\frac{g}{16} = \frac{6}{4}\)

21. \(\frac{14}{49} = \frac{2}{a}\)

22. \(\frac{7}{9} = \frac{8}{t}\)

23. \(\frac{3}{q} = \frac{5}{6}\)

24. \(\frac{m}{6} = \frac{5}{8}\)

25. \(\frac{v}{0.23} = \frac{7}{1.61}\)

26. \(\frac{3}{0.72} = \frac{12}{b}\)

27. \(\frac{6}{n} = \frac{3}{0.51}\)

28. \(\frac{7}{a - 4} = \frac{14}{6}\)

29. \(\frac{3}{12} = \frac{2}{y + 6}\)

30. \(\frac{m - 1}{8} = \frac{2}{4}\)

31. \(\frac{5}{12} = \frac{x + 1}{4}\)

32. \(\frac{r + 2}{7} = \frac{5}{7}\)

33. \(\frac{3}{7} = \frac{x - 2}{6}\)

34. PAINTING Ysidra paints a room that has 400 square feet of wall space in \(2\frac{1}{2}\) hours. At this rate, how long will it take her to paint a room that has 720 square feet of wall space?

35. VACATION PLANS Walker is planning a summer vacation. He wants to visit Petrified National Forest and Meteor Crater, Arizona, the 50,000-year-old impact site of a large meteor. On a map with a scale where 2 inches equals 75 miles, the two areas are about \(1\frac{1}{2}\) inches apart. What is the distance between Petrified National Forest and Meteor Crater?
1. WATER A dripping faucet wastes 3 cups of water every 24 hours. How much water is wasted in a week?

2. GASOLINE In November 2010 the average price of 5 gallons of regular unleaded gasoline in the United States was $14.46. What was the price for 16 gallons of gas?

3. SHOPPING Stevenson’s Market is selling 3 packs of toothpicks for $0.87. How much will 10 packs of toothpicks cost at this price? Round your answer to the nearest cent.

4. BUILDINGS Willis Tower in Chicago is 1450 feet tall. The John Hancock Center in Chicago is 1127 feet tall. Suppose you are asked to build a small-scale replica of each. If you make the Willis Tower 3 meters tall, what would be the approximate height of the John Hancock replica? Round your answer to the nearest hundredth.

5. MAPS A map of Waco, Texas and neighboring towns is shown below.

   a. Use a metric ruler to measure the distances between Robinson and Neale on the map.

   b. Using the scale of the map, find the approximate actual distance by air (not by roads), between Robinson and Neale.

   c. Approximately how many square miles are shown on this map?
Angles of a Triangle
In geometry, many statements about physical space are proven to be true. Such statements are called **theorems**. Here are two examples of geometric theorems.

a. The sum of the measures of the angles of a triangle is 180°.

b. If two sides of a triangle have equal measure, then the two angles opposite those sides also have equal measure.

For each of the triangles, write an equation and then solve for \( x \). (A tick mark on two or more sides of a triangle indicates that the sides have equal measure.)

1. \( 70° \)
   \( x° \)
   \( 50° \)

2. \( x° \)
   \( 90° \)
   \( 45° \)

3. \( 90° \)
   \( x° \)

4. \( (x + 30)° \)
   \( (5x + 10)° \)
   \( x° \)

5. \( 5x° \)
   \( 2x° \)
   \( x° \)

6. \( 4x° \)
   \( 5x° \)
   \( 90° \)

7. \( (x - 15)° \)
   \( (x + 30)° \)

8. \( x° \)
   \( 40° \)

9. \( x° \)

10. \( 30° \)
    \( 2x° \)
    \( x° \)

11. Two angles of a triangle have the same measure. The sum of the measures of these angles is one-half the measure of the third angle. Find the measures of the angles of the triangle.

12. The measure of one angle of a triangle is twice the measure of a second angle. The measure of the third angle is 12 less than the sum of the other two. Find the measures of the angles of the triangle.
Percent of Change

When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

**Example 1**

Find the percent of increase.

**original**: 48  
**new**: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

\[
\frac{12}{48} = \frac{r}{100} \\
(12 \times 100) = 48r \\
1200 = 48r \\
\frac{1200}{48} = \frac{48r}{48} \\
25 = r
\]

The percent of increase is 25%.

**Example 2**

Find the percent of decrease.

**original**: 30  
**new**: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

\[
\frac{8}{30} = \frac{r}{100} \\
(8 \times 100) = 30r \\
800 = 30r \\
\frac{800}{30} = \frac{30r}{30} \\
26 \frac{2}{3} = r \\
\]

The percent of decrease is $26 \frac{2}{3} \%$, or about 27%.

**Exercises**

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. **original**: 50  
   **new**: 80
2. **original**: 90  
   **new**: 100
3. **original**: 45  
   **new**: 20
4. **original**: 77.5  
   **new**: 62
5. **original**: 140  
   **new**: 150
6. **original**: 135  
   **new**: 90
7. **original**: 120  
   **new**: 180
8. **original**: 90  
   **new**: 270
9. **original**: 27.5  
   **new**: 25
10. **original**: 84  
    **new**: 98
11. **original**: 12.5  
    **new**: 10
12. **original**: 250  
    **new**: 500
Lesson 2-7

Study Guide and Intervention (continued)

Percent of Change

Solve Problems  Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is an amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example  SALES A coat is on sale for 25% off the original price. If the original price of the coat is $75, what is the discounted price?

The discount is 25% of the original price.

\[ 25\% \text{ of } 75 = 0.25 \times 75 \quad 25\% = 0.25 \]

\[ = 18.75 \quad \text{Use a calculator.} \]

Subtract $18.75 from the original price.

\[ 75 - 18.75 = 56.25 \]

The discounted price of the coat is $56.25.

Exercises

Find the total price of each item.

1. Shirt: $24.00  
   Sales tax: 4%  
2. CD player: $142.00  
   Sales tax: 5.5%  
3. Celebrity calendar: $10.95  
   Sales tax: 7.5%

Find the discounted price of each item.

4. Compact disc: $16  
   Discount: 15%  
5. Two concert tickets: $28  
   Student discount: 28%  
6. Airline ticket: $248.00  
   Superair discount: 33%

7. VIDEOS  The original selling price of a new sports video was $65.00. Due to the demand the price was increased to $87.75. What was the percent of increase over the original price?

8. SCHOOL  A high school paper increased its sales by 75% when it ran an issue featuring a contest to win a class party. Before the contest issue, 10% of the school’s 800 students bought the paper. How many students bought the contest issue?

9. BASEBALL  Baseball tickets cost $15 for general admission or $20 for box seats. The sales tax on each ticket is 8%. What is the final cost of each type of ticket?
2-7 Skills Practice

Percent of Change

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 25
   new: 10
2. original: 50
   new: 75

3. original: 55
   new: 50
4. original: 25
   new: 28

5. original: 50
   new: 30
6. original: 90
   new: 95

7. original: 48
   new: 60
8. original: 60
   new: 45

Find the total price of each item.

9. dress: $69.00
   tax: 5%
10. binder: $14.50
    tax: 7%

11. hardcover book: $28.95
    tax: 6%
12. groceries: $47.52
    tax: 3%

13. filler paper: $6.00
    tax: 6.5%
14. shoes: $65.00
    tax: 4%

15. basketball: $17.00
    tax: 6%
16. concert tickets: $48.00
    tax: 7.5%

Find the discounted price of each item.

17. backpack: $56.25
    discount: 20%
18. monitor: $150.00
    discount: 50%

19. CD: $15.99
    discount: 20%
20. shirt: $25.50
    discount: 40%

21. sleeping bag: $125
    discount: 25%
22. coffee maker: $102.00
    discount: 45%
2-7 Practice

Percent of Change

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 18
   new: 10
2. original: 140
   new: 160
3. original: 200
   new: 320

4. original: 10
   new: 25
5. original: 76
   new: 60
6. original: 128
   new: 120

7. original: 15
   new: 35.5
8. original: 98.6
   new: 64
9. original: 58.8
   new: 65.7

Find the total price of each item.

10. concrete blocks: $95.00
tax: 6%
11. crib: $240.00
tax: 6.5%
12. jacket: $125.00
tax: 5.5%

13. class ring: $325.00
tax: 6%
14. blanket: $24.99
tax: 7%
15. kite: $18.90
tax: 5%

Find the discounted price of each item.

16. dry cleaning: $25.00
discount: 15%
17. computer game: $49.99
discount: 25%
18. luggage: $185.00
discount: 30%

19. stationery: $12.95
discount: 10%
20. prescription glasses: $149
discount: 20%
21. pair of shorts: $24.99
discount: 45%

Find the final price of each item.

22. television: $375.00
discount: 25%
tax: 6%
23. DVD player: $269.00
discount: 20%
tax: 7%
24. printer: $255.00
discount: 30%
tax: 5.5%

25. INVESTMENTS The price per share of a stock decreased from $90 per share to $36 per share. By what percent did the price of the stock decrease?

26. HEATING COSTS Customers of a utility company received notices in their monthly bills that heating costs for the average customer had increased 125% over last year because of an unusually severe winter. In January of last year, the Garcia’s paid $120 for heating. What should they expect to pay this January if their bill increased by 125%?
1. **SPORTS** A regulation girls’ fast pitch softball diamond has bases that are 60 feet apart. A regulation professional baseball diamond has bases that are 50% farther apart. Label the distance between the bases on the regulation baseball diamond diagram.

2. **SALES TAX** Olivia purchases a DVD movie priced at $21.99. The sales tax is 6.5%. What is the total price of the movie, including tax?

3. **EDUCATION** The ACT is a college entrance exam taken by high school students. The maximum score that can be earned is 36. The average score in the United States was 21.2 during a recent year. The average score for Vermont students that year was 7.5% higher than the national average. What was the average ACT score for Vermont students? Round your answer to the nearest tenth.

4. **CARS** Mr. Thompson plans to purchase a used car priced at $8400. He will receive a 15% employee discount and then will have to pay a 5.5% sales tax. What will be the final price of the car?

5. **MUSIC** The table below shows the total number of CDs, downloaded singles, and music videos sold in 2004, 2006, and 2008.

| Sales of Recorded Music and Music Videos (millions of units) |
|-------------------|---------|---------|---------|
| Format            | 2004    | 2006    | 2008    |
| CD                | 767.0   | 614.9   | 368.4   |
| Downloaded        | 139.4   | 586.4   | 1042.7  |
| Video             | 32.8    | 23.1    | 20.8    |

Source: Recording Industry Association of America

a. Find the percent of change in the number of units sold between 2004 and 2006 and between 2006 and 2008 for each format. Round your answers to the nearest tenth.

b. Tell whether each percent of change in part a is a percent of increase or a percent of decrease.

c. Did these trends change from 2004 to 2008? Explain.
Using Percent

Use what you have learned about percent to solve each problem.

A TV movie had a “rating” of 15 and a 25 “share.” The rating is the percentage of the nation’s total TV households that were tuned in to this show. The share is the percentage of homes with TVs turned on that were tuned to the movie. How many TV households had their TVs turned off at this time?

To find out, let $T$ = the number of TV households and $x$ = the number of TV households with the TV off.

Then $T - x$ = the number of TV households with the TV on.

Since 0.15$T$ and 0.25($T - x$) both represent the number of households tuned to the movie,

$$0.15T = 0.25(T - x)$$

$$0.15T = 0.25T - 0.25x.$$  

Solve for $x$.  

$$0.25x = 0.10T$$  

$$x = \frac{0.10T}{0.25} = 0.40T.$$  

Forty percent of the TV households had their TVs off when the movie was aired.

Answer each question.

1. During that same week, a sports broadcast had a rating of 22.1 and a 43 share. Show that the percent of TV households with their TVs off was about 48.6%.

2. Find the percent of TV households with their TVs turned off during a show with a rating of 18.9 and a 29 share.

3. Show that if $T$ is the number of TV households, $r$ is the rating, and $s$ is the share, then the number of TV households with the TV off is $\frac{(s - r)T}{s}$.

4. If the fraction of TV households with no TV on is $\frac{s - r}{s}$ then show that the fraction of TV households with TVs on is $\frac{r}{s}$.

5. Find the percent of TV households with TVs on during the most watched serial program in history: the last episode of M*A*S*H, which had a 60.3 rating and a 77 share.

6. A local station now has a 2 share. Each share is worth $50,000 in advertising revenue per month. The station is thinking of going commercial free for the three months of summer to gain more listeners. What would its new share have to be for the last 4 months of the year to make more money for the year than it would have made had it not gone commercial free?
2-7 Spreadsheet Activity

Discounts

Electronics Warehouse is having a software clearance sale. The manager is making a discounted price list to distribute to customers as they shop. Different tables will have software that is discounted by a different percent. Use a spreadsheet to show the discounted price of software that is originally priced $5.99, $7.99, $9.99, $15.99, $19.99, $23.99, $27.99, $29.99, $33.99, and $35.99. Compute discounted prices for software that is 10%, 15%, 25%, and 50% off.

If an item is on sale for 10% off, that means that discounted price is \((1 - 0.1)\) or 0.9 times the original price. Use this pattern to complete the spreadsheet.

**Step 1** Use Column A of the spreadsheet for the original prices.

**Step 2** Columns B through E contain the formulas for the discounted prices.

**Step 3** Because these are prices, the numbers must be rounded to 2 digits. You can program the spreadsheet to list the information as dollars and cents by choosing **currency** from the number menu when you format the cells.

<table>
<thead>
<tr>
<th>Discounts.xls</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>10 % off</td>
<td>15 % off</td>
<td>25 % off</td>
<td>50 % off</td>
</tr>
<tr>
<td>2</td>
<td>$5.99</td>
<td>$5.39</td>
<td>$5.09</td>
<td>$4.49</td>
<td>$3.00</td>
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<td>$8.79</td>
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<td>$4.00</td>
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<td>$32.39</td>
<td>$30.59</td>
<td>$26.99</td>
<td>$18.00</td>
</tr>
</tbody>
</table>

**Exercises**

1. For the second week of the sale, the Electronics Warehouse manager is changing the discounts to 20%, 30%, 45%, and 75% off. Use a spreadsheet to create a new discount price list for the software.

2. How should the spreadsheet be altered if the manager wished to show the final prices of the software including the 6% sales tax for their area?
**Lesson 2-8 Study Guide and Intervention**

**Literal Equations and Dimensional Analysis**

**Solve for Variables** Sometimes you may want to solve an equation such as \( V = \ell wh \) for one of its variables. For example, if you know the values of \( V, w, \) and \( h, \) then the equation \( \ell = \frac{V}{wh} \) is more useful for finding the value of \( \ell. \) If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

**Example 1** Solve \( 2x - 4y = 8, \) for \( y. \)

\[
\begin{align*}
2x - 4y &= 8 \\
2x - 4y - 2x &= 8 - 2x \\
-4y &= 8 - 2x \\
\frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\
y &= \frac{8 - 2x}{-4} \quad \text{or} \quad y = \frac{2x - 8}{4}
\end{align*}
\]

The value of \( y \) is \( \frac{2x - 8}{4}. \)

**Example 2** Solve \( 3m - n = km - 8, \) for \( m. \)

\[
\begin{align*}
3m - n &= km - 8 \\
3m - n - km &= km - 8 - km \\
3m - n - km &= -8 \\
3m - n - km + n &= -8 + n \\
m(3 - k) &= -8 + n \\
\frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \\
m &= \frac{-8 + n}{3 - k} \quad \text{or} \quad \frac{n - 8}{3 - k}
\end{align*}
\]

The value of \( m \) is \( \frac{n - 8}{3 - k}. \) Since division by 0 is undefined, \( 3 - k \neq 0, \) or \( k \neq 3. \)

**Exercises**

Solve each equation or formula for the variable indicated.

1. \( ax - b = c, \) for \( x \)
2. \( 15x + 1 = y, \) for \( x \)
3. \( (x + f) + 2 = j, \) for \( x \)
4. \( xy + w = 9, \) for \( y \)
5. \( x(4 - k) = p, \) for \( k \)
6. \( 7x + 3y = m, \) for \( y \)
7. \( 4(r + 3) = t, \) for \( r \)
8. \( 2x + b = w, \) for \( x \)
9. \( x(1 + y) = z, \) for \( x \)
10. \( 16w + 4x = y, \) for \( x \)
11. \( d = rt, \) for \( r \)
12. \( A = \frac{h(a + b)}{2}, \) for \( h \)
13. \( C = \frac{5}{9}(F - 32), \) for \( F \)
14. \( P = 2\ell + 2w, \) for \( w \)
15. \( A = \ell w, \) for \( \ell \)
Use Formulas Many real-world problems require the use of formulas. Sometimes solving a formula for a specified variable will help solve the problem.

**Example** The formula \( C = \pi d \) represents the circumference of a circle, or the distance around the circle, where \( d \) is the diameter. If an airplane could fly around Earth at the equator without stopping, it would have traveled about 24,900 miles. Find the diameter of Earth.

\[
C = \pi d \\
\text{Given formula} \\
\]
\[
d = \frac{C}{\pi} \text{ Solve for } d. \\
\]
\[
d = \frac{24,900}{3.14} \text{ Use } \pi = 3.14. \\
\]
\[
d \approx 7930 \text{ Simplify.} \\
\]

The diameter of Earth is about 7930 miles.

**Exercises**

1. **GEOMETRY** The volume of a cylinder \( V \) is given by the formula \( V = \pi r^2 h \), where \( r \) is the radius and \( h \) is the height.
   
   a. Solve the formula for \( h \).
   
   b. Find the height of a cylinder with volume \( 2500\pi \) cubic feet and radius 10 feet.

2. **WATER PRESSURE** The water pressure on a submerged object is given by \( P = 64d \), where \( P \) is the pressure in pounds per square foot, and \( d \) is the depth of the object in feet.
   
   a. Solve the formula for \( d \).
   
   b. Find the depth of a submerged object if the pressure is 672 pounds per square foot.

3. **GRAPHS** The equation of a line containing the points \((a, 0)\) and \((0, b)\) is given by the formula \( \frac{x}{a} + \frac{y}{b} = 1 \).
   
   a. Solve the equation for \( y \).
   
   b. Suppose the line contains the points \((4, 0)\), and \((0, -2)\). If \( x = 3 \), find \( y \).

4. **GEOMETRY** The surface area of a rectangular solid is given by the formula \( x = 2lw + 2lh + 2wh \), where \( l \) = length, \( w \) = width, and \( h \) = height.
   
   a. Solve the formula for \( h \).
   
   b. The surface area of a rectangular solid with length 6 centimeters and width 3 centimeters is 72 square centimeters. Find the height.
Solve each equation or formula for the variable indicated.

1. \(7t = x\), for \(t\)

2. \(r = wp\), for \(p\)

3. \(q - r = r\), for \(r\)

4. \(4m - t = m\), for \(m\)

5. \(7a - b = 15a\), for \(a\)

6. \(-5c + d = 2c\), for \(c\)

7. \(x - 2y = 1\), for \(y\)

8. \(d + 3n = 1\), for \(n\)

9. \(7f + g = 5\), for \(f\)

10. \(ax - c = b\), for \(x\)

11. \(rt - 2n = y\), for \(t\)

12. \(bc + 3g = 2k\), for \(c\)

13. \(kn + 4f = 9v\), for \(n\)

14. \(8c + 6j = 5p\), for \(c\)

15. \(\frac{x - c}{2} = d\), for \(x\)

16. \(\frac{x - c}{2} = d\), for \(c\)

17. \(\frac{p + 9}{5} = r\), for \(p\)

18. \(\frac{b - 4z}{7} = a\), for \(b\)

19. The volume of a box \(V\) is given by the formula \(V = \ell wh\), where \(\ell\) is the length, \(w\) is the width, and \(h\) is the height.
   a. Solve the formula for \(h\).
   b. What is the height of a box with a volume of 50 cubic meters, length of 10 meters, and width of 2 meters?

20. Trent purchases 44 euros worth of souvenirs while on vacation in France.
   If \$1 U.S. = 0.678 euros, find the cost of the souvenirs in United States dollars.
   Round to the nearest cent.
Literal Equations and Dimensional Analysis

Solve each equation or formula for the variable indicated.

1. \(d = rt\), for \(r\)  
2. \(6w - y = 2z\), for \(w\)

3. \(mx + 4y = 3t\), for \(x\)  
4. \(9s - 5g = -4u\), for \(s\)

5. \(ab + 3c = 2x\), for \(b\)  
6. \(2p = kx - t\), for \(x\)

7. \(\frac{2}{3}m + a = a + r\), for \(m\)  
8. \(\frac{2}{5}h + g = d\), for \(h\)

9. \(\frac{2}{3}y + v = x\), for \(y\)  
10. \(\frac{3}{4}a - q = k\), for \(a\)

11. \(\frac{rx + 9}{5} = h\), for \(x\)  
12. \(\frac{3b - 4}{2} = c\), for \(b\)

13. \(2w - y = 7w - 2\), for \(w\)  
14. \(3\ell + y = 5 + 5\ell\), for \(\ell\)

15. ELECTRICITY The formula for Ohm’s Law is \(E = IR\), where \(E\) represents voltage measured in volts, \(I\) represents current measured in amperes, and \(R\) represents resistance measured in ohms.

   a. Solve the formula for \(R\).

   b. Suppose a current of 0.25 ampere flows through a resistor connected to a 12-volt battery. What is the resistance in the circuit?

16. MOTION In uniform circular motion, the speed \(v\) of a point on the edge of a spinning disk is \(v = \frac{2\pi}{t}r\), where \(r\) is the radius of the disk and \(t\) is the time it takes the point to travel once around the circle.

   a. Solve the formula for \(r\).

   b. Suppose a merry-go-round is spinning once every 3 seconds. If a point on the outside edge has a speed of 12.56 feet per second, what is the radius of the merry-go-round? (Use 3.14 for \(\pi\).)

17. HIGHWAYS Interstate 90 is the longest interstate highway in the United States, connecting the cities of Seattle, Washington and Boston, Massachusetts. The interstate is 4,987,000 meters in length. If 1 mile = 1.609 kilometers, how many miles long is Interstate 90?
1. **INTEREST** Simple interest that you may earn on money in a savings account can be calculated with the formula $I = prt$. $I$ is the amount of interest earned, $p$ is the principal or initial amount invested, $r$ is the interest rate, and $t$ is the amount of time the money is invested for. Solve the formula for $p$.

2. **DISTANCE** The distance $d$ a car can travel is found by multiplying its rate of speed $r$ by the amount of time $t$ that it took to travel the distance. If a car has already traveled 5 miles, the total distance $d$ is found by the formula $d = rt + 5$. Solve the formula for $r$.

3. **ENVIRONMENT** The United States released 5.877 billion metric tons of carbon dioxide into the environment through the burning of fossil fuels in a recent year. If 1 trillion pounds = 0.4536 billion metric tons, how many trillions of pounds of carbon dioxide did the United States release in that year?

4. **PHYSICS** The pressure exerted on an object is calculated by the formula $P = \frac{F}{A}$, where $P$ is the pressure, $F$ is the force, and $A$ is the surface area of the object. Water shooting from a hose has a pressure of 75 pounds per square inch (psi). Suppose the surface area covered by the direct hose spray is 0.442 square inch. Solve the equation for $F$ and find the force of the spray.

5. **GEOMETRY** The regular octagon is divided into 8 congruent triangles. Each triangle has an area of 21.7 square centimeters. The perimeter of the octagon is 48 centimeters.
   a. What is the length of each side of the octagon?
   b. Solve the area of a triangle formula for $h$.
   c. What is the height of each triangle? Round to the nearest tenth.
Compound Interest

In most banks, interest on savings accounts is compounded at set time periods such as three or six months. At the end of each period, the bank adds the interest earned to the account. During the next period, the bank pays interest on all the money in the bank, including interest. In this way, the account earns interest on interest.

Suppose Ms. Tanner has $1000 in an account that is compounded quarterly at 5%. Find the balance after the first two quarters.

Use $I = prt$ to find the interest earned in the first quarter if $p = 1000$ and $r = 5\%$. Why is $t$ equal to $\frac{1}{4}$?

First quarter: $I = 1000 \times 0.05 \times \frac{1}{4}$

$I = 12.50$

The interest, $12.50, earned in the first quarter is added to $1000$. The principal becomes $1012.50$.

Second quarter: $I = 1012.50 \times 0.05 \times \frac{1}{4}$

$I = 12.65625$

The interest in the second quarter is $12.66$.

The balance after two quarters is $1012.50 + 12.66$ or $1025.16$.

**Answer each of the following questions.**

1. How much interest is earned in the third quarter of Ms. Tanner's account?

2. What is the balance in her account after three quarters?

3. How much interest is earned in the fourth quarter?

4. What is the balance in her account after one year?

5. Suppose Ms. Tanner’s account is compounded semiannually. What is the balance at the end of six months?

6. What is the balance after one year if her account is compounded semiannually?
Mixture Problems  Mixture Problems are problems where two or more parts are combined into a whole. They involve weighted averages. In a mixture problem, the weight is usually a price or a percent of something.

| Weighted Average | The weighted average $M$ of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights. |

Example  COOKIES  Delectable Cookie Company sells chocolate chip cookies for $6.95 per pound and white chocolate cookies for $5.95 per pound. How many pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies to obtain a mixture that sells for $6.75 per pound.

Let $w = $ the number of pounds of chocolate chip cookies

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Price per Pound</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate Chip</td>
<td>$w$</td>
<td>6.95</td>
</tr>
<tr>
<td>White Chocolate</td>
<td>4</td>
<td>5.95</td>
</tr>
<tr>
<td>Mixture</td>
<td>$w + 4$</td>
<td>6.75</td>
</tr>
</tbody>
</table>

Equation: $6.95w + 4(5.95) = 6.75(w + 4)$

Solve the equation.

$6.95w + 23.80 = 6.75w + 27$

$6.95w + 23.80 - 6.75w = 6.75w + 27 - 6.75w$

$0.2w + 23.80 = 27$

$0.2w = 3.2$

$w = 16$

16 pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies.

Exercises

1. SOLUTIONS  How many grams of sugar must be added to 60 grams of a solution that is 32% sugar to obtain a solution that is 50% sugar?

2. NUTS  The Quik Mart has two kinds of nuts. Pecans sell for $1.55 per pound and walnuts sell for $1.95 per pound. How many pounds of walnuts must be added to 15 pounds of pecans to make a mixture that sells for $1.75 per pound?

3. INVESTMENTS  Alice Gleason invested a portion of $32,000 at 9% interest and the balance at 11% interest. How much did she invest at each rate if her total income from both investments was $3200.

4. MILK  Whole milk is 4% butterfat. How much skim milk with 0% butterfat should be added to 32 ounces of whole milk to obtain a mixture that is 2.5% butterfat?
2-9 Study Guide and Intervention (continued)

Weighted Averages

Uniform Motion Problems

Motion problems are another application of weighted averages. Uniform motion problems are problems where an object moves at a certain speed, or rate. Use the formula \(d = rt\) to solve these problems, where \(d\) is the distance, \(r\) is the rate, and \(t\) is the time.

**Example**

**DRIVING** Bill Gutierrez drove at a speed of 65 miles per hour on an expressway for 2 hours. He then drove for 1.5 hours at a speed of 45 miles per hour on a state highway. What was his average speed?

\[
M = \frac{65 \cdot 2 + 45 \cdot 1.5}{2 + 1.5}
\]

\[= \frac{130 + 67.5}{3.5}
\]

\[= \frac{197.5}{3.5}
\]

\[\approx 56.4
\]

Bill drove at an average speed of about 56.4 miles per hour.

**Exercises**

1. **TRAVEL** Mr. Anders and Ms. Rich each drove home from a business meeting. Mr. Anders traveled east at 100 kilometers per hour and Ms. Rich traveled west at 80 kilometers per hour. In how many hours were they 100 kilometers apart?

2. **AIRPLANES** An airplane flies 750 miles due west in \(1 \frac{1}{2}\) hours and 750 miles due south in 2 hours. What is the average speed of the airplane?

3. **TRACK** Sprinter A runs 100 meters in 15 seconds, while sprinter B starts 1.5 seconds later and runs 100 meters in 14 seconds. If each of them runs at a constant rate, who is further in 10 seconds after the start of the race? Explain.

4. **TRAINS** An express train travels 90 kilometers per hour from Smallville to Megatown. A local train takes 2.5 hours longer to travel the same distance at 50 kilometers per hour. How far apart are Smallville and Megatown?

5. **CYCLING** Two cyclists begin traveling in the same direction on the same bike path. One travels at 15 miles per hour, and the other travels at 12 miles per hour. When will the cyclists be 10 miles apart?

6. **TRAINS** Two trains leave Chicago, one traveling east at 30 miles per hour and one traveling west at 40 miles per hour. When will the trains be 210 miles apart?
2-9 Skills Practice

Weighted Averages

1. SEASONING A health food store sells seasoning blends in bulk. One blend contains 20% basil. Sheila wants to add pure basil to some 20% blend to make 16 ounces of her own 30% blend. Let \( b \) represent the amount of basil Sheila should add to the 20% blend.

   a. Complete the table representing the problem.

<table>
<thead>
<tr>
<th>Ounces</th>
<th>Amount of Basil</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Basil Blend</td>
<td></td>
</tr>
<tr>
<td>100% Basil</td>
<td></td>
</tr>
<tr>
<td>30% Basil Blend</td>
<td></td>
</tr>
</tbody>
</table>

   b. Write an equation to represent the problem.

   c. How many ounces of basil should Sheila use to make the 30% blend?

   d. How many ounces of the 20% blend should she use?

2. HIKING At 7:00 A.M., two groups of hikers begin 21 miles apart and head toward each other. The first group, hiking at an average rate of 1.5 miles per hour, carries tents, sleeping bags, and cooking equipment. The second group, hiking at an average rate of 2 miles per hour, carries food and water. Let \( t \) represent the hiking time.

   a. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First group of hikers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second group of hikers</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Write an equation using \( t \) that describes the distances traveled.

   c. How long will it be until the two groups of hikers meet?

3. SALES Sergio sells a mixture of Virginia peanuts and Spanish peanuts for $3.40 per pound. To make the mixture, he uses Virginia peanuts that cost $3.50 per pound and Spanish peanuts that cost $3.00 per pound. He mixes 10 pounds at a time.

   a. How many pounds of Virginia peanuts does Sergio use?

   b. How many pounds of Spanish peanuts does Sergio use?
1. **GRASS SEED** A nursery sells Kentucky Blue Grass seed for $5.75 per pound and Tall Fescue seed for $4.50 per pound. The nursery sells a mixture of the two kinds of seed for $5.25 per pound. Let $k$ represent the amount of Kentucky Blue Grass seed the nursery uses in 5 pounds of the mixture.

   a. Complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Price per Pound</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kentucky Blue Grass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tall Fescue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixture</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Write an equation to represent the problem.

   c. How much Kentucky Blue Grass does the nursery use in 5 pounds of the mixture?

   d. How much Tall Fescue does the nursery use in 5 pounds of the mixture?

2. **TRAVEL** Two commuter trains carry passengers between two cities, one traveling east, and the other west, on different tracks. Their respective stations are 150 miles apart. Both trains leave at the same time, one traveling at an average speed of 55 miles per hour and the other at an average speed of 65 miles per hour. Let $t$ represent the time until the trains pass each other.

   a. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$t$</th>
<th>$d = rt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Train</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second Train</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Write an equation using $t$ that describes the distances traveled.

   c. How long after departing will the trains pass each other?

3. **TRAVEL** Two trains leave Raleigh at the same time, one traveling north, and the other south. The first train travels at 50 miles per hour and the second at 60 miles per hour. In how many hours will the trains be 275 miles apart?

4. **JUICE** A pineapple drink contains 15% pineapple juice. How much pure pineapple juice should be added to 8 quarts of the pineapple drink to obtain a mixture containing 50% pineapple juice?
2-9 Word Problem Practice

Weighted Averages

1. DRIVING The drive from New York City to Boston is about 240 miles. It took Samir 5 hours to drive one way, but due to severe weather it took him 6.5 hours for the return trip. What was his average speed round trip? Round the answer to the nearest hundredth.

2. GRADES In math classes at Gorbine High School, all tests are given double the weight of a quiz or homework score when calculating grades. Use Donna’s grade record sheet to determine her average grade so far this term.

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework chapter 1</td>
<td>95</td>
</tr>
<tr>
<td>Quiz chapter 1</td>
<td>80</td>
</tr>
<tr>
<td>Test chapter 1</td>
<td>92</td>
</tr>
<tr>
<td>Homework chapter 2</td>
<td>93</td>
</tr>
<tr>
<td>Quiz chapter 2</td>
<td>96</td>
</tr>
<tr>
<td>Test chapter 2</td>
<td>81</td>
</tr>
</tbody>
</table>

3. MIXTURE Keith wants to create a drink that is 40% juice. How much of a 10% juice solution should he add to 100 milliliters of 100% grape juice to obtain the 40% mixture?

4. BUSINESS Mrs. Winship sells chocolate fudge for $7.50 per pound and peanut butter fudge for $7.00 per pound. The total number of pounds sold on Saturday was 146 and the total amount of money collected was $1065. How many pounds of each type of fudge were sold?

5. TRAINS Two trains are 5000 feet apart, heading toward each other, but on separate parallel straight tracks, Train A is traveling at 45 miles per hour and Train B is traveling at 24 miles per hour.

   a. Change 45 miles per hour and 24 miles per hour into feet per second.

   b. About how far will each train travel before they meet? Round your answers to the nearest hundredth.

   c. In how many seconds will Train A and Train B meet?
Expected Value

Expected value is the average return of an event over repeated trial. Expected value is also a form of weighted average and can be used to determine whether or not you should play a game based on what the expected value of your winnings is. It can also be used to determine the expected length of a game.

Example

You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a one, you get 12 candy bars. If you roll a two or a three, you get 3 candy bars. If you roll a four, five or six, you get 2 candy bars.

Since there are six possibilities for what you can roll:

- \( \frac{1}{6} \) of the time, you will win 12 candy bars.
- \( \frac{2}{6} \) or \( \frac{1}{3} \) of the time, you will win 3 candy bars.
- \( \frac{3}{6} \) or \( \frac{1}{2} \) of the time, you will win 2 candy bars.

The expected value is \( \frac{1}{6}(12) + \frac{1}{3}(3) + \frac{1}{2}(2) \) or 4 candy bars.

Therefore, if you played the game 100 times, you could expect to win about 4 candy bars each time.

Exercises

Find the expected value of each of the following events.

1. You are flipping a coin. If you flip heads, you win $5.00. If you flip tails, you win $1.00.

2. You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a 1, you get 18 candy bars. If you roll a 2 or a 3, you get 6 candy bars. If you roll a 4, 5 or 6, you get 0 candy bars.

3. You are rolling a die to determine the amount of money you will win at a school fair. If you roll a 1, you get $12.00. If you roll a 2 or a 3, you get $3.00. If you roll a 4, 5 or 6, you owe $2.00.

4. You are flipping a coin. If you flip heads, you win $10. If you flip tails, you win $2.

5. You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a 1, you get 24 candy bars. If you roll a 2 or a 3, you get 6 candy bars. If you roll a 4, 5 or 6, you get none.
2 Student Recording Sheet

Use this recording sheet with pages 148–149 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. O O O O
2. O O O O
3. O O O O
4. O O O O
5. O O O O
6. O O O O
7. O O O O
8. O O O O

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

9. ____________
10. ____________(grid in)
11. ____________(grid in)
12. ____________
13. ____________(grid in)
14. ____________

Extended Response

Record your answers for Question 15 on the back of this paper.
**Rubric for Scoring Extended Response**

**General Scoring Guidelines**

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

**Exercise 15 Rubric**

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>A correct solution that is supported by well-developed, accurate explanations. In part a, students show an understanding that ( x ) visits for a yearly member cost ( 120 + 2x ) and ( x ) visits for a nonmember cost ( x(12 + 5 + 5) ) or ( 22x ). The total cost for members and nonmembers will be the same when ( 120 + 2x = 22x ). The equation is solved correctly as ( x = 6 ). In part b, students substitute 6 in either expression and solve to find the total cost to be $132. In part c, students explain that Georgena is better off buying a membership only if she plans to visit more than 6 times.</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
Chapter 2 Quiz 1
(Lessons 2-1 through 2-3)

Translate each sentence into an equation.

1. The difference of the square of \( y \) and twelve is the same as the product of five and \( x \).
   \[ \text{____________} \]

Translate each equation into a sentence.

2. \( 2b - 10 = 4 \) \[ \text{____________} \]
3. \( y + 3x^2 = 5x \) \[ \text{____________} \]

Solve each equation.

4. \( d - 8 = 6 \) \[ \text{____________} \]
5. \( -28 = p + 21 \) \[ \text{____________} \]
6. \( -7x = 63 \) \[ \text{____________} \]
7. \( -\frac{t}{5} = -8 \) \[ \text{____________} \]
8. \( 3x + 8 = 29 \) \[ \text{____________} \]
9. \( \frac{a}{6} - 5 = 9 \) \[ \text{____________} \]

10. **MULTIPLE CHOICE** Solve \( \frac{5r}{2} - 6 = 19 \).

   A 2.5   B 5   C -5.25   D 10 \[ \text{____________} \]

Chapter 2 Quiz 2
(Lessons 2-4 and 2-5)

Solve each equation.

1. \( 7n + 6 = 4n - 9 \) \[ \text{____________} \]
2. \( 3b - 13 + 4b = 7b + 1 \) \[ \text{____________} \]
3. \( 5 - 3(w + 4) = w - 7 \) \[ \text{____________} \]
4. \( 2x - 5(x - 3) = 2(x - 10) \) \[ \text{____________} \]

5. **MULTIPLE CHOICE** Solve \(-6(2r + 8) = -10(r - 3)\).

   A -11   B -156   C -39   D 9 \[ \text{____________} \]

Evaluate each expression if \( h = 6 \) and \( w = -3 \).

6. \( 5 + |2w| \) \[ \text{____________} \]
7. \( |4 - h| - w \) \[ \text{____________} \]
8. \( |2 - w - h| - h \) \[ \text{____________} \]

Solve each equation.

9. \( |7 - 2q| = 3 \) \[ \text{____________} \]
10. \( |4x - 2| = 26 \) \[ \text{____________} \]
Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. \( \frac{5}{7} : \frac{20}{28} \)  
2. \( \frac{11}{13} : \frac{22}{25} \)  
3. \( \frac{4.2}{6.3} : \frac{0.3}{0.5} \)

Solve each proportion.

4. \( \frac{3}{4} = \frac{n}{20} \)  
5. \( \frac{6}{4} = \frac{x}{18} \)  
6. \( \frac{33}{6} = \frac{15}{45} \)

For Questions 7 and 8, state whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change.

7. original: 25  
   new: 18

8. original: 36  
   new: 45

9. The cost of a compact disc is $18. If the sales tax is 6%, find the total price.

10. MULTIPLE CHOICE A store sells a certain digital camera model for $108. During a special promotion, the camera is discounted by 30 percent. What is the discounted price?

   A $32.40  
   B $75.60  
   C $78.00  
   D $140.40

For Questions 1 and 2, solve each equation or formula for the variable specified.

1. \( nx - r = p \), for \( x \)  
2. \( \frac{x - b}{a} = c \), for \( b \)

3. A chemist needs a 30% iron alloy. How many grams of a 70% iron alloy must be mixed with 16 grams of a 20% iron alloy to obtain the required 30% alloy?

4. MULTIPLE CHOICE On Mary’s walk she covered the 3-mile distance to the park in one hour. However, the return trip took her one and a half hours. What was her average speed for the round trip?

   A 2.4 mph  
   B 2.5 mph  
   C 2.8 mph  
   D 3.9 mph

5. The formula \( p = 2\ell + 2w \) represents the perimeter of a rectangle. In this formula, \( \ell \) is the length of the rectangle and \( w \) is the width. Solve the formula for \( \ell \). Find the length when the width is 4 meters and the perimeter is 36 meters.
Chapter 2 Mid-Chapter Test
(Lessons 2-1 through 2-5)

Part I  Write the letter for the correct answer in the blank at the right of each question.

1. Translate the following sentence into an equation.
   The product of five and the sum of a number x and three is twelve.
   A 5 + 3x = 12  C 5x + 3 = 12
   B 5(x + 3) = 12  D 5x + 3 = x  1._____

2. Solve y + (-16) = -12.
   F 192  G -28  H 3/4  J 4  2._____

3. Solve \(-\frac{a}{6} + 5 = 2\).
   A 18  B -6  C \(\frac{1}{2}\)  D -9  3._____

4. Solve \(\frac{3}{5}y = -9\).
   F \(-5\frac{2}{5}\)  G -5  H -15  J \(-\frac{5}{9}\)  4._____

5. Solve \(-6d = -42\).
   A -48  B 7  C -36  D 252  5._____

   F 7  G 11  H 14  J 19  6._____

Part II

7. Translate 4n = x (5 - n) into a sentence.  7.____________________

For Questions 8–10, solve each equation.

8. 5(12 - 3p) = 15p + 60  8.____________________

9. 3(y - 2) = 6(y - 1) - 3y  9.____________________

10. 3a + 21 = 7 - 4a  10.____________________

11. Liza earned some money delivering newspapers. She bought a battery for $1.95, and gave her mother $30. She bought a ring for $7.20, and then spent half of the remaining money on a radio. If Liza has $38.50 left, how much money did she earn delivering newspapers?
   11.____________________

12. Solve |2x - 3| = 7. Then graph the solution set.
   12.____________________
Choose a term from the list above that best matches each phrase.

1. a comparison of two numbers by division
   1. ________________

2. the sum of the product of the number of units and the value per unit divided by the number of units
   2. ________________

3. percent of increase or decrease
   3. ________________

4. a ratio of two measurements having different units of measure
   4. ________________

5. an equation with more than one operation
   5. ________________

6. the number of one item being compared to 1 of another item
   6. ________________

7. equations that are true for all values of variables
   7. ________________

Define each term in your own words.

8. proportion

9. formula
Write the letter for the correct answer in the blank at the right of each question.

1. Translate the following sentence into an equation.
   *Twice a number m minus three equals the sum of m and five.*
   
   A $2(m - 3) = m + 5$
   B $2m - 3 = m + 5$
   C $2m - 3 = 5m$
   D $2(m - 3) = 5m$
   1. _____

2. Translate the following equation into a verbal sentence.
   $x + 5 = 2(7 + x)$
   F The quotient of $x$ and five is two times seven plus $x$.
   G The number $x$ plus five is two times the sum of seven and $x$.
   H The number $x$ plus five is two times seven plus $x$.
   J The product of $x$ and five is the sum of two times seven and $x$.
   2. _____

   A $-21$
   B $21$
   C $-15$
   D $15$
   3. _____

4. Solve $5n = 35$.
   F $30$
   G $7$
   H $40$
   J $165$
   4. _____

5. Solve $\frac{3}{5}x = 15$.
   A $9$
   B $5$
   C $25$
   D $75$
   5. _____

6. Solve $2t + 1 = 3$.
   F $1$
   G $-1$
   H $2$
   J $-2$
   6. _____

7. A number is added to 9. The result is then multiplied by 4 to give a new result of 120. What is the number?
   A $21$
   B $39$
   C $489$
   D $4(n + 9) + 120$
   7. _____

8. Evaluate $|2b - 5| + 1$ if $b = 1$.
   F $-2$
   G $2$
   H $4$
   J $-8$
   8. _____

   A $\emptyset$
   B $\{-4, 4\}$
   C $\{-4, 6\}$
   D $\{-2, 12\}$
   9. _____

10. Which ratio forms a proportion with $\frac{7}{14}$?
    F $\frac{4}{9}$
    G $\frac{5}{12}$
    H $\frac{2}{5}$
    J $\frac{3}{6}$
    10. _____
11. Solve the proportion \( \frac{2}{7} = \frac{x}{42} \).
   A \( \frac{1}{2} \)  B \( 12 \)  C \( \frac{2}{7} \)  D \( 6 \)

12. Solve \( 3t - 6 = t - 2 \).
   F \(-2\)  G \(-4\)  H \(2\)  J \(1\)

13. Solve \( 4(t + 1) = 6t - 1 \).
   A \(2\frac{1}{2}\)  B \(1\)  C \(0\)  D \(1\frac{1}{2}\)

14. Solve \( 5(g - 2) + g = 6(g - 4) \).
   F all numbers  G \(0\)  H \(2\)  J \(0\)

15. Solve \( ax - 5 = b \) for \( a \).
   A \(x(b + 5)\)  B \(\frac{b - 5}{x}\)  C \(\frac{b + 5}{x}\)  D \(x(b - 5)\)

16. Find the percent of change. original: 10 new: 12
   F \(12\%\)  G \(25\%\)  H \(20\%\)  J \(18\%\)

17. A baseball costs $4.00. If the sales tax is 5%, what is the total price?
   A \$3.80  B \$4.20  C \$4.05  D \$4.50

18. How many liters of pure acid must be added to 3 liters of a 50% acid solution to obtain a 75% acid solution?
   F \(1\) L  G \(4.5\) L  H \(1.5\) L  J \(3\) L

19. Joe and Janna leave home at the same time, traveling in opposite directions. Joe drives 45 miles per hour and Janna drives 40 miles per hour. In how many hours will they be 510 miles apart?
   A \(7\) hours  B \(6\) hours  C \(5\) hours  D \(4\) hours

20. TEMPERATURE In Death Valley, California, the highest ground temperature recorded was 94°C on July 15, 1972. In the formula \( C = \frac{5}{9}(F - 32) \), \( C \) represents the temperature in degrees Celsius and \( F \) represents the temperature in degrees Fahrenheit. To the nearest degree, what is the highest ground temperature in Death Valley in Fahrenheit?
   F \(201^\circ\) F  G \(84^\circ\) F  H \(34^\circ\) F  J \(137^\circ\) F

Bonus A concrete mixture is made with 3 parts water and 5 parts cement. If 27 parts of water are being used in the current batch of concrete, how many parts of cement are being used?

B: ____________________
Write the letter for the correct answer in the blank at the right of each question.

1. Translate the following sentence into an equation.
   *The sum of twice a number x and 13 is two less than three times x.*
   A \(2(x + 13) + 3x - 2\)  
   B \(2x + 13 = 2 - 3x\)  
   C \(2x + 13 = 3x - 2\)  
   D \(2x + 13 = 3(x - 2)\)  
   1. _____

2. Translate the following equation into a verbal sentence.
   \(3x - y = 5(y + 2x)\)
   F Three times the difference of \(x\) and \(y\) equals five times the sum of \(y\) and two times \(x\).
   G Three times \(x\) less than \(y\) is five times \(y\) plus two times \(x\).
   H The sum of three times \(x\) and \(y\) is five times \(y\) plus two times \(x\).
   J Three times \(x\) minus \(y\) is five times the sum of \(y\) and two times \(x\).
   2. _____

3. Solve \(m - 13 = 8\).
   A 21  
   B -21  
   C 5  
   D -5  
   3. _____

4. Solve \(5w = -75\).
   F 15  
   G -80  
   H -15  
   J 80  
   4. _____

5. Solve \(-\frac{3}{8}y = -24\).
   A -9  
   B 9  
   C -64  
   D 64  
   5. _____

6. Solve \(5x + 3 = 23\).
   F 4  
   G \(5 \frac{1}{2}\)  
   H 25  
   J 15  
   6. _____

7. Six is subtracted from a number. The result is divided by four. This result is added to 10 to give 30. What is the number?
   A 4  
   B 80  
   C 86  
   D 16  
   7. _____

8. Evaluate \(|2d - 3n| - 4\) if \(n = 2\) and \(d = 3\).
   F -8  
   G -4  
   H 0  
   J 4  
   8. _____

9. Solve \(|3r - 6| = 21\).
   A \{5, -5\}  
   B \{9, -9\}  
   C \{9, -5\}  
   D no solution  
   9. _____

10. Which ratio forms a proportion with \(\frac{25}{35}\)?
    F \(\frac{3}{5}\)  
    G \(\frac{15}{21}\)  
    H \(\frac{24}{34}\)  
    J \(\frac{5}{10}\)  
    10. _____
11. Solve the proportion $\frac{5}{3c} = \frac{1}{6}$.
   A 2  B 10  C 30  D $\frac{11}{3}$  11. ____

12. Solve $2x + 7 = 5x + 16$.
   F $-3$  G $\frac{2}{3}$  H $-\frac{7}{3}$  J 3  12. ____

13. Solve $\frac{2}{3}(6x + 30) = x + 5(x + 4) - 2x$.
   A 6  B 0  C all numbers  D no solution  13. ____

14. Solve $-3(h - 6) = 5(2h + 3)$.
   F $-\frac{3}{13}$  G $\frac{3}{13}$  H $-\frac{9}{13}$  J $\frac{9}{13}$  14. ____

15. Solve $2x - y = y$ for $x$.
   A $2y - 2$  B $y - 2$  C $y$  D 0  15. ____

16. Find the percent of change. original: 80 new: 64
   F 25%  G 20%  H 16%  J 10%  16. ____

17. A calculator costs $32.00. If the sales tax is 6%, what is the total price?
   A $31.40  B $30.08  C $32.60  D $33.92  17. ____

18. How many liters of a 40% acid solution must be added to 12 liters of a
   20% solution to obtain a 25% solution?
   F 4 L  G 1 L  H 16 L  J $\frac{4}{5}$ L  18. ____

19. Mandy begins bicycling west at 30 miles per hour at 11:00 a.m. If Liz leaves
   from the same point 20 minutes later bicycling west at 36 miles per hour,
   when will she catch Mandy?
   A 2:00 p.m.  B 1:00 p.m.  C 1:30 p.m.  D 2:30 p.m.  19. ____

20. GEOMETRY The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2h$ where $V$
   represents the volume, $r$ represents the radius of the base, and $h$ represents
   the height. What is the height of a cone with a volume of 66 cubic
   centimeters and a base with a radius of 3 centimeters?
   F 21 cm  G 69.14 cm  H 7 cm  J 0.78 cm  20. ____

Bonus A mixture of 10% acid and 90% water is added to 5 liters
   of pure acid. The final mixture is 40% water. How many
   liters of water are in the final mixture?  B: ______________
Write the letter for the correct answer in the blank at the right of each question.

1. Translate the following sentence into an equation.
   The product of five and a number \( y \) is two less than the quotient of four and \( y \).
   \[
   A \quad 5 + y = \frac{4}{y} - 2 \\
   B \quad 5y = \frac{4}{y} - 2 \\
   C \quad 5y = 2 - \frac{4}{y} \\
   D \quad 5 + y = 2 - \frac{4}{y}
   \]
   1._____

2. Translate the following equation into a verbal sentence.
   \( x(7 - 5y) = \frac{x}{2} \)
   \[
   F \quad x \text{ times seven minus five times } y \text{ equals } x \text{ divided by two.} \\
   G \quad \text{The product of } x \text{ and seven minus five times } y \text{ equals the quotient of } x \text{ and two.} \\
   H \quad x \text{ times the difference of seven and the product of five and } y \text{ equals the quotient of } x \text{ and two.} \\
   J \quad x \text{ times the sum of seven and five times } y \text{ equals } x \text{ divided by two.}
   \]
   2._____

3. Solve \( x - 12 = 5 \).
   \[
   A \quad -17 \\
   B \quad -7 \\
   C \quad 17 \\
   D \quad 7
   \]
   3._____

4. Solve \( 6z = -84 \).
   \[
   F \quad -90 \\
   G \quad -78 \\
   H \quad -14 \\
   J \quad -504
   \]
   4._____

5. Solve \( -\frac{4}{7}k = -28 \).
   \[
   A \quad 49 \\
   B \quad -49 \\
   C \quad 16 \\
   D \quad -16
   \]
   5._____

6. Solve \( 2 + 7y = 44 \).
   \[
   F \quad 6 \frac{4}{y} \\
   G \quad 35 \\
   H \quad 49 \\
   J \quad 6
   \]
   6._____

7. A number is divided by four. The result is added to five. This result is multiplied by three to give 27. What is the number?
   \[
   A \quad 16 \\
   B \quad 1 \\
   C \quad 21 \frac{1}{2} \\
   D \quad 3 \frac{1}{2}
   \]
   7._____

8. Evaluate \( |3f - 2g| + 2 \) if \( f = -2 \) and \( g = 1 \).
   \[
   F \quad -2 \\
   G \quad 4 \\
   H \quad 6 \\
   J \quad 10
   \]
   8._____

9. Solve \( |2n + 5| = 11 \).
   \[
   A \quad \{3, -3\} \\
   B \quad \{3, -8\} \\
   C \quad \{8, -8\} \\
   D \quad \text{no solution}
   \]
   9._____

10. What ratio forms a proportion with \( \frac{8}{36} \)?
    \[
    F \quad \frac{1}{4} \\
    G \quad \frac{6}{27} \\
    H \quad \frac{7}{30} \\
    J \quad \frac{2}{7}
    \]
    10._____

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11. Solve the proportion $\frac{1}{8} = \frac{7}{2h}$.
   A 4  B 28  C 56  D 16  11. ____

12. Solve $9a + 28 = 4a + 3$.
   F $-30$  G $-20$  H $6\frac{1}{5}$  J $-5$  12. ____

13. Solve $3x + 4(x - 8) - x = \frac{3}{5}(10x + 15)$.
   A 0  B all numbers  C no solution  D 41  13. ____

14. Solve $4(3r - 2) = -3(r + 7)$.
   F $\frac{\text{13}}{\text{15}}$  G $-1\frac{4}{15}$  H $1\frac{14}{15}$  J $-13$  14. ____

15. Solve $3b = 6v - 3b$, for $v$.
   A $6b - 6$  B $b$  C $b - 6$  D 0  15. ____

16. Find the percent of change. original: 45  new: 54
   F $33\frac{1}{3}\%$  G 25%  H $16\frac{2}{3}\%$  J 20%  16. ____

17. Find the discounted price. radio: $45.00$ discount: 30%
   A $15.00$  B $31.50$  C $36.00$  D $42.00$  17. ____

18. Nature Drinks wants to combine orange juice they sell for $0.09$ per ounce with guava juice they sell for $0.14$ per ounce to create an orange-guava drink. How many ounces of orange juice should they use to create a 16-ounce drink that would sell for $1.74$?
   F 10 oz  G 6 oz  H 16 oz  J 0 oz  18. ____

19. Teri begins walking east at 2 miles per hour at 1:00 P.M. If Cindy leaves from the same point 30 minutes later walking east at 3 miles per hour, when will she catch Teri?
   A 2:30 P.M.  B 1:30 P.M.  C 2:00 P.M.  D 3:00 P.M.  19. ____

20. GEOMETRY The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2h$, where $V$ represents the volume, $r$ represents the radius of the base, and $h$ represents the height. What is the height of a cone with a volume of 110 cubic centimeters and a base with a radius of 5 centimeters?
   F 21 cm  G 0.47 cm  H 4.2 cm  J 41.49 cm  20. ____

Bonus In a bag of blue, green, and red marbles, 50% are blue and 30% are green. There are 6 red marbles in the bag. If you increase the number of blue marbles by 40%, how many blue marbles will be in the bag?  B: ___________
1. Translate the following sentence into an equation.
   A number $x$ subtracted from 36 is three times the sum of four and $x$.

2. Translate the following equation into a verbal sentence.
   $3(x + y) = 2y - x$

Solve each equation.

3. $12 + r = 3$
4. $-12 = p - 7$
5. $-7b = -35$
6. $-\frac{5}{8}w = -9$
7. $\frac{9}{25} = \frac{p}{125}$
8. $-3a + 4 = -14$

For Questions 9 and 10, write an equation for each problem.
Then solve the equation.

9. What number decreased by 3.5 equals 12.7?
10. Twelve is added to the product of a number and 5.
    The result is $-3$. Find the number.

11. Julie cashed a paycheck and repaid her brother $10 that she had borrowed from him. She then spent $30 on fuel for her car and half of the remaining money on a new tent for camping. She bought a pair of running shoes for $29.45 and had $17.75 left. How much did Julie receive when she cashed her paycheck?

12. Evaluate $|4t + n|$ if $t = -2$ and $n = 5$.

For Questions 13 and 14, solve each equation.
Then graph the solution set.

13. $|5t - 1| = 6$
14. $|2a + 1| = 1$
15. Use cross products to determine whether the pair of ratios \( \frac{4}{6} \) and \( \frac{14}{21} \) form a proportion. Write yes or no.

16. Solve the proportion \( \frac{12}{15} = \frac{18}{b} \).

For Questions 17–19, solve each equation.

17. \(-x + 4 = x + 6\)

18. \(5n + 7 = 7(n + 1) - 2n\)

19. \(-4(p + 2) + 8 = 2(p - 1) - 7p + 15\)

20. Solve \( \frac{a}{b}x - c = 0 \) for \( x \).

21. State whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change. original: 55 new: 44

22. A shirt costs $12.00. If the sales tax is 7%, find the total cost.

23. How many liters of a 90% acid solution must be added to 6 liters of a 15% acid solution to obtain a 40% acid solution?

24. A freight train leaves a station traveling 60 miles per hour. Thirty minutes later a passenger train leaves the station in the same direction on a parallel track at a speed of 72 miles per hour. How long will it take the passenger train to catch the freight train?

25. GEOMETRY A container company wants to make a cylindrical can with a volume of 1188 cubic inches. The formula \( V = \pi r^2h \) represents the volume of a cylinder. In this formula, \( V \) represents the volume, \( r \) represents the radius of the cylinder’s base, and \( h \) represents the height of the cylinder. Solve for \( h \). What height should the company make the can if the radius of the base must be 6 inches?

Bonus A clown is preparing for a party by inflating one balloon for every invited guest. Just when she has half of the necessary balloons inflated, 3 of them pop. She inflates 5 more balloons, and two pop. Then 6 balloons are carried away by the wind. She finishes by inflating 16 more balloons, and then learns that only 12 guests will attend the party. How many extra balloons did the clown inflate?

B:

Chapter 2 Test, Form 2C (continued)
1. Translate the following sentence into an equation.
   A number n added to 18 is seven times the difference of n and three.

2. Translate the following equation into a verbal sentence.
   \( \frac{3}{y} - 5 = x(y + 7) \)

Solve each equation.

3. \( 7 + t = 11 \)

4. \( -5 = v - 12 \)

5. \( -8x = -56 \)

6. \( -\frac{7}{9}y = -6 \)

7. \( \frac{10}{27} = \frac{a}{135} \)

8. \( 3 - 5b = -32 \)

For Questions 9 and 10, write an equation for each problem. Then solve the equation.

9. What number decreased by 8.1 equals 4.9?

10. Fifteen is added to the product of a number and 6. The result is 9. Find the number.

11. During an evening out, Dean paid a cab driver $20. He then spent $25 on dinner and half of his remaining money on a painting. He bought an umbrella for $23.75 and had $42.15 left. How much money did Dean have at the beginning of the weekend?

12. Evaluate \( |3t - n| \) if \( t = -3 \) and \( n = 2 \).

For Questions 13 and 14, solve each equation. Then graph the solution set.

13. \( |2t + 4| = 2 \)

14. \( |r + 3| = 1 \)
15. Use cross products to determine whether the pair of ratios \( \frac{14}{17} \) and \( \frac{39}{51} \) form a proportion. Write yes or no.

16. Solve the proportion \( \frac{9}{12} = \frac{15}{a} \).

For Questions 17–19, solve each equation.

17. \( 9 - t = t + 3 \)

18. \( 2(y - 6) = 3y + 12 - y \)

19. \( 17 + 3(z - 2) - 11z = -7(z + 2) + 14 \)

20. Solve \( \frac{r}{n} + t = 4v \) for \( r \).

21. State whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change. 
   original: 60, new: 75

22. Find the discounted price.
   flashlight: $18.00  discount: 25%

23. Nature’s Best wants to combine nuts they sell for $3.60 a pound with dried fruit they sell for $2.40 a pound to create a trail mix. How much of each snack should they use to make 10 pounds of trail mix that would sell for $3.30 a pound?

24. Paula leaves home driving 40 miles per hour. One hour later, her brother Dan leaves home, driving in the same direction at a speed of 50 miles per hour. How long will it take Dan to catch up to Paula?

25. GEOMETRY  A container company wants to make a cylindrical cardboard container with a volume of 4752 cubic inches. The formula \( V = \pi r^2 h \) represents the volume of a cylinder. In this formula, \( V \) represents the volume, \( r \) represents the radius of the cylinder’s base, and \( h \) represents the height of the cylinder. Solve for \( h \). What height should the company make the container if the radius of the base must be 9 inches?

Bonus  A store has all board games on sale for 25% off the regular price. A checker set has a sale price of $12. It is then moved to a clearance table where every item is discounted 40% off its regular price. What is the clearance price of the checker set?
1. Translate the following into an equation.
   A number $x$ is decreased by 45. The result is then divided by 12. Then 20 is added to this new result to give a final result of five times the difference of 32 and the number $x$. 

2. Translate the following equation into a verbal sentence.
   $5(2x + 3y) = y^2 - 2x^3$

For Questions 3–7, solve each equation.

3. $n + 39 = 12$
4. $w + (-8) = -21$
5. $-6n = 16$
6. $\frac{3}{4}h = -\frac{45}{52}$
7. $-\frac{a}{6} + 7 = -14$
8. If $x - 5 = 12$, what is the value of $x - 9$?

For Questions 9 and 10, write an equation for each problem. Then solve the equation.

9. Three-fifths of what number equals one?
10. The product of 2 more than a number and 10 is 36 more than 8 times the number. What is the number?

11. Evaluate $2|m - 3x| - p$ if $m = -1$, $x = 2$, and $p = 4$.
12. Solve $2|\frac{x}{2} + 3| = 1$. Then graph the solution set.

13. Shyam invested money in the stock market. In the first year, his stock increased 20%. He paid his stock broker $300 and then lost $450. He withdrew $500, and then his remaining investment doubled. Shyam’s investment is now worth $7100. How much was Shyam’s original investment?

14. Use cross products to determine whether the pair of ratios $\frac{42}{48}$ and $\frac{63}{72}$ form a proportion. Write yes or no.

15. A blueprint for a house states that 2 inches represents 8 feet. If the width of a window is 2.5 inches on the blueprint, what is the width of the actual window?
16. Solve the proportion \( \frac{t + 4}{t - 2} = \frac{1}{4} \).

**For Questions 17 and 18, solve each equation.**

17. \( 6 - 2y = 7y + 13 \)

18. \( 5(7 - a) - 3(a + 4) - 4 = 4(a - 3) + 7 \)

19. Solve \( ax - n = r \) for \( x \).

20. Solve \( \frac{4x + t}{r} = y \) for \( x \).

21. State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change.
   
   original: 75, new: 84

22. A jacket costs $75.00 retail. A warehouse outlet discounts the price by 20%. If the sales tax is 6%, find the final price.

23. Calvin invested $7500 for one year, part at 12% annual interest and the rest at 10% annual interest. His total interest for the year was $890. How much money did he invest at 12%?

24. Two airplanes leave the Atlanta airport at the same time, traveling in opposite directions. One plane travels 30 miles per hour faster than the other. After 3 hours, the planes are 3150 miles apart. What is the rate of each plane?

25. **PHYSICS** A ball is thrown straight up at an initial velocity of 53 feet per second. In the first 1.5 seconds, it travels 42 feet. The formula \( x = \left( \frac{u + t}{n} \right) t \) represents the vertical distance \( x \) that an object travels in \( t \) seconds, where \( u \) represents the initial velocity of the object and \( n \) represents the velocity of the object at the end of \( t \) seconds. Find the velocity of the ball at the end of 1.5 seconds.

**Bonus** Paloma Rey drove to work on Wednesday at 40 miles per hour and arrived one minute late. She left home at the same time on Thursday, drove 45 miles per hour, and arrived one minute early. How far does Ms. Rey drive to work?

B: _________________
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Phrase 1: $x$ times $y$ plus $z$; Phrase 2: $x$ times the sum of $y$ and $z$
   a. Discuss what is different between the two phrases.
   b. Find values for $x, y$ and $z$ that make the two phrases equal.

2. a. Solve $\frac{ry + z}{m} - t = x$ for $y$, and explain each step in your solution.
   b. Would there be any limitations for the value of each variable? If so, explain the limitation.

3. You buy a stereo at a local store. The stereo has been discounted by 10%. The store then charges 10% tax.
   a. Compare the final price with the original price.
   b. Would the final price be different if the tax was added first and then the discount was applied to this new amount?

4. Tony and Ivia started walking south from the same location at the same time. Ivia walked 8 miles and walked 1 mile per hour faster than Tony who walked 6 miles. They each walked for the same amount of time.
   a. Describe how a proportion could be used to find the rate that each person walked.
   b. The next day they both walked 6 miles, and Ivia again walked 1 mile per hour faster than Tony, who walked 3 miles per hour. Determine whether a proportion could be used to find how long each person walked.

5. a. Write four equivalent equations to $x = 8$ using one of the four operations of addition, subtraction, multiplication and division for each equivalent equation. Use each operation only once.
   b. Write an equivalent equation to $x = 8$ that has the variable $x$ on both sides.
   c. Determine if $\frac{n}{6} = \frac{15}{18}$ and $2(n + 1) = 3(n - 1)$ are equivalent equations. Determine if either equation is equivalent to any of the equations created for parts a and b.
1. Write an algebraic expression for the following verbal expression 
the sum of n and 5. (Lesson 1-1)  
A 5n  
B \( \frac{n}{5} \)  
C \( n + 5 \)  
D \( n - 5 \)  
1. ☐ ☐ ☐ ☐

2. Determine which of the following relations is a function. (Lesson 1-7)  
F \{(-4, 3), (-2, -2), (0, 2), (0, 5)\}  
G \{(-3, 1), (-3, 3), (-2, -1), (0, 5)\}  
H \{(-4, -1), (-2, -1), (-1, -1), (3, 3)\}  
J \{(2, -5), (-1, -1), (0, 4), (2, -3)\}  
2. ☐ ☐ ☐ ☐

3. Simplify the expression \( 7(x - y) - 2(y - x) + 4x \). (Lesson 1-4)  
A 13x - 9y  
B 9x - 5y  
C 9x - 9y  
D 13x - 5y  
3. ☐ ☐ ☐ ☐

4. Evaluate \( a(b - c^2) \) if \( a = \frac{2}{3}, b = \frac{3}{4}, \text{ and } c = \frac{1}{2} \). (Lesson 1-2)  
F \( \frac{1}{6} \)  
G \( \frac{1}{3} \)  
H \( \frac{1}{4} \)  
J \( \frac{2}{3} \)  
4. ☐ ☐ ☐ ☐

5. Solve the proportion \( \frac{a}{25} = \frac{9}{45} \). (Lesson 2-6)  
A 7.8  
B 16.2  
C 125  
D 5  
5. ☐ ☐ ☐ ☐

6. Jackson’s meal cost $15.40. How much money should he leave 
for a 15% tip? (Lesson 2-7)  
F $0.23  
G $2.31  
H $23.10  
J $231.00  
6. ☐ ☐ ☐ ☐

7. Solve \( -\frac{3}{4}y = \frac{8}{20} \). (Lesson 2-2)  
A \( \frac{2}{5} \)  
B \( -\frac{3}{10} \)  
C \( \frac{8}{15} \)  
D \( -\frac{8}{15} \)  
7. ☐ ☐ ☐ ☐

8. Which equation has a solution of \(-2?\) (Lesson 2-3)  
F \( 4n + 3 = 11 \)  
G \( 4 = 3n - 2 \)  
H \( 5(1 + n) = -5 \)  
J \( 3(n + 1) = 2 \)  
8. ☐ ☐ ☐ ☐

9. A car dealership has 180 cars on their lot. If they increase their 
inventory by 25%, how many cars will be on the lot? (Lesson 2-7)  
A 230  
B 225  
C 135  
D 205  
9. ☐ ☐ ☐ ☐
10. Translate the following sentence into an equation.
   *The quotient of 24 and x equals 14 minus 2 times x.*  (Lesson 2-1)
   \[ \frac{24}{x} = 14 - 2x \]
   - F \[ 24x = 14 - 2x \]
   - H \[ \frac{24}{x} = 14 - 2x \]
   - G \[ 24x = 2x - 14 \]
   - J \[ \frac{24}{x} = 2x - 14 \]

11. Evaluate \( 2^6 \).  (Lesson 1-2)
   - A 12
   - B 32
   - C 64
   - D 128

12. Which pair of ratios forms a proportion?  (Lesson 2-6)
   - F \[ \frac{2}{3} \text{ and } \frac{4}{9} \]
   - G \[ \frac{5}{15} \text{ and } \frac{4}{12} \]
   - H \[ \frac{4}{12} \text{ and } \frac{6}{24} \]
   - J \[ \frac{1}{9} \text{ and } \frac{9}{10} \]

13. Evaluate \( 14 - \left( \frac{1}{4} \right)(17 - 5) \).  (Lesson 1-2)
   - A 17
   - B 34
   - C 11
   - D 120

14. Evaluate \( 21 \div 3 + 4 \cdot 2 \).  (Lesson 1-2)
   - F 15
   - G 22
   - H 1.9
   - J 9

**Part 2: Gridded Response**

**Instructions:** Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

15. Solve \( 2 + 5x = 10 - 3x \).  (Lesson 2-4)

16. Sasha drove from her house to the grocery store in 20 minutes. Her return trip took 15 minutes. If the store is 7 miles from her house, what was Sasha’s average speed for the round trip, in miles per hour?  (Lesson 2-9)
17. Evaluate $3y - x^2z$ if $x = 2$, $y = 14$, and $z = 5$. (Lesson 1-2)

18. Simplify $2(u + 3x) + 3(u + x)$. (Lesson 1-3)

19. Miguel was riding his bike to school. He got halfway there and realized he had forgotten his backpack. He turned around, went home, retrieved his backpack, and continued his ride to school. Sketch a reasonable graph to show his distance from school from the time he started to the time he arrived at school. Assume his rate is always the same. (Lesson 1-6)

20. Translate the following sentence into an algebraic equation. 
   Nine times a number $y$ subtracted from 85 is seven times the sum of four and $y$. (Lesson 2-1)

21. Solve the following problem by working backward. Three is added to a number. The result is divided by two, and then the new result is added to eighteen. The final result is 35. What is the number? (Lesson 2-3)

For Questions 22–24, solve each equation. (Lessons 2-2 through 2-4)

22. $-27 = -6 - 3p$

23. $7a + 2 = 3a - 10$

24. $2(x - 3) + 6x = 3(9 - x)$

25. Solve $t = \frac{m}{x} + p$ for $m$. (Lesson 2-8)

26. How many liters of water must be added to 7 liters of a 20% acid solution to obtain a 10% acid solution? (Lesson 2-9)

27. Stacy bought a skateboard at a local sporting goods store. The skateboard was on sale for 10% off the original price. The store then charges 10% sales tax. (Lesson 2-7)

   a. How does the original price compare to the final price?

   b. Would the final price be different if the 10% sales tax was added first and then the 10% discount was applied to this new amount?
1. Write a verbal expression for $4r + 9$.

2. Write an algebraic expression for the difference of 5 and $n$ cubed.

3. Evaluate $2x + 5y^2 - 3z$ if $x = 6$, $y = 4$, and $z = 7$.

4. Name the property used in the equation $1 = 6n$. Then find the value of $n$.

For Questions 5–7, simplify each expression.

5. $2t^2 + 5t^2 + 3t$

6. $7(r + 2t) - 5t$

7. $5(4a + b) + 3a + b$

8. Find the solution set for $3b - 4 = 8$ if the replacement set is {1, 2, 3, 4, 5}.

For Questions 9–10, determine whether each relation is a function.

9. {$(1, 5), (2, 4), (3, 5), (4, 9)$}

10. $x = -2$

11. If $f(n) = 6 - 2n$, find $f(-1)$.

12. True or False: A linear graph can have a maximum or a minimum.

13. Draw a reasonable graph showing the relationship between the temperature of a pizza as it is removed from an oven and placed on a counter at room temperature, and time.

14. The sides of an equilateral triangle measure $(2x + 4)$ units. What is the perimeter?

15. Translate $m^2 - 4 = 2r + 1$ into a sentence.

16. Write a problem based on the given information.
   
   $h =$ the height of a math textbook;
   $h + 2 =$ the height of a science textbook
   $4h + 2 (h + 2)$
Solve each equation.

17. \( m - 5 = -23 \)

18. \(-4 = 8 + k\)

19. \( \frac{a}{2} + 9 = 30 \)

20. \( -\frac{2}{7}x = -16 \)

21. \( 5(c + 3) = 15 + 2(2c - 1) \)

22. \( 10(a + 1) - 14a = 9 - (4a - 1) \)

23. \( \frac{7}{10} = \frac{3}{x + 1} \)

For Questions 24 and 25, evaluate each expression if \( a = 3, b = 4, \) and \( c = 9. \)

24. \( 2|a - b| + |c| \)

25. \( c - b |1 - a| \)

26. Solve \( |2x - 1| = 5. \) Then graph the solution set.

27. Determine whether \( \frac{4}{9} \) and \( \frac{20}{45} \) are equivalent ratios. Write \( yes \) or \( no. \)

28. A magazine is on sale for 15% off the original price. If the original price of the magazine is $4.60, what is the discounted price?

29. Solve \( \frac{t - v}{r} = k, \) for \( v. \)

30. How many pounds of peanuts costing $3.00 a pound should be mixed with 4 pounds of cashews costing $4.50 a pound to obtain a mixture costing $3.50 a pound?
Chapter 2 Anticipation Guide
Linear Equations

Step 1 Before you begin Chapter 2
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>When writing equations the phrases as much as, is, and is identical to, all suggest the equals sign.</td>
<td>A</td>
</tr>
<tr>
<td>2.</td>
<td>The solving an equation strategy cannot be used if an equation is not given in the problem.</td>
<td>D</td>
</tr>
<tr>
<td>3.</td>
<td>Given a true equation, any value can be added or subtracted to both sides resulting in a true equation.</td>
<td>A</td>
</tr>
<tr>
<td>4.</td>
<td>Since the equation ( t - 23 = 54 ) involves subtraction, subtraction would be used to solve for ( t ).</td>
<td>D</td>
</tr>
<tr>
<td>5.</td>
<td>To solve ( 21 = -7y ), you could divide by (-7 ) or multiply by (-\frac{1}{7} ).</td>
<td>A</td>
</tr>
<tr>
<td>6.</td>
<td>To solve equations with more than one operation, undo operations in the same order as the Order of Operations.</td>
<td>D</td>
</tr>
<tr>
<td>7.</td>
<td>Equations with the variable on both sides have no solution.</td>
<td>D</td>
</tr>
<tr>
<td>8.</td>
<td>3 to 5, 3/5, and ( \frac{3}{5} ) are all examples of ratios.</td>
<td>A</td>
</tr>
<tr>
<td>9.</td>
<td>Because ( 5 \cdot 12 = 15 \cdot 4 \cdot \frac{12}{15} ) is in proportion to ( \frac{4}{5} ).</td>
<td>A</td>
</tr>
<tr>
<td>10.</td>
<td>A percent of change is found by dividing the amount of change by the original amount.</td>
<td>A</td>
</tr>
<tr>
<td>11.</td>
<td>Equations containing two variables cannot be solved since variables cannot be added or subtracted from each side of the equation.</td>
<td>D</td>
</tr>
<tr>
<td>12.</td>
<td>To find a weighted average, extremely high or low values are not included.</td>
<td>D</td>
</tr>
</tbody>
</table>

Step 2 After you complete Chapter 2
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

2-1 Study Guide and Intervention
Writing Equations

Write Equations Writing equations is one strategy for solving problems. You can use a variable to represent an unspecified number or measure referred to in a problem. Then you can write a verbal expression as an algebraic expression.

Example 1 Translate each sentence into an equation or a formula.

a. Ten times a number \( x \) is equal to 2.8 times the difference \( y - z \).
   \[ 10x = 2.8(y - z) \]

b. A number \( m \) minus 8 is the same as a number \( n \) divided by 2.
   \[ m - 8 = n \div 2 \]

c. The area of a rectangle equals the length times the width. Translate this sentence into a formula.
   Let \( A = \) area, \( ℓ = \) length, and \( w = \) width.
   Formula: Area equals length times width.
   \[ A = ℓw \]

Example 2 Use the Four-Step Problem-Solving Plan.
POPULATION The population of the United States in July 2007 was about 301,000,000, and the land area of the United States is about 3,500,000 square miles. Find the average number of people per square mile in the United States.

Step 1 Read You know that there are 301,000,000 people. You want to know the number of people per square mile.

Step 2 Plan Write an equation to represent the situation. Let \( p \) represent the number of people per square mile.

Step 3 Solve \[ \frac{301,000,000}{3,500,000} = p \]

Step 4 Check If there are 86 people per square mile, \( 3,500,000 \times \frac{86}{3,500,000} = 86 \).

Exercises
Translate each sentence into an equation or formula.

1. Three times a number \( t \) minus twelve equals forty.
   \[ 3t - 12 = 40 \]

2. One-half of the difference of \( a \) and \( b \) is 54.
   \[ \frac{1}{2}(a - b) = 54 \]

3. Three times the sum of \( d \) and 4 is 32.
   \[ 3(d + 4) = 32 \]

4. The area \( A \) of a circle is the product of \( π \) and the radius \( r \) squared.
   \[ A = πr^2 \]

5. WEIGHT LOSS Lou wants to lose weight to audition for a part in a play. He weighs 160 pounds now. He wants to weigh 150 pounds.
   a. If \( p \) represents the number of pounds he wants to lose, write an equation to represent this situation.
      \[ 160 - p = 150 \]
   b. How many pounds does he need to lose to reach his goal? 10 lb
Chapter 2

2-1 Study Guide and Intervention (continued)

Writing Equations

Write Verbal Sentences You can translate equations into verbal sentences.

Example: Translate each equation into a sentence.

a. \(4n - 8 = 12\)
   
   Four times \(n\) minus eight equals twelve.

b. \(a^2 + b^2 = c^2\)
   
   The sum of the squares of \(a\) and \(b\) is equal to the square of \(c\).

Exercises

Translate each equation into a sentence.

1. \(4n - 8 = 23\)
   
   4 times \(n\) minus 8 is equal to 23.

2. \(10 + k = 4k\)
   
   The sum of 10 and \(k\) is equal to 4 times \(k\).

3. \(6x^2 = 24\)
   
   6 times the product of \(x\) and \(y\) is equal to 24.

4. \(a^2 + b^2 = c^2\)
   
   The sum of the squares of \(a\) and \(b\) is equal to the square of \(c\).

5. \(p + 3 = 2p\)
   
   The sum of \(p\) and 3 is equal to 2 times \(p\).

6. \(b = \frac{1}{3}(h - 1)\)
   
   \(b\) is \(\frac{1}{3}\) of the difference of \(h\) and 1.

7. \(100 - 2x = 80\)
   
   100 minus 2 times \(x\) is equal to 80.

8. \(3(g + h) = 12\)
   
   3 times the sum of \(g\) and \(h\) is 12.

9. \(p^2 - 2p = 9\)
   
   The square of \(p\) minus 2 times \(p\) is equal to 9.

10. \(C = \frac{5}{9}(F - 32)\)
    
    \(C\) is equal to \(\frac{5}{9}\) of the difference of \(F\) and 32.

11. \(V = \frac{1}{3}Bh\)
    
    \(V\) is equal to \(\frac{1}{3}\) of the product of \(B\) and \(h\).

12. \(A = \frac{1}{2}bh\)
    
    \(A\) is equal to \(\frac{1}{2}\) of the product of \(b\) and \(h\).

Chapter 2

2-1 Skills Practice

Writing Equations

Translate each sentence into an equation.

1. Two added to three times a number \(m\) is the same as 18. \(3m + 2 = 18\)

2. Twice \(a\) increased by the cube of \(a\) equals \(b\). \(2a + a^3 = b\)

3. Seven less than the sum of \(p\) and \(t\) is as much as 6. \((p + t) - 7 = 6\)

4. The sum of \(x\) and its square is equal to \(y\) times \(z\). \(x + x^2 = yz\)

5. Four times the sum of \(f\) and \(g\) is identical to six times \(g\). \(4(f + g) = 6g\)

Translate each sentence into a formula.

6. The perimeter \(P\) of a square equals four times the length of a side \(s\). \(P = 4s\)

7. The area \(A\) of a square is the length of a side \(s\) squared. \(A = s^2\)

8. The perimeter \(P\) of a triangle is equal to the sum of the lengths of sides \(a\), \(b\), and \(c\). \(P = a + b + c\)

9. The area \(A\) of a circle is \(\pi\) times the radius \(r\). \(A = \pi r^2\)

10. The volume \(V\) of a rectangular prism equals the product of the length \(L\), the width \(W\), and the height \(H\). \(V = LWH\)

Translate each equation into a sentence.

11. \(g + 10 = 3g\)
    
    \(g\) plus 10 is the same as 3 times \(g\).

12. \(2p + 4t = 20\)
    
    Twice \(p\) plus 4 times \(t\) is 20.

13. \(4a + b = 5a\)
    
    4 times \(a\) plus \(b\) is 5 times \(a\).

14. \(8 - 6a = 4 + 2x\)
    
    8 minus 6 times \(a\) is 4 plus 2 times \(x\).

15. \(\frac{1}{2}(f + y) = f - 5\)
    
    Half of the sum of \(f\) and \(y\) is \(f\) minus 5.

16. \(k^2 - n^2 = 25\)
    
    \(k\) squared minus \(n\) squared is 25.

Write a problem based on the given information.

17. \(c = \text{cost per pound of plain coffee beans}\)
    \(c + 3 = \text{cost per pound of flavored coffee beans}\)
    \(2c + (c + 3) = 21\)

    Sample answer: The cost of two pounds of plain coffee beans plus one pound of flavored beans is $21. How much does 1 pound of plain coffee beans cost?

18. \(p = \text{cost of dinner}\)
    \(0.15p = \text{cost of a 15% tip}\)
    \(p + 0.15p = 23\)

    Sample answer: The cost of dinner plus a 15% tip was $23. How much was the dinner?
2-1 Practice

Writing Equations

Translate each sentence into an equation.

1. Fifty-three plus four times b is as much as 21. \(53 + 4b = 21\)
2. The sum of five times h and twice g is equal to 23. \(5h + 2g = 23\)
3. One fourth the sum of r and ten is identical to r minus 4. \(\frac{1}{4}(r + 10) = r - 4\)
4. Three plus the sum of the squares of w and x is 32. \(3 + (w^2 + x^2) = 32\)

Translate each sentence into a formula.

5. Degrees Kelvin \(K\) equals 273 plus degrees Celsius \(C\). \(K = 273 + C\)
6. The total cost \(C\) of gas is the price \(p\) per gallon times the number of gallons \(g\). \(C = pg\)
7. The sum \(S\) of the measures of the angles of a polygon is equal to 180 times the difference of the number of sides \(n\) and 2. \(S = 180(n - 2)\)

Translate each equation into a sentence.

8. \(r = (4 + p) = \frac{3}{3}r\) minus the sum of 4 and \(p\) equals \(\frac{3}{3}\) times \(r\).
9. \(\frac{2}{3}t + 2 = t\) Two more than \(\frac{3}{3}\) of \(t\) equals \(t\).
10. \(9(y^2 + x) = 18\) 9 times the sum of \(y\) squared and \(x\) is 18.
11. \(2(m - n) = x + 7\) Twice the quantity \(m\) minus \(n\) is \(x\) plus 7.

Write a problem based on the given information.

12. \(a = \text{cost of one adult's ticket to zoo}\) \(a - 4 = \text{cost of one children's ticket to zoo}\) \(2a + 4(a - 4) = 38\)

Sample answer: The cost of two adult's tickets and 4 children's tickets to the zoo is $38. How much is an adult's ticket?

13. \(c = \text{regular cost of one airline ticket}\) \(0.20c = \text{amount of 20\% promotional discount}\) \(3c - 0.20c = 330\)

Sample answer: The cost of three airline tickets that are discounted 20\% is $330. What is the regular cost of a ticket?

14. GEOGRAPHY About 15\% of all federally-owned land in the 48 contiguous states of the United States is in Nevada. If \(F\) represents the area of federally-owned land in these states and \(N\) represents the portion in Nevada, write an equation for this situation. \(0.15F = N\)

15. FITNESS Deanna and Pietra each go for walks around a lake a few times per week. Last week, Deanna walked 7 miles more than Pietra.

a. If \(F\) represents the number of miles Pietra walked, write an equation that represents the total number of miles \(T\) the two girls walked. \(T = F + (F + 7)\)

b. If Pietra walked 9 miles during the week, how many miles did Deanna walk? 16 mi

c. If Pietra walked 11 miles during the week, how many miles did the two girls walk together? 29 mi

2-1 Word Problem Practice

Writing Equations

1. HOUSES The area of the Hartstein's kitchen is 182 square feet. This is 20\% of the area of the first floor of their house. Let \(P\) represent the area of the first floor. Write an equation to represent the situation. \(182 = 0.2P\)

2. FAMILY Katie is twice as old as her sister Mara. The sum of their ages is 24. Write a one-variable equation to represent the situation. \(m + 2m = 24\) or \(3m = 24\)

3. GEOMETRY The formula \(F + V = E + 2\) shows the relationship between the number of faces \(F\), edges \(E\), and vertices \(V\) of a polyhedron, such as a pyramid. Write the formula in words.

4. WIRELESS PHONE Spinfrog wireless phone company bills on a monthly basis. Each bill includes a $29.95 service fee for 1000 minutes plus a $2.95 federal communication tax. Additionally, there is a charge of $0.05 for each minute used over 1000. Let \(m\) represent the number of minutes over 1000 used during the month. Write an equation to describe the cost \(p\) of the wireless phone service per month.

\[ p = 32.90 + 0.05m \]

5. TEMPERATURE The table below shows the temperature in Fahrenheit for some corresponding temperatures in Celsius.

<table>
<thead>
<tr>
<th>Celsius</th>
<th>Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20\°C</td>
<td>-4\°F</td>
</tr>
<tr>
<td>-10\°C</td>
<td>14\°F</td>
</tr>
<tr>
<td>0\°C</td>
<td>32\°F</td>
</tr>
<tr>
<td>10\°C</td>
<td>50\°F</td>
</tr>
<tr>
<td>20\°C</td>
<td>68\°F</td>
</tr>
<tr>
<td>30\°C</td>
<td>86\°F</td>
</tr>
</tbody>
</table>

a. Write a formula for converting Celsius temperatures to Fahrenheit temperatures.

\[ F = 1.8C + 32 \]

b. Find the Fahrenheit equivalents for 29\°C and 35\°C. 77\°F and 95\°F
Guess the Number

Think of a number. Add five to your number. Now, double your result. Double your result again. Divide your answer by four. Finally, subtract your original number. Your result is five.

How is it possible to know what the answer is without knowing the original number? Write the steps listed above as an expression in equation form. Then use algebra to show why this trick works.

Think of a number:

\[ x \]

Add five to your number:

\[ x + 5 \]

Double your result:

\[ 2(x + 5) \]

Double your result again:

\[ 2(2(x + 5)) \]

Divide your answer by four:

\[ \frac{2(2(x + 5))}{4} = x \]

Subtract your original number:

\[ \frac{2(2(x + 5))}{4} - x = x \]

Simplify the final expression:

\[ \frac{4(x + 5)}{4} - x = \frac{x + 5 - x}{5} \]

So, the result will always be five, no matter what the starting number is.

Write variable expressions to determine why each number trick works.

1. Think of a number. Add eight. Double your result. Next, subtract 16. Finally, divide your result by 2. You get your original number back.

\[ \frac{2(x + 8) - 16}{2} = x \]

2. Think of a number. Multiply by 10. Add 5 to your result. Next, subtract 3. Then add 2. Next, subtract 4. Divide your result by 5. Finally, subtract your original number. Your result is your original number.

\[ \frac{5(x + 2) - 5 - 2 - x}{5} = x \]

3. Think of a number. Add 1. Multiply your result by 6. Now, double your result. Next, divide your result by 12. Finally, subtract your original number. Your result is 1.

\[ \frac{2(6(x + 1))}{12} - x = 1 \]

4. Think of a number. Multiply by 5. Add five to your result. Now, divide by 5. Subtract 1 from your result. Finally, subtract your original number. Your final result is 0.

\[ \frac{5x + 5}{5} - 1 - x = 0 \]


\[ \frac{2(30(x + 30))}{6} - x = 30 \]
2-2 Study Guide and Intervention (continued)

Solving One-Step Equations

Solve Equations Using Multiplication and Division. If each side of an equation is multiplied by the same number, the resulting equation is equivalent to the given one. You can use the property to solve equations involving multiplication and division. To solve equations with multiplication and division, you can also use the Division Property of Equality. If each side of an equation is divided by the same number, the resulting equation is true.

**Multiplication Property of Equality**

For any numbers \(a\), \(b\), and \(c\), if \(a = b\), then \(ac = bc\).

**Division Property of Equality**

For any numbers \(a\), \(b\), and \(c\), with \(c \neq 0\), if \(a = b\), then \(\frac{a}{c} = \frac{b}{c}\).

**Example 1** Solve \(3\frac{1}{2}p = 1\frac{1}{2}\).

- Original equation: \(3\frac{1}{2}p = 1\frac{1}{2}\)
- Rewrite each mixed number as an improper fraction: \(\frac{7}{2}p = \frac{3}{2}\)
- Multiply each side by \(\frac{2}{7}\):
  \[\frac{2}{7} \cdot \frac{7}{2}p = \frac{2}{7} \cdot \frac{3}{2}\]
  \[p = \frac{3}{7}\]

The solution is \(\frac{3}{7}\).

**Example 2** Solve \(-5n = 60\).

- Original equation: \(-5n = 60\)
- Divide each side by \(-5\):
  \[\frac{-5n}{-5} = \frac{60}{-5}\]
  \[n = -12\]

The solution is \(-12\).

**Exercises**

Solve each equation. Check your solution.

1. \(\frac{b}{3} = -2\) \(-6\)
2. \(\frac{1}{8}m = 6\) 48
3. \(\frac{1}{5}p = \frac{3}{5}\) 3
4. \(5 = \frac{y}{12}\) 60
5. \(-\frac{1}{4}x = -2.5\) 10
6. \(-m = \frac{5}{8}\) -5
7. \(-1\frac{1}{2} = 4\) \(-8\)
8. \(-12 = -3\frac{3}{2}\) 8
9. \(\frac{5}{3} = \frac{2}{5}\) \(\frac{1}{5}\)
10. \(-3\frac{1}{2} = 5\) \(-1\frac{1}{2}\)
11. \(\frac{7}{2}m = 10\) 14 \(\frac{2}{7}\)
12. \(\frac{2}{5} = \frac{1}{4}\) \(-\frac{1}{4}\)
13. \(3b = -42\) \(-14\)
14. \(8m = 16\) 2
15. \(-3t = 51\) \(-17\)
16. \(-3x = -24\) 8
17. \(8k = -64\) \(-8\)
18. \(-2m = 16\) \(-8\)
19. \(12h = 4\) \(\frac{1}{3}\)
20. \(-2.4p = 7.2\) \(-3\)
21. \(0.5j = 5\) 10
22. \(-25 = 5m\) \(-5\)
23. \(6m = 15\) \(2\frac{1}{2}\)
24. \(-15p = -75\) 50

**Answers**

1. \(w + 14 = -8\) \(-22\)
2. \(p - 4 = 6\) 10
3. \(y - 32 = -1\) 31
4. \(-13 = 5 + x\) \(-18\)
5. \(b = 34\) 64
6. \(y = 32\) \(-1\)
7. \(n + (-28) = 0\) 28
8. \(y = 32\)
9. \(t = -19\) 18
10. \(j = -17\) 36
11. \(d + (-10) = 24\)
12. \(a + (-5) = -15\)
13. \(n - 16 = y\) 27
14. \(c - 3 = 100\) 97
15. \(w = (-8)\) 39
16. \(x = -74\) \(-22\)
17. \(4 - (-t) = 68\) 64
18. \(56 = 20 - (-p)\) \(-76\)
19. \(12z = 108\)
20. \(-7t = 49\)
21. \(18p = -216\)
22. \(-22 = 11v\)
23. \(-6z = -42\)
24. \(96 = -24a\)
25. \(\frac{c}{4} = 16\) 64
26. \(\frac{a}{16} = 9\) 144
27. \(-84 = \frac{d}{3}\) \(-252\)
28. \(-\frac{d}{7} = -13\)
29. \(\frac{t}{4} = -13\)
30. \(31. \) \(-9\)
31. \(-9\)
32. \(\frac{q}{3} = -4\)
33. \(\frac{5}{9}p = -10\)
34. \(\frac{d}{10} = \frac{2}{5}\) 4

Glencoe Algebra 1
2-2 Practice

Solving One-Step Equations

Solve each equation. Check your solution.

1. \( d - 8 = 17 \)
   \[ d = 25 \]

2. \( v + 12 = -5 \)
   \[ v = -17 \]

3. \( b - 2 = -11 \)
   \[ b = -9 \]

4. \( -16 = 3m + 71 \)
   \[ m = -25 \]

5. \( 29 = a - 76 \)
   \[ a = 105 \]

6. \( -14 + y = -2 \)
   \[ y = 12 \]

7. \( 8 - (-n) = 1 \)
   \[ n = 7 \]

8. \( 78 + r = -15 \)
   \[ r = -93 \]

9. \( f + (-3) = -9 \)
   \[ f = -6 \]

10. \( \frac{8}{y} = 96 \)
    \[ y = \frac{1}{12} \]

11. \( -13z = -39 \)
    \[ z = 3 \]

12. \( -180 = \frac{15n}{m} \)
    \[ n = -12 \]

13. \( 243 = 27w \)
    \[ w = 9 \]

14. \( \frac{7}{9} = -8 \)
    \[ x = -72 \]

15. \( \frac{6}{2} - \frac{1}{6} = 8 \)
    \[ x = 96 \]

16. \( \frac{a}{5} = \frac{4}{5} \)
    \[ a = 12 \]

17. \( \frac{8}{27} = \frac{2}{9} \)
    \[ x = 6 \]

18. \( \frac{q}{9} = \frac{1}{6} \)
    \[ x = 4 \]

Write an equation for each sentence. Then solve the equation.

19. Negative nine times a number equals -117.
    \[-9n = -117; \quad n = 13 \]

20. Negative one eighth of a number is \(-\frac{3}{4} \times \frac{1}{8} = -\frac{3}{4}
    \[ n = -\frac{3}{4} \]

21. Five sixths of a number is \(\frac{5}{9} \times \frac{6}{n} = \frac{5}{9} - \frac{2}{3}
    \[ n = 9 \]

22. 2.7 times a number equals 8.37.
    \[ 2.7n = 8.37; \quad n = 3.1 \]

23. HURRICANES The day after a hurricane, the barometric pressure in a coastal town has risen to 29.7 inches of mercury, which is 2.9 inches of mercury higher than the pressure when the eye of the hurricane passed over.

   a. Write an addition equation to represent the situation. \(b + 2.9 = 29.7\)

   b. What was the barometric pressure when the eye passed over? 26.8 in. of mercury

24. ROLLER COASTERS Kingda Ka in New Jersey is the tallest and fastest roller coaster in the world. Riders travel at an average speed of 61 feet per second for 3118 feet. They reach a maximum speed of 187 feet per second.

   a. If \(x\) represents the total time that the roller coaster is in motion for each ride, write an expression to represent the situation. (Hint: Use the distance formula \(d = rt\).) \(61x = 3118\)

   b. How long is the roller coaster in motion? 51.1 seconds

2-2 Word Problem Practice

Solving One-Step Equations

1. SUPREME COURT Chief Justice William Rehnquist served on the Supreme Court for 33 years until his death in 2005. Write and solve an equation to determine the year he was confirmed as a justice on the Supreme Court.
   \[ 2005 - x = 33; \quad x = 1972 \]

2. SALARY In a recent year, the annual salary of the Governor of New York was $179,000. During the same year, the annual salary of the Governor of Tennessee was $94,000 less. Write and solve an equation to determine the annual salary of the Governor of Tennessee in that year.
   \[ 179,000 - 94,000 = s \quad \text{or} \quad 179,000 - s = 94,000; \quad s = 85,000 \]

3. WEATHER On a cold January day, Mavis noticed that the temperature dropped 21 degrees over the course of the day to -9°C. Write and solve an equation to determine what the temperature was at the beginning of the day.
   \[ x - 21 = -9; \quad x = 12°C \]

4. FARMING Mr. Hill’s farm is 126 acres. Mr. Hill’s farm is \(\frac{1}{3}\) the size of Mr. Miller’s farm. How many acres is Mr. Miller’s farm?
   \[ 504 \text{ acres} \]

5. NAUTICAL On the sea, distances are measured in nautical miles rather than miles.

   a. If a boat travels 16 knots in 1 hour, how far will it have traveled in feet?
      Write and solve an equation.
      \[ 6080 \times 16 = x; \quad x = 97,280 \text{ ft} \]

   b. About how fast was the boat traveling in miles per hour? Round your answer to the nearest hundredth.
      \[ 97,280 \div 5280 = 18.42 \text{ mph} \]
Elevator Puzzle
José gets on the elevator and rides without pushing any buttons. First, the elevator goes up 4 floors where Bob gets on. Bob goes down 6 floors and gets off. At that same floor Florence gets on and goes up one floor before getting off. The elevator then moves down 8 floors to pick up the Hartt family who ride down 3 floors and get off. Then the elevator goes up one floor, picks up Kris, and goes down 6 floors to the street level where José exits the elevator.

1. Suppose \( x \) is your starting point. Write an equation that represents José's elevator ride.

\[ x + 4 - 6 + 1 - 8 - 3 + 1 - 6 = 0 \]

2. At what floor did José get on the elevator? 17th floor

Now that you know the starting point of José, the starting point of every other person who rode the elevator can be determined.

3. At what floor did Bob get on the elevator? At what floor did Bob get off?

21st floor; 15th floor

4. At what floor did Florence get on the elevator? At what floor did Florence get off?

15th floor; 16th floor

5. At what floor did the Hartt family get on the elevator? At what floor did the Hartt family get off?

8th floor; 5th floor

6. At what floor did Kris get on the elevator? At what floor did Kris get off?

6th floor; street level floor

Example 2
A number is divided by 2, and then 8 is subtracted from the quotient. The result is 16. What is the number?
Solve the problem by working backward.
The final number is 16. Undo subtracting 8 by adding 8 to get 24. To undo dividing 24 by 2, multiply 24 by 2 to get 48.
The original number is 48.

Example 2
A bacteria culture doubles each half hour. After 3 hours, there are 6400 bacteria. How many bacteria were there to begin with?
Solve the problem by working backward.
The bacteria have grown for 3 hours. Since there are 2 one-half hour periods in one hour, in 3 hours there are 6 one-half hour periods. Since the bacteria culture has grown for 6 time periods, it has doubled 6 times. Undo the doubling by halving the number of bacteria 6 times.

\[ \frac{6400}{1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \frac{6400 \times \frac{1}{64}}{100} \]

There were 100 bacteria to begin with.

Exercises
Solve each problem by working backward.

1. A number is divided by 3, and then 4 is added to the quotient. The result is 8. Find the number. 12

2. A number is multiplied by 5, and then 3 is subtracted from the product. The result is 12. Find the number. 3

3. Eight is subtracted from a number, and then the difference is multiplied by 2. The result is 24. Find the number. 20

4. Three times a number plus 3 is 24. Find the number. 7

5. CAR RENTAL Angela rented a car for $29.99 a day plus a one-time insurance cost of $5.00. Her bill was $124.96. For how many days did she rent the car? 4 days

6. MONEY Mike withdrew an amount of money from his bank account. He spent one fourth for gasoline and had $90 left. How much money did he withdraw? $120
2-3 Study Guide and Intervention (continued)

Solving Multi-Step Equations

To solve equations with more than one operation, often called multi-step equations, undo operations by working backward. Reverse the usual order of operations as you work.

Example

Solve $5x + 3 = 23$.

Original equation

$5x + 3 = 23$

Subtract 3 from each side.

$5x = 20$

Simplify.

$\frac{5x}{5} = \frac{20}{5}$

Divide each side by 5.

$x = 4$

Simplify.

Exercises

Solve each equation. Check your solution.

1. $5x + 2 = 27$  
2. $6x + 9 = 27$  
3. $5x + 16 = 51$  
4. $14x - 8 = 34$  
5. $0.6x - 1.5 = 1.8$  
6. $\frac{7}{8}p - 4 = 10$  
7. $16 = \frac{d - 12}{4}$  
8. $8 + \frac{30}{12} = 13$  
9. $\frac{6}{x} + 3 = -13$  
10. $4b + 8 = 10$  
11. $0.2x - 8 = -2$  
12. $3.2y - 1.8 = 3$  
13. $-4 = \frac{7x - (-1)}{8}$  
14. $8 = -12 + k$  
15. $0 = 10y - 40$

Write an equation and solve each problem.

16. Find three consecutive integers whose sum is 96. $n + (n + 1) + (n + 2) = 96; 31, 32, 33$

17. Find two consecutive odd integers whose sum is 176. $n + (n + 2) = 176; 87, 89$

18. Find three consecutive integers whose sum is −93. $n + (n + 1) + (n + 2) = -93; -32, -31, -30$

2-3 Skills Practice

Solving Multi-Step Equations

Solve each problem by working backward.

1. A number is divided by 2, and then the quotient is added to 8. The result is 33. Find the number. 50

2. Two is subtracted from a number, and then the difference is divided by 3. The result is 30. Find the number. 92

3. A number is multiplied by 2, and then the product is added to 9. The result is 49. What is the number? 20

4. ALLOWANCE After Ricardo received his allowance for the week, he went to the mall with some friends. He spent $15 of his allowance on a new paperback book. Then he bought himself a snack for $1.25. When he arrived home, he had $5.00 left. How much was his allowance? $12.50

Solve each equation. Check your solution.

5. $5x + 3 = 23$  
6. $4 = 3a - 14$  
7. $2y + 5 = 19$

8. $6 + 5x = -29 - 7$  
9. $8 - 5w = -37$  
10. $18 - 4w = 42 - 6$

11. $\frac{n}{3} - 8 = -2$  
12. $5 + \frac{k}{4} = 1 - 16$  
13. $-\frac{b}{3} - 4 = 13 - 51$

14. $-\frac{d}{6} + 12 = -7$  
15. $\frac{5g}{3} - 2 = 9$  
16. $\frac{d}{7} + 3 = -1$

17. $\frac{3}{4}q - 7 = 8$  
18. $\frac{2}{3}g + 6 = -12 - 27$  
19. $\frac{3}{2}x - 8 = -3$

20. $\frac{1}{5}m + 2 = 6$  
21. $\frac{5-5}{4} = 3$  
22. $\frac{b+1}{3} = 2$

Write an equation and solve each problem.

23. Twice a number plus four equals 6. What is the number? $2n + 4 = 6; 1$

24. Sixteen is seven plus three times a number. Find the number. $16 = 7 + 3n; 3$

25. Find two consecutive integers whose sum is 35. $n + (n + 1) = 35; 17, 18$

26. Find three consecutive integers whose sum is 36. $n + (n + 1) + (n + 2) = 36; 11, 12, 13$
2-3 Practice

Solving Multi-Step Equations

Solve each problem by working backward.

1. Three is added to a number, and then the sum is multiplied by 4. The result is 16. Find the number. 1

2. A number is divided by 4, and the quotient is added to 3. The result is 24. What is the number? 8

3. Two is subtracted from a number, and then the difference is multiplied by 5. The result is 30. Find the number. 8

4. BIRD WATCHING While Michelle sat observing birds at a bird feeder, one-fourth of the birds flew away when they were startled by a noise. Two birds left the feeder to go to another station a few feet away. Three more birds flew into the branches of a nearby tree. Four birds remained at the feeder. How many birds were at the feeder initially? 12

Solve each equation. Check your solution.

5. \(-2n - 19 = 77\) \(-8\)

6. \(17 + \frac{3}{4}n = 14\) \(-1\)

7. \(15x + 4 = 49\) \(3\)

8. \(\frac{n}{5} + 6 = 2\) \(-20\)

9. \(\frac{9}{4} + 3 = 15\) \(-48\)

10. \(\frac{5}{3} - 6 = -2\) \(12\)

11. \(\frac{1}{2}y - \frac{1}{3} = \frac{7}{8}\) \(2\)

12. \(-32 - \frac{3}{5}f = -17\) \(-25\)

13. \(8 - \frac{3}{8}k = -4\) \(32\)

14. \(r + \frac{13}{12} = 1\) \(-1\)

15. \(\frac{15 - a}{3} = -9\) \(42\)

16. \(\frac{33}{7} = 16\) \(29\)

17. \(\frac{5}{7} - 0.5 = 2.5\) \(21\)

18. \(2.5g + 0.45 = 0.95\) \(0.2\)

19. \(0.4m - 0.7 = 0.22\) \(2.3\)

Write an equation and solve each problem.

20. Seven less than four times a number equals 13. What is the number? \(4n - 7 = 13\) \(5\)

21. Find two consecutive odd integers whose sum is 116. \(n + (n + 2) = 116\) \(57, 59\)

22. Find two consecutive even integers whose sum is 136. \(n + (n + 2) = 126\) \(62, 64\)

23. Find three consecutive odd integers whose sum is 117. \(n + (n + 2) + (n + 4) = 117\) \(37, 39, 41\)

24. COIN COLLECTING Jung has a total of 92 coins in his coin collection. This is 8 more than three times the number of quarters in the collection. How many quarters does Jung have in his collection? \(28\)
Consecutive Integer Problems

Many types of problems and puzzles involve the idea of consecutive integers. Knowing how to represent these integers algebraically can help to solve the problem.

Example 1
Find four consecutive odd integers whose sum is –80.

An odd integer can be written as 2n + 1, where n is any integer. If 2n + 1 is the first odd integer, then add 2 to get the next largest odd integer, and so on.

Now write an equation to solve this problem.

(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) = –80

Exercise
Write an equation for each problem. Then solve.

1. Complete the solution to the problem in the example.
   –23, –21, –19, –17

2. Find three consecutive even integers whose sum is 132.
   2n + (2n + 2) + (2n + 4) = 132; n = 21; 42, 44, 46

3. Find two consecutive integers whose sum is 19.
   n + (n + 1) = 19; 9, 10

4. Find two consecutive integers whose sum is 100.
   n + (n + 1) = 100; no solution

5. The lesser of two consecutive even integers is 10 more than one-half the greater. Find the integers.
   2n = 10 + 1/2 (2n + 2); 22 and 24

6. The greater of two consecutive even integers is 6 less than three times the lesser. Find the integers.
   2n + 2 = 3(2n) – 6; 4, 6

7. Find four consecutive integers such that twice the sum of the two greater integers exceeds three times the first by 91.
   2[(n + 2) + (n + 3)] = 3n + 91; 81, 82, 83, 84

8. Find a set of four consecutive positive integers such that the greatest integer in the set is twice the least integer in the set.
   n + 3 = 2n; 3, 4, 5, 6
2-4 Study Guide and Intervention (continued)

Solving Equations with the Variable on Each Side

Grouping Symbols When solving equations that contain grouping symbols, first use the Distributive Property to eliminate grouping symbols. Then solve.

Example

Solve \(4(2a - 1) = 10(a - 5)\).

\[
4(2a - 1) = 10(a - 5) \\
8a - 4 = 10a - 50 \text{ Simplify.} \\
8a - 4 + 10a = 10a - 50 + 10a \text{ Add } 10a \text{ to each side.} \\
18a - 4 = 20a - 50 \text{ Simplify.} \\
18a - 4 + 4 = 20a - 50 + 4 \text{ Add } 4 \text{ to each side.} \\
18a = 54 \text{ Simplify.} \\
\frac{18a}{18} = \frac{54}{18} \text{ Divide each side by } 18. \\
a = 3 \text{ Simplify.}
\]

The solution is 3.

Exercises

Solve each equation. Check your solution.

1. \(-3x + 5 = 3(x - 1)\)
   \[-2\]
2. \(2(7 + 3t) = -t\)
   \[-2\]
3. \(3(x + 1) - 5 = 3x - 2\)
   \[all \ numbers\]
4. \(75 - 9g = 5(-4 + 2g)\)
   \[\frac{5}{g} = \frac{2}{f(x - 3) + f(x - 3)}\]
5. \(5(f + 2) = 2(x - 3)\)
   \[6. \ 4p + 3 = 36\]
6. \(\frac{5}{p} + 3 = 9\)
   \[6\]
7. \(18 = 3(2t + 2)\)
   \[2\]
8. \(3d = 8\)
   \[3d = 8\]
9. \(5p + 3 = 9 = 3p - 2 = 6\)
   \[-12\]
10. \(4k - 2 = 2(5 - b)\)
    \[2\]
11. \(4 \qquad 3(x - 2) = 2 - x\)
    \[2\]
12. \(\frac{x + y}{2} = \frac{y}{x}\)
    \[-2\]
13. \(\frac{a - 8}{12} = \frac{2k + 3}{4}\)
    \[-6\]
14. \(2(4 + 2k) + 10 = k\)
    \[-1\]
15. \(2w - 1 + 4 = 4(w + 1)\)
    \[1\]
16. \(6n - 1 = 2(2m + 4)\)
    \[7\]
17. \(2[3 + (y - 1)] = 22\)
    \[4\]
18. \(-4r + 2 = 4(2 - 4r)\)
    \[1\]
19. \(-3x - 8 = 24\)
    \[0\]
20. \(4(4 - 4k) = 10 - 16k\)
    \[\text{no solution}\]
21. \(6(2 - 2y) = 5(2y - 2)\)
    \[1\]

Solve each equation. Check your solution.

1. \(2m + 12 = 3m - 31\)
   \[43\]
2. \(5\)
   \[2\]
3. \(t = 3 - 2t = 3\)
   \[3\]
4. \(6.2h - 8 = h + 17\)
   \[25\]
5. \(a = 3 - 2d = 3\)
   \[3\]
6. \(4h - 12 = 12 - 4h = 3\)
   \[3\]
7. \(4r - 9 = 7r + 12 = -7\)
   \[8\]
8. \(6r - 3 = 3 - 6r = -6\)
   \[no \ solution\]
9. \(5 + 3r = 5r - 19 = 12\)
   \[10\]
10. \(-9 + 8r = 7 + 4r = 4\)
    \[4\]
11. \(8y + 12 = 4y + 2y = 12\)
    \[\text{all \ numbers}\]
12. \(3(5 + 2) = 3(5 + 6) = -2\)
    \[1\]
13. \(6 - 3z - 1 \equiv z - 2z - 2\)
    \[\text{no \ solution}\]
14. \(-7(2b - 4) = 5 - 2b + 6 = -0.5 or -1\)
    \[2\]
15. \(3b - 3b = 0\)
    \[1\]
16. \(3(3b + 7) = 4 - 3 - 2b\)
    \[13\]
17. \(8(3y - 2) = 7(3y + 2) = 0\)
    \[15\]
18. \(5(-6 - 3d) = 3(8 + 7d) = -1.5 or 1\)
    \[2\]
19. \(6m - 1 = 3(3x + 5) = -7\)
    \[2\]
20. \(7(1 - y + 3) = 8(3y - 2) = 2\)
    \[3\]
21. \(\frac{1}{2} + \frac{5}{8} = \frac{7}{8} + \frac{7}{2} = -2\)
    \[9\]
2-4 Practice

Solving Equations with the Variable on Each Side

Solve each equation. Check your solution.

1. \(5x - 3 = 13 - 3x\) \(\Rightarrow \) \(2x = 10\) \(\Rightarrow \) \(x = 5\)

2. \(-4r - 11 = 4r + 21\) \(\Rightarrow \) \(-8r = 32\) \(\Rightarrow \) \(r = -4\)

3. \(1 - m = 6 - 6m\) \(\Rightarrow \) \(5m = 5\) \(\Rightarrow \) \(m = 1\)

4. \(14 + 5n = -4n + 17\) \(\Rightarrow \) \(9n = 3\) \(\Rightarrow \) \(n = \frac{1}{3}\)

5. \(\frac{1}{2}k - 3 = 2 - \frac{3}{4}k\) \(\Rightarrow \) \(\frac{11}{4}k = 5\) \(\Rightarrow \) \(k = \frac{20}{11}\)

6. \(\frac{1}{2}(6 - y) = y\) \(\Rightarrow \) \(6 - y = 2y\) \(\Rightarrow \) \(3y = 6\) \(\Rightarrow \) \(y = 2\)

7. \(3x - 2 - 3x = -9x - 4\) no solution

8. \(4(4 - w) = 3(2w + 2)\) \(\Rightarrow \) \(16 - 4w = 6w + 6\) \(\Rightarrow \) \(10w = 10\) \(\Rightarrow \) \(w = 1\)

9. \(9(6 - 1) = 2(96 + 3)\) \(\Rightarrow \) \(276 = 201\) \(\Rightarrow \) no solution

10. \(3(6 + 5y) = 2(-5 + 4y)\) \(\Rightarrow \) \(18 + 15y = -10 + 8y\) \(\Rightarrow \) \(7y = -28\) \(\Rightarrow \) \(y = -4\)

11. \(-5x - 10 = 2 - (x + 4)\) \(\Rightarrow \) \(-5x = 8\) \(\Rightarrow \) \(x = -\frac{8}{5}\)

12. \(6 + 2(3y - 3) = 4(1 + j)\) \(\Rightarrow \) \(6 + 6y - 6 = 4 + 4j\) \(\Rightarrow \) \(12 = 4j + 6 + 6y\) \(\Rightarrow \) \(j = \frac{6 - 6y}{2}\)

13. \(\frac{5}{2}x - t = 3 + \frac{3}{2}t\) no solution

14. \(1.4f + 1.1 = 8.3 - f\) \(\Rightarrow \) \(2.5f = 7.2\) \(\Rightarrow \) \(f = \frac{72}{25}\)

15. \(\frac{2}{3}x - \frac{1}{6} = \frac{1}{2}x + \frac{5}{6}\) \(\Rightarrow \) \(\frac{1}{3}x = \frac{1}{2}\) \(\Rightarrow \) \(x = \frac{3}{2}\)

16. \(2 - \frac{3}{4}k = \frac{1}{8}k + 9\) \(\Rightarrow \) \(-\frac{13}{4}k = 7\) \(\Rightarrow \) \(k = -\frac{28}{13}\)

17. \(\frac{1}{2}(3g - 2) = \frac{g}{2}\) \(\Rightarrow \) \(\frac{3g - 2}{2} = \frac{g}{2}\) \(\Rightarrow \) \(3g = g + 2\) \(\Rightarrow \) \(2g = 2\) \(\Rightarrow \) \(g = 1\)

18. \(\frac{1}{3}(n + 1) = \frac{1}{6}(3n - 5)\)

19. \(\frac{1}{2}(5 - 2h) = \frac{1}{2} + \frac{2}{3}h\) \(\Rightarrow \) \(\frac{5 - 2h}{2} = \frac{1}{2} + \frac{2}{3}h\) \(\Rightarrow \) \(5 - 2h = 1 + 4h\) \(\Rightarrow \) \(6h = 4\) \(\Rightarrow \) \(h = \frac{2}{3}\)

20. \(\frac{1}{5}(2m - 16) = \frac{3}{2}(m + 4)\) \(\Rightarrow \) \(\frac{2m - 16}{5} = \frac{3m + 12}{2}\) \(\Rightarrow \) \(4(2m - 16) = 15m + 60\) \(\Rightarrow \) \(8m - 64 = 15m + 60\) \(\Rightarrow \) \(-7m = 124\) \(\Rightarrow \) \(m = -\frac{124}{7}\)

21. \(3(d - 8) - 5 = 9(d + 2) + 1 - 8\) \(\Rightarrow \) \(3d - 24 - 5 = 9d + 18 + 1 - 8\) \(\Rightarrow \) \(3d - 29 = 9d + 11\) \(\Rightarrow \) \(-6d = 40\) \(\Rightarrow \) \(d = -\frac{20}{3}\)

22. \(2a - 8 + 7 = 5(a + 2) - 3a - 19\) \(\Rightarrow \) \(2a - 1 = 5a + 10 - 3a\) \(\Rightarrow \) \(2a - 1 = 2a + 10\) \(\Rightarrow \) \(-1 = 10\) \(\Rightarrow \) no solution

23. NUMBERS Two thirds of a number reduced by 11 is equal to 3 more than the number. Find the number. \(-9\)

24. NUMBERS Five times the sum of a number and 3 is the same as 3 times muliplied by 1 less than twice the number. What is the number? \(18\)

25. NUMBER THEORY Tripling the greater of two consecutive even integers gives the same result as subtracting 10 from the lesser even integer. What are the integers? \(-8, -6\)

26. GEOMETRY The formula for the perimeter of a rectangle is \(P = 2l + 2w\), where \(l\) is the length and \(w\) is the width. A rectangle has a perimeter of 24 cm. Find its dimensions if its length is 3 inches greater than its width. \(4.5\) in. by \(7.5\) in.
Chapter 2

2-4 Enrichment

Identities

An equation that is true for every value of the variable is called an identity. When you try to solve an identity, you end up with a statement that is always true. Here is an example.

Example

Solve $8 - (5 - 6x) = 3(1 + 2x)$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 - 5 - 6x = 3 + 2x$</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>$8 - (-6x) = 3 + 2x$</td>
<td>Add</td>
</tr>
</tbody>
</table>

Exercises

State whether each equation is an identity. If it is not, find its solution.

1. $2x - 3(x) = 3(x + x) + 4$
2. $5m + 1 + 6 = 3(4 + m) + (2m - 1)$
3. $(5x - 9) - (9x - 13) = 2(1 + x)$
4. $14 - 6 - 3x = 4c - c$
5. $3y - 2y + 19 = 9y - 3y - 3$
6. $3(z - 1) = 4(h + 3)$
7. Use the true equation $3x - 2 = 3x - 2$ to create an identity of your own. Sample answer: $6x - 4 = 2(3x - 2)$
8. Use the false equation $1 = 2$ to create an equation with no solution. Sample answer: $2x + 1 = 2x + 2$
9. Create an equation whose solution is $x = 3$. Sample answer: $4x + 2(x + 1) = 20$

2-4 Graphing Calculator Activity

Solving Linear Equations

One way to solve a linear equation in one variable is to graph each side of the equation as a separate entity. The intersection of these two graphs identifies the $x$ value for which the equation is true.

Example 1

Solve $5x - 9 = -3x + 7$.

Enter the left side of the equation into $Y_1$ and the right side into the $Y_2$.

Keystrokes: $Y = $ $5 \times \text{X,T,} \text{\textasciitilde} + 9 \times \text{ENTER} \quad 3 \times \text{TARK} + 1 \times 7$

Graph the equations.

Keystrokes: $\text{ZOOM} \quad 0 \quad \text{ZOOM} \quad 8 \times \text{ENTER}$

Use the TRACE key to find the intersection.

Notice the coordinates at the bottom of the screen do not change as you toggle up and down. This indicates a common solution. 2. Substitute 2 into the equation to check the answer.

Example 2

Solve $\frac{3a}{4} + 16 = 2 - \frac{1}{4}a$.

Enter each side of the equation into $Y_1$.

Keystrokes: $Y = $ $\frac{3a}{4} \times \text{X,T,} \text{\textasciitilde} + 16 \times \text{ENTER} \quad 2 \times \text{X,T,} \text{\textasciitilde} + 1 	imes 1$

 TRACE to locate the point of intersection at $x = -16$. So, the solution to the equation is $a = -16$.

Exercises

Solve each equation.

1. $7x - 3 = 7$
2. $5 - \frac{1}{2}(x - 6) = 4$
3. $\frac{3}{2}(7 + 2x) = -\frac{5}{6}$
4. $2x - 2 = 11x + 262.6$
5. $1.03x - 4 = -2.15x + 8.72$
6. $\frac{x + 1}{3} = \frac{2-c}{5}$
7. $6x - 2x - 3 = 4x + 1$
8. $5(x - 2) + 2x = 7(x + 4) - 3$
9. $\frac{2}{3} = 2x$
10. Not all equations have integer solutions. Investigate using [CALC] 1 to solve these equations.
2-5 Study Guide and Intervention

Solving Equations Involving Absolute Value

Absolute Value Expressions
Expressions with absolute values define an upper and lower range in which a value must lie. Absolute value expressions can be evaluated using the given value for the variable.

Example
Evaluate $|t - 5| - 7$ if $t = 3$.

$$|t - 5| - 7 = |3 - 5| - 7 = | -2 | - 7 = 2 - 7 = 1 - 3 = 2 = -5$$

Simplify.

Exercises
Evaluate each expression if $r = -2$, $n = -3$, and $t = 3$.

1. $|8 - r| + 3$  8  
2. $|t - 3| - 7 - 7$  11  
3. $5 + |3 - n| - 2$  10

Evaluate each expression if $n = 2$, $q = -1.5$, $r = -3$, $v = -8$, $w = 4.5$, and $x = 4$.

4. $|r + n| - 7 - 2$  10  
5. $|n - r| + 4$  10  
6. $-|r + n + t|$  10

Example 1
Solve $|x + 4| = 1$. Then graph the solution set.

Write $|x + 4| = 1$ as $x + 4 = 1$ or $x + 4 = -1$.

$x + 4 = 1$ or $x + 4 = -1$

$x = 5$ or $x = -5$

The solution set is $[5, -5]$.

The graph is shown below.

Example 2
Write an equation involving absolute value for each graph.

Example 3
Evaluate $y = |x - 1|$ when $x = 3$.

$y = |3 - 1| = 2$

Exercises
Solve each equation. Then graph the solution set.

1. $|y| = 3$ [-3, 3]  
2. $|x - 4| = 4$ [0, 8]  
3. $|y + 3| = 2$ [-5, -1]

Write an equation involving absolute value for each graph.

13. $|x| = 4$  
14. $|x - 1| = 2$  
15. $|x + 3| = 4$

Answers (Lesson 2-5)
**2-5 Skills Practice**

**Solving Equations Involving Absolute Value**

Evaluate each expression if \( a = 2, b = -3, \) and \( c = -4. \)

1. \(|a - 5| - 1\)
2. \(|b + 1| + 8\)
3. \(5 - |c + 1|\)
4. \(|a + b| - c\)

Solve each equation. Then graph the solution set.

5. \(|w + 1| = 5\)
6. \(|c - 3| = 1\)
7. \(|n + 2| = 1\)
8. \(|t + 6| = 4\)
9. \(|w - 3| = 2\)
10. \(|a - 5| = 4\)

Write an equation involving absolute value for each graph.

11. \(|x| = 1\)
12. \(|x + 3| = 2\)
13. \(|x - 4| = 1\)
14. \(|x| = 4\)

**2-5 Practice**

**Solving Equations Involving Absolute Value**

Evaluate each expression if \( x = -1, y = 3, \) and \( z = -4. \)

1. \(16 - |2x + 1|\)
2. \(|x - y| + 4\)
3. \(|-3y + x| - x\)
4. \(3z - |x| + |2 - y|\)

Solve each equation. Then graph the solution set.

5. \(|2x - 9| = 1\)
6. \(|3 - 2r| = 7\)
7. \(|2 + 6| = 9\)
8. \(|2g - 5| = 9\)
9. \(|x - 6| = 5\)
10. \(|x + 4| = 2\)
11. \(|x + 3| = 4\)
12. \(|x - 2| = 4\)

Write an equation involving absolute value for each graph.

13. **FITNESS** Taisha uses the elliptical cross-trainer at the gym. Her general goal is to burn 280 Calories per workout, but she varies by as much as 25 Calories from this amount on any given day. Write and solve an equation to find the maximum and minimum number of Calories Taisha burns on the cross-trainer.

\[ |c - 280| = 25; \ \text{min} = 255; \ \text{max} = 305 \]

14. **TEMPERATURE** A thermometer is guaranteed to give a temperature no more than \( 1.2\)°F from the actual temperature. If the thermometer reads \( 28\)°F, write and solve an equation to find the maximum and minimum temperatures it could be.

\[ |t - 28| = 1.2; \ \text{min} = 26.8\)°F; \ \text{max} = 29.2\)°F \]
2-5 Word Problem Practice

Solving Open Sentences Involving Absolute Value

1. ENGINEERING Tolerance in engineering is an allowance made for imperfections in a manufactured object. The manufacturer of an oven specifies a temperature tolerance of ±15°F. This means that the temperature inside the oven will be within 15°F of the temperature that it is set on. Write an absolute value expression to represent the maximum and minimum temperatures inside the oven when the thermostat is set on 400°F.

\[ |t - 400| = 15 \]

2. AVIATION The circle graph shows the results of a survey that asked 4300 students ages 7 to 18 what they thought would be the most important benefit of air travel in the future. There are about 40 million students in the United States. If the margin of error is ±3%, what is the range of the number of students ages 7 to 18 who would likely say that “Finding new resources for Earth” is the most important benefit of future flight?

The Next Frontier for Flight

Finding new resources for Earth: 21%
Exploring other planets in our solar system: 43%
Flying faster from one continent to another: 10%
Scientific experiments in space: 26%

Source: The World Almanac between 7,200,000 and 9,600,000

3. COLLEGE A certain scholarship and student loan fund uses a formula to determine whether or not a student qualifies for college funding. The formula is \( |k - 6| = 15 \), where \( k \) is a need score determined by an interview. What are the possible need scores?

\( \{-7, 3\} \)

4. STATISTICS The most familiar statistical measure is the arithmetic mean, or average. A second important statistical measure is the standard deviation, which is a measure of how far the individual scores are from the mean. For example, the mean score on the Wechsler IQ test is 100 and the standard deviation is 15. This means that people within one deviation of the mean have IQ scores that are 15 points higher or lower than the mean.

a. Write an absolute value equation to find the maximum and minimum test scores if the mean was 80 and the standard deviation was 12.

\[ |t - 80| = 12 \]

b. What is the range of Wechsler IQ test scores ±3 standard deviations from the mean?

55 to 145

5. The actual measurement is between 12.5 cm and 13.5 cm.

6. The actual measurement is between 23.05 mm and 23.15 mm.

For each measurement, give the smallest unit used and the absolute error.

7. The actual measurement is between 12.5 cm and 13.5 cm. 1 cm; ±0.5 cm

8. The actual measurement is between 12 3/8 in. and 12 3/8 in. 1/8 in.; ±1/8 in.

9. The actual measurement is between 56 3/8 in. and 57 3/2 in. 1 in.; ±1/2 in.

10. The actual measurement is between 23.05 mm and 23.15 mm. 0.1 mm; ±0.05 mm
### Ratios and Proportions

A ratio is a comparison of two numbers by division. The ratio of $x$ to $y$ can be expressed as $x : y$, $\frac{x}{y}$, or $x:y$. Ratios are usually expressed in simplest form. An equation stating that two ratios are equal is called a proportion. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

#### Example 1
Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ are equivalent ratios. Write yes or no. Justify your answer.

- $\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.
- $\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

#### Example 2
Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.

- $10 \div 18 = \frac{5}{9}$
- $10(45) = 450$ (Cross products)
- $25 \div 45 = \frac{5}{9}$
- $450 = 450$ (Simplify)

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

### Exercises

Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. $\frac{1}{2} : \frac{3}{8}$
   - yes
2. $\frac{5}{8} : \frac{10}{15}$
   - no
3. $\frac{10}{20} : \frac{35}{49}$
   - no
4. $\frac{25}{35} : \frac{15}{20}$
   - no
5. $\frac{12}{20} : \frac{3}{5}$
   - yes
6. $\frac{15}{20} : \frac{20}{27}$
   - yes
7. $\frac{5}{8} : \frac{9}{12}$
   - yes
8. $\frac{20}{30} : \frac{3}{4}$
   - yes
9. $\frac{25}{45} : \frac{5}{9}$
   - yes
10. $\frac{30}{50} : \frac{30}{5} \div 2$ (simplify)
11. $\frac{50}{44} : \frac{15}{33}$
12. $\frac{5}{8} : \frac{6}{9}$
13. $\frac{0.05}{1.1} : \frac{1}{20}$
14. $\frac{0.45}{0.9}$
15. $\frac{0.5}{0.8}$
16. $\frac{5}{8} : \frac{6}{9}$
17. $\frac{15}{12}$
18. $\frac{3}{2} : \frac{4}{2}$

#### Solve Proportions

If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{5}{6} = \frac{10}{12}$, $x$ and $13$ are called extremes. They are the first and last terms of the proportion. $5$ and $10$ are called means. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

---

**Means-Extremes Property of Proportions**

For any numbers $a$, $b$, $c$, and $d$, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

#### Example
Solve $\frac{x}{5} = \frac{10}{13}$.

- $\frac{x}{5} = \frac{10}{13}$ (Original proportion)
- $13x = 50$ (Cross products)
- $x = 4 \div 13$ (Simplify)

**Exercises**

Solve each proportion. If necessary, round to the nearest hundredth.

1. $\frac{3}{x} = \frac{2}{8}$
2. $\frac{2}{y} = \frac{3}{5}$
3. $\frac{0.1}{3} = \frac{5}{x}$
4. $\frac{x}{4} = \frac{1}{4}$
5. $\frac{2}{3} = \frac{8}{x}$
6. $\frac{x}{21} = \frac{3}{63}$
7. $\frac{7}{9} = \frac{y}{1}$
8. $\frac{2}{3} = \frac{1}{2}$
9. $\frac{5}{12} = \frac{y}{24}$
10. $\frac{4}{b} = \frac{2}{3}$
11. $\frac{1}{3} = \frac{12}{x}$
12. $\frac{3+y}{y} = \frac{3}{8}$
13. $\frac{a}{18} = \frac{15}{78}$
14. $\frac{12}{6} = \frac{a}{24}$
15. $\frac{2+1}{3} = \frac{a}{12}$

Use a proportion to solve each problem.

16. **MODELS** To make a model of the Guadeloupe River bed, Hermie used 1 inch of clay for 5 miles of the river’s actual length. His model river was 50 inches long. How long is the Guadeloupe River? 250 mi

17. **EDUCATION** Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems? 3 h
Determine whether each pair of ratios are equivalent ratios. Write yes or no.

1. \( \frac{4}{5} : \frac{20}{25} \) yes
2. \( \frac{5}{9} : \frac{7}{11} \) no
3. \( \frac{6}{7} : \frac{24}{28} \) yes
4. \( \frac{8}{7} : \frac{8}{81} \) yes
5. \( \frac{7}{16} : \frac{42}{90} \) no
6. \( \frac{13}{29} : \frac{26}{38} \) yes
7. \( \frac{3}{21} : \frac{14}{98} \) yes
8. \( \frac{12}{50} : \frac{85}{85} \) no

Solve each proportion. If necessary, round to the nearest hundredth.

9. \( \frac{1}{a} = \frac{2}{14} \) 7
10. \( \frac{5}{b} = \frac{3}{9} \) 15
11. \( \frac{9}{e} = \frac{15}{10} \) 6
12. \( \frac{3}{a} = \frac{1}{2} \) 18
13. \( \frac{6}{2} = \frac{3}{5} \) 10
14. \( \frac{5}{f} = \frac{5}{21} \) 3
15. \( \frac{12}{7} = \frac{26}{m} \) 21
16. \( \frac{6}{20} = \frac{y}{60} \) 18
17. \( \frac{42}{56} = \frac{6}{f} \) 8
18. \( \frac{7}{9} = \frac{1}{9} \) 63
19. \( \frac{10}{14} = \frac{30}{m} \) 42
20. \( \frac{11}{29} = \frac{n}{60} \) 44
21. \( \frac{9}{c} = \frac{27}{39} \) 13
22. \( \frac{5}{12} = \frac{20}{b} \) 48
23. \( \frac{4}{21} = \frac{y}{84} \) 16
24. \( \frac{22}{3} = \frac{11}{30} \) 60
25. BOATING Hue’s boat used 5 gallons of gasoline in 4 hours. At this rate, how many gallons of gasoline will the boat use in 10 hours? 12.5 gal

Solve each proportion. If necessary, round to the nearest hundredth.

10. \( \frac{5}{a} = \frac{20}{54} \) 9
11. \( \frac{b}{4} = \frac{34}{23} \) 68
12. \( \frac{40}{56} = \frac{k}{7} \) 5
13. \( \frac{24}{49} = \frac{4}{w} \) 7
14. \( \frac{3}{a} = \frac{27}{162} \) 18
15. \( \frac{5}{3} = \frac{48}{9} \) 16
16. \( \frac{2}{a} = \frac{10}{60} \) 12
17. \( \frac{5}{11} = \frac{35}{x} \) 77
18. \( \frac{3}{5} = \frac{3}{17} \) 1
19. \( \frac{6}{61} = \frac{12}{b} \) 122
20. \( \frac{e}{6} = \frac{6}{4} \) 24
21. \( \frac{3}{4} = \frac{2}{a} \) 7
22. \( \frac{7}{9} = \frac{5}{10} \) 7
23. \( \frac{3}{a} = \frac{3}{5} \) 3
24. \( \frac{m}{5} = \frac{5}{8} \) 3
25. \( \frac{7}{a} = \frac{0.25}{0.4} \) 1
26. \( \frac{3}{0.72} = \frac{12}{3} \) 2.88
27. \( \frac{5}{a} = \frac{3}{0.51} \) 1.02
28. \( \frac{7}{a} = \frac{4}{11} \) 7
29. \( \frac{3}{12} = \frac{2}{y} \) 6
30. \( \frac{m}{2} = \frac{3}{4} \) 5
31. \( \frac{5}{12} = \frac{x + 1}{4} \) 2
32. \( z + 2 = \frac{3}{7} \) 3
33. \( \frac{z}{2} = \frac{x - 2}{6} \) 4

34. PAINTING Ysidra paints a room that has 400 square feet of wall space in 292 hours. At this rate, how long will it take her to paint a room that has 720 square feet of wall space? 412 h

35. VACATION PLANS Walker is planning a summer vacation. He wants to visit Petrified National Forest and Meteor Crater, Arizona, the 50,000-year-old impact site of a large meteor. On a map with a scale where 2 inches equals 75 miles, the two areas are about 13 inches apart. What is the distance between Petrified National Forest and Meteor Crater? About 56.25 mi
### 2-6 Word Problem Practice

#### Ratios and Proportions

1. **WATER** A dripping faucet wastes 3 cups of water every 24 hours. How much water is wasted in a week?
   - 21 cups

2. **GASOLINE** In November 2010 the average price of 5 gallons of regular unleaded gasoline in the United States was $14.46. What was the price for 16 gallons of gas?
   - $46.27

3. **SHOPPING** Stevenson’s Market is selling 3 packs of toothpicks for $0.87. How much will 10 packs of toothpicks cost at this price? Round your answer to the nearest cent.
   - $2.90

4. **BUILDINGS** Willis Tower in Chicago is 1450 feet tall. The John Hancock Center in Chicago is 1127 feet tall. Suppose you are asked to build a small-scale replica of each. If you make the Willis Tower 3 meters tall, what would be the approximate height of the John Hancock replica? Round your answer to the nearest hundredth.
   - 2.33 meters

5. **MAPS** A map of Waco, Texas and neighboring towns is shown below.

![Map of Waco, Texas and neighboring towns]

   - **a.** Use a metric ruler to measure the distances between Robinson and Neale on the map. 2 cm

   - **b.** Using the scale of the map, find the approximate actual distance by air (not by roads), between Robinson and Neale. 6.67 miles

   - **c.** Approximately how many square miles are shown on this map? $23.3 \times 23.3 = 543$ square miles

### 2-6 Enrichment

#### Angles of a Triangle

In geometry, many statements about physical space are proven to be true. Such statements are called **theorems**. Here are two examples of geometric theorems.

- **a.** The sum of the measures of the angles of a triangle is 180°.
- **b.** If two sides of a triangle have equal measure, then the two angles opposite those sides also have equal measure.

For each of the triangles, write an equation and then solve for $x$. (A tick mark on two or more sides of a triangle indicates that the sides have equal measure.)

1. $70 + 50 + x = 180$; $x = 60°$
2. $x + 45 + 90 = 180$; $x = 45°$
3. $x + x + 90 = 180$; $x = 45°$
4. $(x + 30) + x + (5x + 10) = 180$; $x = 20°$
5. $2x + 5x + x = 180$; $x = 22.5°$
6. $4x + 5x + 90 = 180$; $x = 10°$
7. $3x + (x - 15) + (x + 30) = 180$; $x = 33°$
8. $40 + 40 + x = 180$; $x = 100°$
9. $x + x + x = 180$; $x = 60°$
10. $30 + 2x + x = 180$; $x = 50°$
11. Two angles of a triangle have the same measure. The sum of the measures of these angles is one-half the measure of the third angle. Find the measures of the angles of the triangle. $30°, 30°, 120°$
12. The measure of one angle of a triangle is twice the measure of a second angle. The measure of the third angle is 12° less than the sum of the other two. Find the measures of the angles of the triangle. $64°, 32°, 84°$
2-7 Study Guide and Intervention

Percent of Change

When an increase or decrease in an amount is expressed as a percent, the percent is called the percent of change. If the new number is greater than the original number, the percent of change is a percent of increase. If the new number is less than the original number, the percent of change is the percent of decrease.

Example 1 Find the percent of increase.
original: 48
new: 60
First, subtract to find the amount of increase. The amount of increase is 60 − 48 = 12.

\[
\frac{12}{48} = \frac{r}{100} \quad \text{Percent proportion}
\]

12(100) = 48r
1200 = 48r
48 = r
Divide each side by 48.

The percent of increase is 25%.

Example 2 Find the percent of decrease.
original: 30
new: 22
First, subtract to find the amount of decrease. The amount of decrease is 30 − 22 = 8.

\[
\frac{8}{30} = \frac{r}{100} \quad \text{Percent proportion}
\]

8100 = 30r
800 = 30r
30 = 30r
Divide each side by 30.

\[
26.6 \text{ or about } 27%.
\]

Exercises

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 50
   new: 80
   increase; 60%
2. original: 90
   new: 100
   increase; 11%
3. original: 45
   new: 20
   decrease; 56%
4. original: 77.5
   new: 62
   decrease; 20%
5. original: 140
   new: 150
   increase; 7%
6. original: 135
   new: 90
   decrease; 33%
7. original: 120
   new: 180
   increase; 50%
8. original: 90
   new: 270
   increase; 200%
9. original: 27.5
   new: 25
   decrease; 9%
10. original: 54
    new: 98
    increase; 77%
11. original: 12.5
    new: 10
    decrease; 20%
12. original: 250
    new: 500
    increase; 100%

Example
SALES A coat is on sale for 25% off the original price. If the original price of the coat is $75, what is the discounted price?

The discount is 25% of the original price.

\[
25\% \text{ of } 75 = \frac{25}{100} \times 75 = 18.75 \quad \text{Use a calculator}
\]

Subtract 18.75 from the original price.

\[
75 - 18.75 = 56.25
\]

The discounted price of the coat is $56.25.

Exercises

Find the total price of each item.

1. Shirt: $24.00
   Sales tax: 4%
   $24.96
2. CD player: $142.00
   Sales tax: 5.5%
   $149.81
3. Celebrity calendar: $10.95
   Sales tax: 7.5%
   $11.77
4. Compact disc: $16
   Discount: 15%
   $13.60
5. Two concert tickets: $28
   Student discount: 28%
   $20.16
6. Airline ticket: $248.00
   Superair discount: 33%
   $166.16
7. VIDEOS The original selling price of a new sports video was $65.00. Due to the demand the price was increased to $87.75. What was the percent of increase over the original price? 35%

8. SCHOOL A high school paper increased its sales by 75% when it ran an issue featuring a contest to win a class party. Before the contest issue, 10% of the school’s 800 students bought the paper. How many students bought the contest issue? 140 students

9. BASEBALL Baseball tickets cost $15 for general admission or $20 for box seats. The sales tax on each ticket is 8%. What is the final cost of each type of ticket? $16.20; $21.60
State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

1. original: 25
   new: 10
   decrease; 60%

2. original: 50
   new: 75
   increase; 50%

3. original: 55
   new: 50
   decrease; 9%

4. original: 25
   new: 28
   increase; 12%

5. original: 50
   new: 95
   increase; 6%

6. original: 55
   new: 60
   increase; 9%

7. original: 48
   new: 45
   decrease; 6%

8. original: 60
   new: 50
   decrease; 15%

Find the total price of each item.

9. dress: $69.00
   tax: 5%
   $72.45

10. binder: $14.50
    tax: 7%
        $15.25

11. hardcover book: $28.95
    tax: 6%
        $30.69

12. groceries: $47.52
    tax: 3%
        $48.95

13. filler paper: $6.00
    tax: 6.5%
        $6.39

14. shoes: $65.00
    tax: 4%
        $67.60

15. basketball: $17.00
    tax: 6%
        $18.02

Find the discounted price of each item.

16. backpack: $56.25
    discount: 20%
        $45.00

17. CD: $15.99
    discount: 20%
        $12.79

18. shirt: $25.50
    discount: 40%
        $15.30

19. coffee maker: $102.00
    discount: 45%
        $56.10

Find the final price of each item.

20. computer game: $49.99
    discount: 15%
        $42.49

21. prescription glasses: $149
    discount: 20%
        $119.20

22. pair of shorts: $24.99
    discount: 45%
        $13.74

23. DVD player: $269.00
    discount: 20%
        $215.20

24. printer: $255.00
    discount: 30%
        $178.50

25. INVESTMENTS The price per share of a stock decreased from $90 per share to $36 per share. By what percent did the price of the stock decrease? 60%

26. HEATING COSTS Customers of a utility company received notices in their monthly bills that heating costs for the average customer had increased 125% over last year because of an unusually severe winter. In January of last year, the Garcia’s paid $120 for heating. What should they expect to pay this January if their bill increased by 125%? $270
2-7 Word Problem Practice

Percent of Change

1. SPORTS A regulation girls’ fast pitch softball diamond has bases that are 60 feet apart. A regulation professional baseball diamond has bases that are 90% farther apart. Label the distance between the bases on the regulation baseball diamond diagram.

5. MUSIC The table below shows the total number of CDs, downloaded singles, and music videos sold in 2004, 2006, and 2008.

| Sales of Recorded Music and Music Videos (millions of units) |
|---------------------------------|---|---|---|
| Format                      | 2004 | 2006 | 2008 |
| CD                          | 7670 | 6149 | 3684 |
| Downloaded                  | 1394 | 5866 | 10427 |
| Video                       | 328  | 231  | 208  |

Source: Recording Industry Association of America

a. Find the percent of change in the number of units sold between 2004 and 2006 and between 2006 and 2008 for each format. Round your answers to the nearest tenth.

CD: 19.8%; 40.1%
Downloaded: 320.6%; 77.8%
Video: 29.6%; 10.0%

b. Tell whether each percent of change in part a is a percent of increase or a percent of decrease.

CD decrease; decrease
Downloaded increase; increase
DVD video increase; increase

c. Did these trends change from 2004 to 2008? Explain.

The CD sales and video sales decreased in both time periods and downloaded sales increased both time periods.

2. SALES TAX Olivia purchases a DVD movie priced at $21.99. The sales tax is 6.5%. What is the total price of the movie, including tax?

$23.42

3. EDUCATION The ACT is a college entrance exam taken by high school students. The maximum score that can be earned is 36. The average score in the United States was 21.2 during a recent year. The average score for Vermont students that year was 7.3% higher than the national average. What was the average ACT score for Vermont students? Round your answer to the nearest tenth.

22.8

4. CARS Mr. Thompson plans to purchase a used car priced at $8400. He will receive a 15% employee discount and then will have to pay a 5.5% sales tax. What will be the final price of the car?

$7532.70

2-7 Enrichment

Using Percent

Use what you have learned about percent to solve each problem.

A TV movie had a “rating” of 15 and a 25 “share.” The rating is the percentage of the nation’s total TV households that were tuned in to this show. The share is the percentage of homes with TVs turned on that were tuned to the movie. How many TV households had their TVs turned off at this time?

To find out, let \( T \) = the number of TV households and \( x \) = the number of TV households with the TV off.

Then \( T - x \) = the number of TV households with the TV on.

Since 0.15\( T \) and 0.25\( T - x \) both represent the number of households tuned to the movie,

\[ 0.15T = 0.25(T - x) \]

\[ 0.15T = 0.25T - 0.25x \]

Solve for \( x \).

\[ 0.25x = 0.10T \]

\[ x = \frac{0.10T}{0.25} \] or \( 0.40T \)

Forty percent of the TV households had their TVs off when the movie was aired.

Answer each question.

1. During that same week, a sports broadcast had a rating of 22.1 and a 43 share. Show that the percent of TV households with their TVs off was about 48.6%.

\[ 0.221T = 0.43T - 0.43x \]

\[ 0.43x = 0.221T \]

\[ x = \frac{0.221T}{0.43} \]

\[ x = 0.468T \]

2. Find the percent of TV households with their TVs turned off during a show with a rating of 18.9 and a 29 share. 34.8%

3. Show that if \( T \) is the number of TV households, \( r \) is the rating, and \( s \) is the share, then the number of TV households with the TV off is \( \frac{r - s}{s} \).

Solve \( rT = s(T - x) \) for \( x \).

4. If the fraction of TV households with no TV on is \( \frac{n}{r} \) then show that the fraction of TV households with TVs on is \( \frac{r - n}{r} \).

5. Find the percent of TV households with TVs on during the most watched serial program in history: the last episode of M*A*S*H, which had a 60.3 rating and a 77 share. 60.3/77 = 78.3%

6. A local station now has a 2 share. Each share is worth $50,000 in advertising revenue per month. The station is thinking of going commercial free for the three months of summer to gain more listeners. What would its new share have to be for the last 4 months of the year to make more money for the year than it would have made if it had not gone commercial free? greater than 3.5
2-7 Spreadsheet Activity

Discounts

Electronics Warehouse is having a software clearance sale. The manager is making a discounted price list to distribute to customers as they shop. Different tables will have software that is discounted by a different percent. Use a spreadsheet to show the discounted price of software that is originally priced $5.99, $7.99, $9.99, $15.99, $19.99, $23.99, $27.99, $29.99, $33.99, and $35.99. Compute discounted prices for software that is 10%, 15%, 25%, and 30% off.

If an item is on sale for 10% off, that means that discounted price is \((1 - 0.1)\) or 0.9 times the original price. Use this pattern to complete the spreadsheet.

**Step 1** Use Column A of the spreadsheet for the original prices.

**Step 2** Columns B through E contain the formulas for the discounted prices.

**Step 3** Because these are prices, the numbers must be rounded to 2 digits. You can program the spreadsheet to list the information as dollars and cents by choosing currency from the number menu when you format the cells.

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
<td>4.49</td>
</tr>
<tr>
<td>3</td>
<td>7.99</td>
<td>7.19</td>
<td>6.73</td>
<td>5.99</td>
</tr>
<tr>
<td>4</td>
<td>9.99</td>
<td>9.09</td>
<td>8.99</td>
<td>7.89</td>
</tr>
<tr>
<td>5</td>
<td>15.99</td>
<td>14.33</td>
<td>11.99</td>
<td>9.89</td>
</tr>
<tr>
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<td>17.99</td>
<td>15.89</td>
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<tr>
<td>8</td>
<td>27.99</td>
<td>25.19</td>
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<tr>
<td>9</td>
<td>31.99</td>
<td>29.19</td>
<td>23.99</td>
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</tr>
<tr>
<td>10</td>
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<td>30.59</td>
<td>28.99</td>
<td>23.49</td>
</tr>
<tr>
<td>11</td>
<td>35.99</td>
<td>32.39</td>
<td>30.59</td>
<td>25.99</td>
</tr>
</tbody>
</table>
```

The formula in cell E6 is \(0.5 \times A6\)

**Exercises**

1. For the second week of the sale, the Electronics Warehouse manager is changing the discounts to 20%, 30%, 45%, and 75% off. Use a spreadsheet to create a new discount price list for the software. **See students’ work.**

2. How should the spreadsheet be altered if the manager wished to show the final prices of the software including the 6% sales tax for their area? **Multiply each discounted price by 1.06.**

2-8 Study Guide and Intervention

LITERAL EQUATIONS AND DIMENSIONAL ANALYSIS

Solve for Variables Sometimes you may want to solve an equation such as \(V = \frac{1}{2} bh\) for one of its variables. For example, if you know the values of \(V, w,\) and \(h,\) then the equation \(t = \frac{V}{wh}\) is more useful for finding the value of \(t.\) If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

**Example 1** Solve \(2x - 4y = 8,\) for \(y.\)

\[
2x - 4y = 8 \quad \text{or} \quad 2x - 8 = 4y
\]

The value of \(y\) is \(\frac{2x - 8}{4}.

**Example 2** Solve \(3m - n = km - 8,\) for \(m.\)

\[
3m - n - km = -8
\]

The value of \(m\) is \(\frac{n - 8}{3 - k}.

**Exercises**

Solve each equation or formula for the variable indicated.

1. \(ax - b = c,\) for \(x\)

\[
x = \frac{c + b}{a}, \quad a \neq 0
\]

2. \(15x + 1 = y,\) for \(x\)

\[
x = \frac{y - 1}{15}
\]

3. \((x + f) + 2 = j,\) for \(x\)

\[
x = j - 2 - f
\]

4. \(xy + w = 9,\) for \(y\)

\[
y = \frac{9 - w}{x}, \quad x \neq 0
\]

5. \(s(4 - k) = p,\) for \(k\)

\[
k = \frac{4 - p}{s}, \quad s \neq 0
\]

6. \(7x + 3y = m,\) for \(y\)

\[
y = \frac{m - 7x}{3}
\]

7. \(4(r + 3) = t,\) for \(r\)

\[
r = \frac{t - 12}{4}, \quad t \neq 0
\]

8. \(2x + h = w,\) for \(x\)

\[
x = \frac{w - h}{2}
\]

9. \(s(1 + y) = x,\) for \(x\)

\[
x = \frac{z}{1+y}, \quad y \neq -1
\]

10. \(16w + 4x = y,\) for \(x\)

\[
x = \frac{y - 16w}{4}
\]

11. \(d = rt,\) for \(r\)

\[
r = \frac{d}{t}, \quad t \neq 0
\]

12. \(A = \frac{k(a + b)}{2},\) for \(h\)

\[
h = \frac{2A}{a + b}, \quad a \neq -b
\]

13. \(C = \frac{1}{2}(F - 32),\) for \(F\)

\[
F = \frac{2}{3}C + 32
\]

14. \(P = 2w + 2m,\) for \(w\)

\[
W = \frac{p - 2f}{2}
\]

15. \(A = cw,\) for \(c\)

\[
c = \frac{A}{w}, \quad w \neq 0
\]
2-8 Study Guide and Intervention (continued)

Literal Equations and Dimensional Analysis

Use Formulas Many real-world problems require the use of formulas. Sometimes solving a formula for a specified variable will help solve the problem.

Example The formula $C = \pi d$ represents the circumference of a circle, or the distance around the circle, where $d$ is the diameter. If an airplane could fly around Earth at the equator without stopping, it would have traveled about 24,900 miles. Find the diameter of Earth.

$C = \pi d$ Given formula
$d = \frac{C}{\pi}$ Solve for $d$
$d = \frac{24,900}{3.14}$ Use $\pi \approx 3.14$
$d = 7931$ Simplify.

The diameter of Earth is about 7930 miles.

Exercises

1. GEOMETRY The volume of a cylinder $V$ is given by the formula $V = \pi r^2 h$, where $r$ is the radius and $h$ is the height.

   a. Solve the formula for $h$. $h = \frac{V}{\pi r^2}$

   b. Find the height of a cylinder with volume 2500 cubic feet and radius 10 feet. 25 ft

2. WATER PRESSURE The water pressure on a submerged object is given by $P = 64d$, where $P$ is the pressure in pounds per square foot, and $d$ is the depth of the object in feet.

   a. Solve the formula for $d$. $d = \frac{P}{64}$

   b. Find the depth of a submerged object if the pressure is 672 pounds per square foot. 10.5 ft

3. GRAPHS The equation of a line containing the points $(a, 0)$ and $(0, b)$ is given by the formula $\frac{x}{a} + \frac{y}{b} = 1$.

   a. Solve the equation for $y$. $y = b\left(1 - \frac{x}{a}\right)$

   b. Suppose the line contains the points $(4, 0)$, and $(0, -2)$. If $x = 3$, find $y$. $y = -1.5$

4. GEOMETRY The surface area of a rectangular solid is given by the formula $A = 2lw + 2wh + 2lh$, where $l$ is length, $w$ is width, and $h$ is height.

   a. Solve the formula for $h$. $h = \frac{x - 2lw}{2l + 2w}$

   b. The surface area of a rectangular solid with length 6 centimeters and width 3 centimeters is 72 square centimeters. Find the height. 2 cm

2-8 Skills Practice

Literal Equations and Dimensional Analysis

Solve each equation or formula for the variable indicated.

1. $7t = x$, for $t$. $t = \frac{x}{7}$

2. $r = \frac{p}{w}$, for $p$. $p = r \cdot w$

3. $q - r = r$, for $r$. $r = \frac{q}{2}$

4. $4m - t = m$, for $m$. $m = \frac{t}{3}$

5. $7a - b = 15a$, for $a$. $a = \frac{b}{8}$

6. $-5c + d = 2c$, for $c$. $c = \frac{d}{7}$

7. $x - 2y = 1$, for $y$. $y = \frac{x - 1}{2}$

8. $d + 3n = 1$, for $n$. $n = \frac{1 - d}{3}$

9. $\frac{9r + g}{5} = 5$, for $f$. $f = \frac{5 - g}{7}$

10. $ax - c = b$, for $x$. $x = \frac{b + c}{a}$; $a \neq 0$

11. $nt - 2n = y$, for $t$. $t = \frac{2n + y}{r}$; $r \neq 0$

12. $2bc + 3g = 2k$, for $c$. $c = \frac{2k - 3g}{b}$; $b \neq 0$

13. $kn + 4f = 9c$, for $n$. $n = \frac{9c - 4f}{k}$; $k \neq 0$

14. $8c + 6j = 5p$, for $c$. $c = \frac{5p - 6j}{8}$

15. $\frac{x - c}{2} = d$, for $x$. $x = c + 2d$

16. $\frac{h - k}{2} = d$, for $c$. $c = h - 2d$

17. $\frac{p + 9}{5} = r$, for $p$. $p = 5r - 9$

18. $a = \frac{b - 4z}{h}$, for $b$. $b = 7a + 4z$

19. The volume of a box $V$ is given by the formula $V = \ell wh$, where $\ell$ is the length, $w$ is the width, and $h$ is the height.

   a. Solve the formula for $h$. $h = \frac{V}{\ell w}$

   b. What is the height of a box with a volume of 50 cubic meter, length of 10 meters, and width of 2 meters? 2.5 m

20. Trent purchases 44 euros worth of souvenirs while on vacation in France. If \$1 U.S. = \$0.678 euros, find the cost of souvenirs in United States dollars. Round to the nearest cent. \$64.90
2-8 Practice

Literal Equations and Dimensional Analysis

Solve each equation or formula for the variable indicated.

1. \(d = rt\), for \(r\) \(\quad r = \frac{d}{t}\)

2. \(6w - y = 2z, \quad for \quad w = \frac{2x + y}{6}\)

3. \(mx + 4y = 3t, \quad for \quad x = \frac{3t - 4y}{m}; \quad m \neq 0\)

4. \(4.9s - 5g = -4u, \quad for \quad s = \frac{-4u + 5g}{9}\)

5. \(ab + 3c = 2x, \quad for \quad b = \frac{2x - 3c}{a}; \quad a \neq 0\)

6. \(2p = kx - t, \quad for \quad x = \frac{2p + t}{k}; \quad k \neq 0\)

7. \(\frac{2}{3}m + a = a + r, \quad for \quad m = \frac{3}{2}r\)

8. \(\frac{2}{3}h + g = d, \quad for \quad h = \frac{5}{2}(d - g)\)

9. \(\frac{3}{4}v + w = x, \quad for \quad y = \frac{3}{2}(x - v)\)

10. \(\frac{3}{4}a - q = h, \quad for \quad a = \frac{4}{3}(k + q)\)

11. \(\frac{rs + 9}{5} = h, \quad for \quad x = \frac{5h - 9}{r}; \quad r \neq 0\)

12. \(\frac{3h - 4}{2} = c, \quad for \quad b = \frac{3c + 4}{3}\)

13. \(3x - y = 7w - 2, \quad for \quad w = \frac{2 - y}{5}\)

14. \(3t + y = 5 + 6s, \quad for \quad t = \frac{y - 5}{2}\)

15. ELECTRICITY The formula for Ohm’s Law is \(E = IR\), where \(E\) represents voltage measured in volts, \(I\) represents current measured in amperes, and \(R\) represents resistance measured in ohms.

a. Solve the formula for \(R\). \(R = \frac{E}{I}\)

b. Suppose a current of 0.25 amperes flows through a resistor connected to a 12-volt battery. What is the resistance in the circuit? 48 ohms

16. MOTION In uniform circular motion, the speed \(v\) of a point on the edge of a spinning disk is \(v = \frac{2\pi r}{t}\), where \(r\) is the radius of the disk and \(t\) is the time it takes the point to travel once around the circle.

a. Solve the formula for \(r\). \(r = \frac{2\pi v}{2\pi}\)

b. Suppose a merry-go-round is spinning once every 3 seconds. If a point on the outside edge has a speed of 12.58 feet per second, what is the radius of the merry-go-round? (Use 3.14 for \(\pi\).) 6 ft

17. HIGHWAYS Interstate 90 is the longest interstate highway in the United States, connecting the cities of Seattle, Washington and Boston, Massachusetts. The interstate is 4,897,000 meters in length. If 1 mile = 1.609 kilometers, how many miles long is Interstate 90? 3099 mi

2-8 Word Problem Practice

Literal Equations and Dimensional Analysis

1. INTEREST Simple interest that you may earn on money in a savings account can be calculated with the formula \(I = prt\). \(I\) is the amount of interest earned, \(p\) is the principal or initial amount invested, \(r\) is the interest rate, and \(t\) is the amount of time the money is invested for. Solve the formula for \(p\).

\(P = \frac{I}{rt}\)

2. DISTANCE The distance \(d\) a car can travel is found by multiplying its rate of speed \(r\) by the amount of time \(t\) that it took to travel the distance. If a car has already traveled 5 miles, the total distance \(d\) is found by the formula \(d = rt + 5\). Solve the formula for \(r\).

\(r = \frac{d - 5}{t}\)

3. ENVIRONMENT The United States released 5.877 billion metric tons of carbon dioxide into the environment through the burning of fossil fuels in a recent year. If 1 trillion pounds = 4.536 billion metric tons, how many trillions of pounds of carbon dioxide did the United States release in that year?

\(\approx 12.96\)

4. PHYSICS The pressure exerted on an object is calculated by the formula \(P = \frac{F}{A}\), where \(P\) is the pressure, \(F\) is the force, and \(A\) is the surface area of the object. Water shooting from a hose has a pressure of 75 pounds per square inch (psi). Suppose the surface area covered by the direct hose spray is 0.442 square inch. Solve the equation for \(F\) and find the force of the spray.

33.15 pounds

5. GEOMETRY The regular octagon is divided into 8 congruent triangles. Each triangle has an area of 21.7 square centimeters. The perimeter of the octagon is 48 centimeters.

a. What is the length of each side of the octagon? 6 centimeters

b. Solve the area of a triangle formula for \(h\).

\(h = \frac{2A}{b}\)

c. What is the height of each triangle? Round to the nearest tenth. 7.2 centimeters
**Enrichment**

**Compound Interest**

In most banks, interest on savings accounts is compounded at set time periods such as three or six months. At the end of each period, the bank adds the interest earned to the account. During the next period, the bank pays interest on all the money in the bank, including interest.

In this way, the account earns interest on interest.

Suppose Ms. Tanner has $1000 in an account that is compounded quarterly at 5%. Find the balance after the first two quarters.

Use \( I = prt \) to find the interest earned in the first quarter if \( p = 1000 \) and \( r = 5\% \). Why is \( t \) equal to \( \frac{1}{4} \)?

First quarter: \( I = 1000 \times 0.05 \times \frac{1}{4} \)

\[ I = 12.50 \]

The interest, $12.50, earned in the first quarter is added to $1000.

The principal becomes $1012.50.

Second quarter: \( I = 1012.50 \times 0.05 \times \frac{1}{4} \)

\[ I = 12.6625 \]

The interest in the second quarter is $12.66.

The balance after two quarters is $1012.50 + 12.66 or $1025.16.

**Weighted Averages**

Mixture Problems are problems where two or more parts are combined into a whole. They involve weighted averages. In a mixture problem, the weight is usually a price or a percent of something.

**Example**

**COOKIES**

Delectable Cookie Company sells chocolate chip cookies for $6.95 per pound and white chocolate cookies for $5.95 per pound. How many pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies to obtain a mixture that sells for $6.75 per pound.

Let \( w \) be the number of pounds of chocolate chip cookies.

**Answers**

1. How much interest is earned in the third quarter of Ms. Tanner’s account? \( I = 12.81 \)

2. What is the balance in her account after three quarters? $1037.97

3. How much interest is earned in the fourth quarter? \( I = 12.97 \)

4. What is the balance in her account after one year? $1050.94

5. Suppose Ms. Tanner’s account is compounded semiannually. What is the balance at the end of six months? $1025.00

6. What is the balance after one year if her account is compounded semiannually? $1050.63

**Exercises**

1. **SOLUTIONS** How many grams of sugar must be added to 60 grams of a solution that is 32% sugar to obtain a solution that is 50% sugar? \( 21.6 \) g

2. **NUTS** The Quik Mart has two kinds of nuts. Pecans sell for $1.55 per pound and walnuts sell for $1.95 per pound. How many pounds of walnuts must be added to 15 pounds of pecans to make a mixture that sells for $1.75 per pound? \( 15 \) lb

3. **INVESTMENTS** Alice Gleason invested a portion of $32,000 at 9% interest and the balance at 11% interest. How much did she invest at each rate if her total income from both investments was $3200. \( \$16,000 \text{ at } 9\% \text{ and } \$16,000 \text{ at } 11\% \)

4. **MILK** Whole milk is 4% butterfat. How much skim milk with 0% butterfat should be added to 32 ounces of whole milk to obtain a mixture that is 2.5% butterfat? \( 19.2 \) oz
Uniform Motion Problems. Motion problems are another application of weighted averages. Uniform motion problems are problems where an object moves at a certain speed, or rate. Use the formula \( d = rt \) to solve these problems, where \( d \) is the distance, \( r \) is the rate, and \( t \) is the time.

Example

**Driving**  Bill Gutierrez drove at a speed of 65 miles per hour on an expressway for 2 hours. He then drove for 1.5 hours at a speed of 45 miles per hour on a state highway. What was his average speed?

\[
M = \frac{65 \cdot 2 + 45 \cdot 1.5}{2 + 1.5}
\]

Definition of weighted average

\[
= 56.4
\]

Simplify

Bill drove at an average speed of about 56.4 miles per hour.

Exercises

1. **Travel**  Mr. Anders and Ms. Rich each drove home from a business meeting. Mr. Anders traveled east at 100 kilometers per hour and Ms. Rich traveled west at 80 kilometers per hour. In how many hours were they 100 kilometers apart? \( \frac{5}{9} \) h

2. **Airplanes**  An airplane flies 750 miles due west in 1 \( \frac{1}{2} \) hours and 750 miles due south in 2 hours. What is the average speed of the airplane? About 429 mph

3. **Track**  Sprinter A runs 100 meters in 15 seconds, while sprinter B starts 1.5 seconds later and runs 100 meters in 14 seconds. If each of them runs at a constant rate, who is farther in 10 seconds after the start of the race? Explain.

Sprinter A; since sprinter A runs 100 m in 15 s, this sprinter runs at a rate of \( \frac{200}{15} \) m/s. In 10 seconds, sprinter A will have run \( \frac{100}{15} \cdot 10 = 66.7 \) m.

Sprinter B's rate is \( \frac{100}{14} \). In 10 seconds, with the delayed start, sprinter B has run \( \frac{100}{14} \cdot (10 - 1.5) = 60.7 \) m.

4. **Trains**  An express train travels 90 kilometers per hour from Smallville to Megatown. A local train takes 2.5 hours longer to travel the same distance at 50 kilometers per hour. How far apart are Smallville and Megatown? 281.25 km

5. **Cycling**  Two cyclists begin traveling in the same direction on the same bike path. One travels at 15 miles per hour, and the other travels at 12 miles per hour. When will the cyclists be 10 miles apart? \( 3 \frac{1}{3} \) h

6. **Trains**  Two trains leave Chicago, one traveling east at 30 miles per hour and the other traveling west at 40 miles per hour. When will the trains be 210 miles apart? \( 3 \) h

---

### 2-9 Skills Practice

**Weighted Averages**

1. **Seasoning**  A health food store sells seasoning blends in bulk. One blend contains 20% basil. Sheila wants to add pure basil to some 20% blend to make 16 ounces of her own 30% blend. Let \( b \) represent the amount of basil Sheila should add to the 20% blend.

   a. Complete the table representing the problem.

<table>
<thead>
<tr>
<th>Ounces</th>
<th>Amount of Basil</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>( 0.20(16 - b) )</td>
</tr>
<tr>
<td>( b )</td>
<td>1.00b</td>
</tr>
<tr>
<td>16</td>
<td>( 0.30(16) )</td>
</tr>
</tbody>
</table>

   b. Write an equation to represent the problem. \( 0.20(16 - b) + 1.00b = 0.30(16) \)

   c. How many ounces of basil should Sheila use to make the 30% blend? \( 2 \) oz

   d. How many ounces of the 20% blend should she use? \( 14 \) oz

2. **Hiking**  At 7:00 A.M., two groups of hikers begin 21 miles apart and head toward each other. The first group, hiking at an average rate of 1.5 miles per hour, carries tents, sleeping bags, and cooking equipment. The second group, hiking at an average rate of 2 miles per hour, carries food and water. Let \( t \) represent the hiking time.

   a. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First group of hikers</td>
<td>1.5</td>
<td>( t )</td>
<td>1.5( t )</td>
</tr>
<tr>
<td>Second group of hikers</td>
<td>2</td>
<td>( t )</td>
<td>2( t )</td>
</tr>
</tbody>
</table>

   b. Write an equation using \( t \) that describes the distances traveled. \( 1.5t + 2t = 21 \)

   c. How long will it be until the two groups of hikers meet? \( 6 \) h

3. **Sales**  Sergio sells a mixture of Virginia peanuts and Spanish peanuts for $3.40 per pound. To make the mixture, he uses Virginia peanuts that cost $3.50 per pound and Spanish peanuts that cost $3.00 per pound. He mixes 10 pounds at a time.

   a. How many pounds of Virginia peanuts does Sergio use? \( 8 \) lb

   b. How many pounds of Spanish peanuts does Sergio use? \( 2 \) lb
2-9 Practice

Weighted Averages

1. GRASS SEED A nursery sells Kentucky Blue Grass seed for $5.75 per pound and Tall Fescue seed for $4.50 per pound. The nursery sells a mixture of the two kinds of seed for $5.25 per pound. Let \( k \) represent the amount of Kentucky Blue Grass seed the nursery uses in 5 pounds of the mixture.

a. Complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Price per Pound</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kentucky Blue Grass</td>
<td>( k )</td>
<td>( 5.75k )</td>
</tr>
<tr>
<td>Tall Fescue</td>
<td>( 5 - k )</td>
<td>( 4.50(5 - k) )</td>
</tr>
<tr>
<td>Mixture</td>
<td>5</td>
<td>( 5.25(5) )</td>
</tr>
</tbody>
</table>

b. Write an equation to represent the problem. \( 5.75k + 4.50(5 - k) = 5.25(5) \)

c. How much Kentucky Blue Grass does the nursery use in 5 pounds of the mixture? \( 3 \) lb

d. How much Tall Fescue does the nursery use in 5 pounds of the mixture? \( 2 \) lb

2. TRAVEL Two commuter trains carry passengers between two cities, one traveling east, and the other west, on different tracks. Their respective stations are 150 miles apart. Both trains leave at the same time, one traveling at an average speed of 55 miles per hour and the other at an average speed of 65 miles per hour. Let \( t \) represent the time until the trains pass each other.

a. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>( r )</th>
<th>( t )</th>
<th>( d = vt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Train</td>
<td>55</td>
<td>( t )</td>
</tr>
<tr>
<td>Second Train</td>
<td>65</td>
<td>( t )</td>
</tr>
</tbody>
</table>

b. Write an equation using \( t \) that describes the distances traveled. \( 55t + 65t = 150 \)

c. How long after departing will the trains pass each other? \( 1.25 \) h

3. TRAVEL Two trains leave Raleigh at the same time, one traveling north, and the other south. The first train travels at 50 miles per hour and the second at 60 miles per hour. In how many hours will the trains be 270 miles apart? \( 2.5 \) h

4. JUICE A pineapple drink contains 15% pineapple juice. How much pure pineapple juice should be added to 8 quarts of the pineapple drink to obtain a mixture containing 50% pineapple juice? \( 5.6 \) qt

2-9 Word Problem Practice

Weighted Averages

1. DRIVING The drive from New York City to Boston is about 240 miles. It took Samir 5 hours to drive one way, but due to severe weather it took him 6.5 hours for the return trip. What was his average speed round trip? Round the answer to the nearest hundredth.

41.74 miles per hour

2. GRADES In math classes at Gorbine High School, all tests are given double weight. Donna's average grade so far this term is 90. Donna got a 95 on the latest test and a quiz. Donna's homework count is 50% of the total score. Donna got 92 on the quiz chapter 1 and 81 on the homework chapter 2. Determine Donna's average grade so far this term.

88.75

3. MIXTURE Keith wants to create a drink that is 40% juice. How much of a 10% juice solution should be added to 200 mL of 100% grape juice to obtain the 40% mixture?

49.4 seconds
Expected Value

Expected value is the average return of an event over repeated trial. Expected value is also a form of weighted average and can be used to determine whether or not you should play a game based on what the expected value of your winnings is. It can also be used to determine the expected length of a game.

Example

You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a one, you get 12 candy bars. If you roll a two or a three, you get 3 candy bars. If you roll a four, five or six, you get 2 candy bars.

Since there are six possibilities for what you can roll:

- \( \frac{1}{6} \) of the time, you will win 12 candy bars.
- \( \frac{2}{6} \) or \( \frac{1}{3} \) of the time, you will win 3 candy bars.
- \( \frac{3}{6} \) or \( \frac{1}{2} \) of the time, you will win 2 candy bars.

The expected value is\( \frac{1}{6}(12) + \frac{1}{3}(3) + \frac{1}{2}(2) \) or 4 candy bars.

Therefore, if you played the game 100 times, you could expect to win about 4 candy bars each time.

Exercises

Find the expected value of each of the following events.

1. You are flipping a coin. If you flip heads, you win $5.00. If you flip tails, you win $1.00.
   \[ \text{Expected Value} = \frac{1}{2}(5) + \frac{1}{2}(1) = $3.00 \]

2. You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a one, you get 18 candy bars. If you roll a two or a three, you get 6 candy bars. If you roll a four, five or six, you get 0 candy bars.
   \[ \text{Expected Value} = \frac{1}{6}(18) + \frac{2}{3}(6) + \frac{1}{2}(0) = 4 \text{ candy bars} \]

3. You are rolling a die to determine the amount of money you will win at a school fair. If you roll a one, you get $12.00. If you roll a two or a three, you get $3.00. If you roll a four, five or six, you owe $2.00.
   \[ \text{Expected Value} = \frac{1}{6}(12) + \frac{2}{3}(3) - \frac{1}{2}(2) = $2.00 \]

4. You are flipping a coin. If you flip heads, you win $10. If you flip tails, you win $2.
   \[ \text{Expected Value} = \frac{1}{2}(10) + \frac{1}{2}(2) = $6.00 \]

5. You are rolling a die to determine the number of candy bars you will win at a school fair. If you roll a one, you get 24 candy bars. If you roll a two or a three, you get 6 candy bars. If you roll a four, five or six, you get none.
   \[ \text{Expected Value} = \frac{1}{6}(24) + \frac{2}{3}(6) + \frac{1}{2}(0) = 6 \text{ candy bars} \]
Chapter 2 Assessment Answer Key

Quiz 1 (Lessons 2-1 through 2-3)
Page 63

1. \( y^2 - 12 = 5x \)
   Two times \( b \) minus
   10 equals 4.

2. The sum of \( y \) and the
   product of 3 and the
   square of \( x \) is 5 times \( x \).

3. \( 14 \)

4. \( -49 \)

5. \( -9 \)

6. \( 40 \)

7. \( 7 \)

8. \( 84 \)

9. \( D \)

10. \( \) no solution

Quiz 2 (Lessons 2-4 and 2-5)
Page 63

1. \( -5 \)

2. no solution

3. \( 0 \)

4. \( 7 \)

5. \( C \)

6. \( 11 \)

7. \( 5 \)

8. \( -5 \)

9. \( \{2, 5\} \)

10. \( \{-6, 7\} \)

Quiz 3 (Lessons 2-6 and 2-7)
Page 64

1. yes

2. no

3. no

4. \( 15 \)

5. \( 27 \)

6. \( 99 \)

7. decrease; 28%

8. increase; 25%

9. \( $19.08 \)

10. \( B \)

Quiz 4 (Lessons 2-8 and 2-9)
Page 64

1. \( x = \frac{p + r}{n} \)

2. \( b = x - ac \)

3. \( 4 \) g

4. \( A \)

5. \( \ell = \frac{p - 2w}{2} ; 14 \) m

Mid Chapter Test
Page 65

Part I

1. \( B \)

2. \( J \)

3. \( A \)

4. \( H \)

5. \( B \)

6. \( G \)

Part II

Four times \( n \)
equals \( x \) times the

difference of five and \( n \).

7. \( \) no solution

8. \( 0 \)

9. all numbers

10. \( -2 \)

11. \( $116.15 \)

12. \( \{-2, 5\} \)
<table>
<thead>
<tr>
<th>Vocabulary Test</th>
<th>Form 1</th>
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<th>Page 68</th>
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</thead>
<tbody>
<tr>
<td>Page 66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. ratio  
2. weighted average  
3. percent of change  
4. rate  
5. multi-step equation  
6. unit rate  
7. identity  

Sample answer: A proportion is an equation stating that two ratios are equal.  
8.  

Sample answer: A formula is an equation that states a rule for the relationship between certain quantities.  
9.  
10. j  

B: 45
## Chapter 2 Assessment Answer Key

<table>
<thead>
<tr>
<th>Form 2A</th>
<th>Form 2B</th>
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</thead>
<tbody>
<tr>
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<td>Page 70</td>
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<tr>
<td><strong>1.</strong> C</td>
<td><strong>1.</strong> B</td>
</tr>
<tr>
<td><strong>2.</strong> J</td>
<td><strong>12.</strong> F</td>
</tr>
<tr>
<td><strong>3.</strong> A</td>
<td><strong>13.</strong> C</td>
</tr>
<tr>
<td><strong>4.</strong> H</td>
<td><strong>14.</strong> G</td>
</tr>
<tr>
<td><strong>5.</strong> D</td>
<td><strong>15.</strong> C</td>
</tr>
<tr>
<td><strong>6.</strong> F</td>
<td><strong>16.</strong> G</td>
</tr>
<tr>
<td><strong>7.</strong> C</td>
<td><strong>17.</strong> D</td>
</tr>
<tr>
<td><strong>8.</strong> G</td>
<td><strong>18.</strong> F</td>
</tr>
<tr>
<td><strong>9.</strong> C</td>
<td><strong>19.</strong> B</td>
</tr>
<tr>
<td><strong>10.</strong> G</td>
<td><strong>10.</strong> G</td>
</tr>
<tr>
<td>B: 4 L</td>
<td>B: 21</td>
</tr>
</tbody>
</table>

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Chapter 2 Assessment Answer Key

Form 2C
Page 73

1. \(36 - x = 3(4 + x)\)
   Three times the sum of \(x\) and \(y\) equals two times \(y\) minus \(x\).

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

14. 

15. yes

16. 22.5

17. -1

18. all numbers

19. 13

20. \(x = \frac{bc}{a}\)

21. decrease; 20%

22. $12.84

23. 3 L

24. 2.5 h

25. \(h = \frac{V}{\pi r^2}; 10.5\) in.

B: 8
Chapter 2 Assessment Answer Key

Form 2D
Page 75

1. 18 + n = 7(n - 3)
   Three divided by y minus five equals x times the sum of y and 7.

2. 4

3. 7

4. 7

5. 7

6. 7.5

7. 50

8. 7

9. n - 8.1 = 4.9; 13

10. 6n + 15 = 9; -1

11. $176.80

12. 11

13. \{-3, -1\}

14. \{-4, -2\}

15. no

16. 20

17. 3

18. no solution

19. 11

20. r = n(4v - t)

21. increase; 25%

22. $13.50

23. 7.5 lb of nuts, 2.5 lb of dried fruit

24. 4 h

25. \( h = \frac{V}{\pi r^2}; 18.67 \text{ in.} \)

B: $9.60

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Chapter 2 Assessment Answer Key

Form 3 Page 77

1. \( \frac{x - 45}{12} + 20 = 5(32 + x) \)
   Five times the sum of two times \( x \) and three times \( y \) equals the square of \( y \) minus two times the cube of \( x \).

2. \(-27\)

3. \(-13\)

4. \(-2\frac{2}{3}\)

5. \(-1\frac{2}{13}\)

6. 126

7. 8

8. \( \frac{3x}{5} = 1; \frac{5}{3} \)

9. \((x + 2)10 = 8x + 36; 8\)

10. 10

11. \(-7, -5\)

12. \$4000

13. yes

14. 10 ft

15. B: 12 mi

Page 78

16. -6

17. \( \frac{7}{9} \)

18. 2

19. \( x = \frac{r + n}{a} \)
   \( x = \frac{ry - t}{4} \)

20. increase; 12%

21. $63.60

22. $7000

23. 510 mph, 540 mph

24. 1.95 ft/s

25. 12 mi
# Scoring Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
| 4     | **Superior**       | - Shows thorough understanding of the concepts of translating between verbal sentences and equations, solving equations, percents of increase and decrease, uniform motion problems, and proportions.  
- Uses appropriate strategies to solve problems.  
- Computations are correct.  
- Written explanations are exemplary.  
- Goes beyond requirements of some or all problems. |
| 3     | **Satisfactory**   | - Shows an understanding of the concepts of translating between verbal sentences and equations, solving equations, percents of increase and decrease, uniform motion problems, and proportions.  
- Uses appropriate strategies to solve problems.  
- Computations are mostly correct.  
- Written explanations are effective.  
- Satisfies all requirements of problems. |
| 2     | **Nearly Satisfactory** | - Shows an understanding of most of the concepts of translating between verbal sentences and equations, solving equations, percents of increase and decrease, uniform motion problems, and proportions.  
- May not use appropriate strategies to solve problems.  
- Computations are mostly correct.  
- Written explanations are satisfactory.  
- Satisfies the requirements of most of the problems. |
| 1     | **Nearly Unsatisfactory** | - Final computation is correct.  
- No written explanations or work is shown to substantiate the final computation.  
- Satisfies minimal requirements of some of the problems. |
| 0     | **Unsatisfactory** | - Shows little or no understanding of most of the concepts of translating between verbal sentences and equations, solving equations, percents of increase and decrease, uniform motion problems, and proportions.  
- Does not use appropriate strategies to solve problems.  
- Computations are incorrect.  
- Written explanations are unsatisfactory.  
- Does not satisfy requirements of problems.  
- No answer may be given. |
Chapter 2 Assessment Answer Key

Page 79, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A40, the following sample answers may be used as guidance in evaluating extended-response assessment items.

1a. The student should explain that the first phrase is a product of \( x \) and \( y \) and then the addition of \( z \), while the second phrase is a product of \( x \) and the quantity \( y + z \).

1b. Check that the student’s values for \( x \), \( y \), and \( z \) satisfy both \( xy + z \) and \( x(y + z) \). One example is \( x = 1, y = 2, \) and \( z = 3 \). Another example is \( x = 2, y = 3, \) and \( z = 0 \).

2a. \[
\frac{ry + z}{m} - t = x
\]
   (Original equation)
   \[
\frac{ry + z}{m} - t + t = x + t
\]  
   (Add \( t \) to each side.)
   \[
\frac{ry + z}{m} = x + t
\]  
   (Simplify.)
   \[
m\left(\frac{ry + z}{m}\right) = m(x + t)
\]  
   (Multiply each side by \( m \).)
   \[
ry + z = mx + mt
\]  
   (Simplify.)
   \[
ry + z - z = mx + mt - z
\]  
   (Subtract \( z \) from each side.)
   \[
ry = mx + mt - z
\]  
   (Simplify.)
   \[
\frac{ry}{r} = \frac{mx + mt - z}{r}
\]  
   (Divide each side by \( r \).)
   \[
y = \frac{mx + mt - z}{r}
\]  
   (Simplify.)
   The value of \( y \) is \( \frac{mx + mt - z}{r} \).

2b. Division by 0 is undefined, so in the original equation \( m \neq 0 \), and in the final equation \( r \neq 0 \).

3a. The student should conclude that a 10% decrease followed by a 10% increase results in a net decrease of 1%. Thus, the final cost would be 99% of the original price.

3b. Since multiplication is commutative, multiplying by 1.1 and then 0.9 would yield the same result as multiplying by 0.9 and then 1.1. The student should conclude that a 10% increase followed by a 10% decrease yields the same result as a 10% decrease followed by a 10% increase.

4a. Since the two people walked for the same amount of time and time can be calculated as distance divided by rate, the proportion
\[
\frac{\text{Tony's distance}}{\text{Tony's rate}} = \frac{\text{Ivia's distance}}{\text{Ivia's rate}}
\]
can be used to solve this problem.

4b. The length of Tony’s walk can be calculated directly from his rate of 3 miles per hour and his distance of 6 miles. Ivia walked 1 mile per hour faster, so her rate is 4 miles per hour. The length of Ivia’s walk can be calculated directly from her rate of 4 miles per hour and distance of 6 miles. Thus, a proportion would not be used to solve this problem.

5a. Sample answer: \( x + 2 = 10, x - 2 = 6 \)
   \[
2x = 16, \quad \frac{x}{2} = 4
\]

5b. Sample answer: \( 2x + 1 = x + 9 \)

5c. These two equations are equivalent. The solution to both equations is 5. The student should recognize that the solution to all equations in parts a and b is 8. Therefore neither of these two equations could be equivalent to any of the equations created for parts a and b.
Chapter 2 Assessment Answer Key

Standardized Test Practice

Page 80

1. ⬜️ ⬜️ ⬜️ ⬜️

2. ⬜️ ⬜️ ⬜️ ⬜️

3. ⬜️ ⬜️ ⬜️ ⬜️

4. ⬜️ ⬜️ ⬜️ ⬜️

5. ⬜️ ⬜️ ⬜️ ⬜️

6. ⬜️ ⬜️ ⬜️ ⬜️

7. ⬜️ ⬜️ ⬜️ ⬜️

8. ⬜️ ⬜️ ⬜️ ⬜️

9. ⬜️ ⬜️ ⬜️ ⬜️

10. ⬜️ ⬜️ ⬜️ ⬜️

11. ⬜️ ⬜️ ⬜️ ⬜️

12. ⬜️ ⬜️ ⬜️ ⬜️

13. ⬜️ ⬜️ ⬜️ ⬜️

14. ⬜️ ⬜️ ⬜️ ⬜️

15.

16.
Chapter 2 Assessment Answer Key

Standardized Test Practice
Page 82

17. 22
18. 5u + 9x
19. Sample answer:

20. 85 − 9y = 7(4 + y)

21. 31
22. 7
23. −3
24. 3
25. m = x(t − p)

26. 7 L

The final price would be 99% of the original price.

27a. No; since multiplication is commutative, a 10% increase followed by a 10% decrease would have the same result as a 10% decrease followed by a 10% increase.
### Chapter 2 Assessment Answer Key

**Unit 1 Test**

**Page 83**

1. Sample answer: the sum of four times $r$ and 9

2. $5 - n^3$

3. Multiplicative Inverse Property; $\frac{1}{6}$

4. $7t^2 + 3t$

5. $7r + 9t$

6. $23a + 6b$

7. Sample answer: the difference of $m$ squared and 4 is equal to the sum of 2 times $r$ plus 1

8. $\{4\}$

9. yes

10. no

11. 8

12. False

13. ![Temperature vs. Time Graph](image)

14. $6x + 12$

15. Sample answer: Four math textbooks are stacked on top of two science textbooks. Write an expression for the total height of the stack of books.

16. 8 lb

17. $-18$

18. $-12$

19. 42

20. 56

21. $-2$

22. all real numbers

23. $3\frac{2}{7}$

24. 11

25. $\{-2, 3\}$

26. yes

27. $\{4\}$

28. $\$3.91$

29. $v = t - kr$

29. 8 lb