CONSUMABLE WORKBOOKS  Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
<th>MHID</th>
<th>ISBN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-660292-3</td>
<td>978-0-07-660292-6</td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660291-5</td>
<td>978-0-07-660291-9</td>
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<tr>
<td>Spanish Version</td>
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<td>Homework Practice Workbook</td>
<td>0-07-660294-X</td>
<td>978-0-07-660294-0</td>
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Answers For Workbooks  The answers for Chapter 3 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED  All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

Spanish Assessment Masters  (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 3 Test Form 2A and Form 2C.
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Teacher’s Guide to Using the
Chapter 3 Resource Masters

The Chapter 3 Resource Masters includes the core materials needed for Chapter 3. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

**Student-Built Glossary** (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 3-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

**Anticipation Guide** (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

**Study Guide and Intervention** These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Practice** This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Word Problem Practice** This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

**Enrichment** These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

**Graphing Calculator, TI-Nspire, or Spreadsheet Activities** These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the Chapter 3 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

• Form 1 contains multiple-choice questions and is intended for use with below grade level students.
• Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

• The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
• Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 3. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>arithmetic sequence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant of variation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>direct variation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>family of functions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inductive reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>parent function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rate of change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td></td>
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<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition/Description/Example</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>standard form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$-intercept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$-intercept</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## 3 Anticipation Guide

### Linear Functions

#### Step 1  Before you begin Chapter 3

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The equation $6x + 2xy = 5$ is a linear equation because each variable is to the first power.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>The graph of $y = 0$ has more than one $x$-intercept.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>The zero of a function is located at the $y$-intercept of the function.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>All horizontal lines have an undefined slope.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>The slope of a line can be found from any two points on the line.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>A direct variation, $y = kx$, will always pass through the origin.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>In a direct variation $y = kx$, if $k &lt; 0$ then its graph will slope upward from left to right.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>A sequence is arithmetic if the difference between all consecutive terms is the same.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Each number in a sequence is called a factor of that sequence.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Making a conclusion based on a pattern of examples is called inductive reasoning.</td>
<td></td>
</tr>
</tbody>
</table>

#### Step 2  After you complete Chapter 3

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
### Antes de comenzar el Capítulo 3

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>La ecuación $6x + 2xy = 5$ es una ecuación lineal porque cada variable está elevada a la primera potencia.</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>La gráfica de $y = 0$ tiene más de una intersección $x$.</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>El cero de una función está ubicado en la intersección $y$ de la función.</td>
<td>A</td>
</tr>
<tr>
<td>4</td>
<td>Todas las rectas horizontales tienen una pendiente indefinida.</td>
<td>A</td>
</tr>
<tr>
<td>5</td>
<td>La pendiente de una recta se puede encontrar a partir de cualquier par de puntos en la recta.</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>Una variación directa, $y = kx$, siempre pasará por cero.</td>
<td>A</td>
</tr>
<tr>
<td>7</td>
<td>En una variación directa $y = kx$, si $k &lt; 0$, entonces su gráfica se inclinará hacia arriba de izquierda a derecha.</td>
<td>D</td>
</tr>
<tr>
<td>8</td>
<td>Una sucesión es aritmética si la diferencia entre todos los términos consecutivos es la misma.</td>
<td>A</td>
</tr>
<tr>
<td>9</td>
<td>Cada número en una sucesión se llama factor de la sucesión.</td>
<td>A</td>
</tr>
<tr>
<td>10</td>
<td>Sacar una conclusión en base a un patrón de ejemplos se llama razonamiento inductivo.</td>
<td>D</td>
</tr>
</tbody>
</table>

### Después de completar el Capítulo 3

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
3-1 Study Guide and Intervention

Graphing Linear Equations

Identify Linear Equations and Intercepts A linear equation is an equation that can be written in the form $Ax + By = C$. This is called the standard form of a linear equation.

| Standard Form of a Linear Equation | $Ax + By = C$, where $A \geq 0$, $A$ and $B$ are not both zero, and $A$, $B$, and $C$ are integers with a greatest common factor of 1 |

Example 1 Determine whether $y = 6 - 3x$ is a linear equation. Write the equation in standard form.

First rewrite the equation so both variables are on the same side of the equation.

$y = 6 - 3x$  
$y + 3x = 6 - 3x + 3x$  
$3x + y = 6$  

The equation is now in standard form, with $A = 3$, $B = 1$ and $C = 6$. This is a linear equation.

Example 2 Determine whether $3xy + y = 4 + 2x$ is a linear equation. Write the equation in standard form.

Since the term $3xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

Exercises Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. $2x = 4y$  
2. $6 + y = 8$  
3. $4x - 2y = -1$

4. $3xy + 8 = 4y$  
5. $3x - 4 = 12$  
6. $y = x^2 + 7$

7. $y - 4x = 9$  
8. $x + 8 = 0$  
9. $-2x + 3 = 4y$

10. $2 + \frac{1}{2}x = y$  
11. $\frac{1}{4}y = 12 - 4x$  
12. $3xy - y = 8$

13. $6x + 4y - 3 = 0$  
14. $yx - 2 = 8$  
15. $6x - 2y = 8 + y$

16. $\frac{1}{4}x - 12y = 1$  
17. $3 + x + x^2 = 0$  
18. $x^2 = 2xy$
Graph Linear Equations  The graph of a linear equation represents all the solutions of the equation. An x-coordinate of the point at which a graph of an equation crosses the x-axis in an x-intercept. A y-coordinate of the point at which a graph crosses the y-axis is called a y-intercept.

**Example 1**  Graph $3x + 2y = 6$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$ and solve for $x$. The x-intercept is 2. The graph intersects the x-axis at (2, 0).

To find the y-intercept, let $x = 0$ and solve for $y$. The y-intercept is 3. The graph intersects the y-axis at (0, 3).

Plot the points (2, 0) and (0, 3) and draw the line through them.

**Example 2**  Graph $y - 2x = 1$ by making a table.

Solve the equation for $y$.

$y - 2x = 1$

$y - 2x + 2x = 1 + 2x$

$y = 2x + 1$

Select five values for the domain and make a table. Then graph the ordered pairs and draw a line through the points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2x + 1$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2(-2) + 1</td>
<td>-3</td>
<td>(-2, -3)</td>
</tr>
<tr>
<td>-1</td>
<td>2(-1) + 1</td>
<td>-1</td>
<td>(-1, -1)</td>
</tr>
<tr>
<td>0</td>
<td>2(0) + 1</td>
<td>1</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>1</td>
<td>2(1) + 1</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>2(2) + 1</td>
<td>5</td>
<td>(2, 5)</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each equation by using the x- and y-intercepts.

1. $2x + y = -2$

2. $3x - 6y = -3$

3. $-2x + y = -2$

Graph each equation by making a table.

4. $y = 2x$

5. $x - y = -1$

6. $x + 2y = 4$
3-1 Skills Practice

Graphing Linear Equations

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. \( xy = 6 \)  
2. \( y = 2 - 3x \)  
3. \( 5x = y - 4 \)

4. \( y = 2x + 5 \)  
5. \( y = -7 + 6x \)  
6. \( y = 3x^2 + 1 \)

7. \( y - 4 = 0 \)  
8. \( 5x + 6y = 3x + 2 \)  
9. \( \frac{1}{2}y = 1 \)

Find the \( x \)- and \( y \)-intercepts of each linear function.

10.  

11.  

12.  

Graph each equation by making a table.

13. \( y = 4 \)  
14. \( y = 3x \)  
15. \( y = x + 4 \)

Graph each equation by using the \( x \)- and \( y \)-intercepts.

16. \( x - y = 3 \)  
17. \( 10x = -5y \)  
18. \( 4x = 2y + 6 \)
### 3-1 Practice

**Graphing Linear Equations**

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form and determine the x- and y-intercepts.

1. $4xy + 2y = 9$
2. $8x - 3y = 6 - 4x$
3. $7x + y + 3 = y$

4. $5 - 2y = 3x$
5. $\frac{x}{4} - \frac{y}{3} = 1$
6. $\frac{5}{x} - \frac{2}{y} = 7$

Graph each equation.

7. $\frac{1}{2}x - y = 2$
8. $5x - 2y = 7$
9. $1.5x + 3y = 9$

10. **COMMUNICATIONS** A telephone company charges $4.95 per month for long distance calls plus $0.05 per minute. The monthly cost $c$ of long distance calls can be described by the equation $c = 0.05m + 4.95$, where $m$ is the number of minutes.
   
   a. Find the y-intercept of the graph of the equation.
   
   b. Graph the equation.
   
   c. If you talk 140 minutes, what is the monthly cost?

11. **MARINE BIOLOGY** Killer whales usually swim at a rate of 3.2–9.7 kilometers per hour, though they can travel up to 48.4 kilometers per hour. Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance $d$ the whale has traveled in $t$ hours can be predicted by the equation $d = 4.5t$.
   
   a. Graph the equation.
   
   b. Use the graph to predict the time it takes the killer whale to travel 30 kilometers.
1. **FOOTBALL** One football season, the Carolina Panthers won 4 more games than they lost. This can be represented by \( y = x + 4 \), where \( x \) is the number of games lost and \( y \) is the number of games won. Write this linear equation in standard form.

2. **TOWING** Pick-M-Up Towing Company charges $40 to hook a car and $1.70 for each mile that it is towed. The equation \( y = 1.7x + 40 \) represents the total cost \( y \) for \( x \) miles towed. Determine the \( y \)-intercept. Describe what the value means in this context.

3. **SHIPPING** The *OOCL Shenzhen*, one of the world’s largest container ships, carries 8063 TEUs (1280 cubic feet containers). Workers can unload a ship at a rate of a TEU every minute. Using this rate, write and graph an equation to determine how many hours it will take the workers to unload half of the containers from the *Shenzhen*.

4. **BUSINESS** The equation \( y = 1000x - 5000 \) represents the monthly profits of a start-up dry cleaning company. Time in months is \( x \) and profit in dollars is \( y \). The first date of operation is when time is zero. However, preparation for opening the business began 3 months earlier with the purchase of equipment and supplies. Graph the linear function for \( x \)-values from \(-3\) to 8.

5. **BONE GROWTH** The height of a woman can be predicted by the equation \( h = 81.2 + 3.34r \), where \( h \) is her height in centimeters and \( r \) is the length of her radius bone in centimeters.
   
   a. Is this a linear function? Explain.

   b. What are the \( r \)- and \( h \)-intercepts of the equation? Do they make sense in the situation? Explain.

   c. Use the function to find the approximate height of a woman whose radius bone is 25 centimeters long.
Translating Linear Graphs

Linear graphs can be translated on the coordinate plane. This means that the graph moves up, down, right, or left without changing its direction.

Translating the graphs up or down affects the $y$-coordinate for a given $x$ value. Translating the graph right or left affects the $x$-coordinate for a given $y$-value.

**Example**

Translate the graph of $y = 2x + 2$, 3 units up.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Add 3 to each $y$-value.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

**Exercises**

Graph the function and the translation on the same coordinate plane.

1. $y = x + 4$, 3 units down

2. $y = 2x - 2$, 2 units left

3. $y = -2x + 1$, 1 unit right

4. $y = -x - 3$, 2 units up
3-1 Spreadsheet Activity

Linear Equations

In addition to organizing data, a spreadsheet can be used to represent data graphically.

Example

An internet retailer charges $1.99 per order plus $0.99 per item to ship books and CDs. Graph the equation \( y = 1.99 + 0.99x \), where \( x \) is the number of items ordered and \( y \) is the shipping cost.

Step 1 Use column A for the numbers of items and column B for the shipping costs.

Step 2 Create a graph from the data. Select the data in columns A and B and select Chart from the Insert menu. Select an XY (Scatter) chart to show the data points connected with line segments.

Exercises

1. A photo printer offers a subscription for digital photo finishing. The subscription costs $4.99 per month. Each standard size photo a subscriber prints costs $0.19. Use a spreadsheet to graph the equation \( y = 4.99 + 0.19x \), where \( x \) is the number of photos printed and \( y \) is the total monthly cost.

2. A long distance service plan includes a $8.95 per month fee plus $0.05 per minute of calls. Use a spreadsheet to graph the equation \( y = 8.95 + 0.05x \), where \( x \) is the number of minutes of calls and \( y \) is the total monthly cost.
Solve by Graphing

You can solve an equation by graphing the related function. The solution of the equation is the \(x\)-intercept of the function.

**Example**

Solve the equation \(2x - 2 = -4\) by graphing.

First set the equation equal to 0. Then replace 0 with \(f(x)\). Make a table of ordered pair solutions. Graph the function and locate the \(x\)-intercept.

\[
\begin{align*}
2x - 2 & = -4 & \text{Original equation} \\
2x - 2 + 4 & = -4 + 4 & \text{Add 4 to each side.} \\
2x + 2 & = 0 & \text{Simplify.} \\
f(x) & = 2x + 2 & \text{Replace 0 with } f(x).
\end{align*}
\]

To graph the function, make a table. Graph the ordered pairs.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 2x + 2)</th>
<th>(f(x))</th>
<th>([x, f(x)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(f(1) = 2(1) + 2)</td>
<td>4</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>-1</td>
<td>(f(-1) = 2(-1) + 2)</td>
<td>0</td>
<td>(-1, 0)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 2(-2) + 2)</td>
<td>-2</td>
<td>(-2, -2)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \((-1, 0)\).

The solution to the equation is \(x = -1\).

**Exercises**

Solve each equation.

1. \(3x - 3 = 0\)  
2. \(-2x + 1 = 5 - 2x\)  
3. \(-x + 4 = 0\)

4. \(0 = 4x - 1\)  
5. \(5x - 1 = 5x\)  
6. \(-3x + 1 = 0\)
Estimate Solutions by Graphing

Sometimes graphing does not provide an exact solution, but only an estimate. In these cases, solve algebraically to find the exact solution.

Example

WALKING You and your cousin decide to walk the 7-mile trail at the state park to the ranger station. The function \(d = 7 - 3.2t\) represents your distance \(d\) from the ranger station after \(t\) hours. Find the zero of this function. Describe what this value means in this context.

Make a table of values to graph the function.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(d = 7 - 3.2t)</th>
<th>(d)</th>
<th>((t, d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(d = 7 - 3.2(0))</td>
<td>7</td>
<td>(0, 7)</td>
</tr>
<tr>
<td>1</td>
<td>(d = 7 - 3.2(1))</td>
<td>3.8</td>
<td>(1, 3.8)</td>
</tr>
<tr>
<td>2</td>
<td>(d = 7 - 3.2(2))</td>
<td>0.6</td>
<td>(2, 0.6)</td>
</tr>
</tbody>
</table>

The graph intersects the \(t\)-axis between \(t = 2\) and \(t = 3\), but closer to \(t = 2\). It will take you and your cousin just over two hours to reach the ranger station.

You can check your estimate by solving the equation algebraically.

Exercises

1. MUSIC Jessica wants to record her favorite songs to one CD. The function \(C = 80 - 3.22n\) represents the recording time \(C\) available after \(n\) songs are recorded. Find the zero of this function. Describe what this value means in this context.

2. GIFT CARDS Enrique uses a gift card to buy coffee at a coffee shop. The initial value of the gift card is $20. The function \(n = 20 - 2.75c\) represents the amount of money still left on the gift card \(n\) after purchasing \(c\) cups of coffee. Find the zero of this function. Describe what this value means in this context.
Solve each equation.

1. \(2x - 5 = -3 + 2x\)  
2. \(-3x + 2 = 0\)  
3. \(3x + 2 = 3x - 1\)  
4. \(4x - 1 = 4x + 2\)  
5. \(4x - 1 = 0\)  
6. \(0 = 5x + 3\)  
7. \(0 = -2x + 4\)  
8. \(-3x + 8 = 5 - 3x\)  
9. \(-x + 1 = 0\)  

10. **GIFT CARDS** You receive a gift card for trading cards from a local store. The function \(d = 20 - 1.95c\) represents the remaining dollars \(d\) on the gift card after obtaining \(c\) packages of cards. Find the zero of this function. Describe what this value means in this context.
Solve each equation.

1. \( \frac{1}{2}x - 2 = 0 \)
2. \( -3x + 2 = -1 \)
3. \( 4x - 2 = -2 \)

4. \( \frac{1}{3}x + 2 = \frac{1}{3}x - 1 \)
5. \( \frac{2}{3}x + 4 = 3 \)
6. \( \frac{3}{4}x + 1 = \frac{3}{4}x - 7 \)

7. \( 13x + 2 = 11x - 1 \)
8. \( -9x - 3 = -4x - 3 \)
9. \( \frac{1}{3}x + 2 = \frac{2}{3}x - 1 \)

10. **DISTANCE** A bus is driving at 60 miles per hour toward a bus station that is 250 miles away. The function \( d = 250 - 60t \) represents the distance \( d \) from the bus station the bus is \( t \) hours after it has started driving. Find the zero of this function. Describe what this value means in this context.
1. **PET CARE** You buy a 6.3-pound bag of dry cat food for your cat. The function \( c = 6.3 - 0.25p \) represents the amount of cat food \( c \) remaining in the bag when the cat is fed the same amount each day for \( p \) days. Find the zero of this function. Describe what this value means in this context.

2. **SAVINGS** Jessica is saving for college using a direct deposit from her paycheck into a savings account. The function \( m = 3045 - 52.5t \) represents the amount of money \( m \) still needed after \( t \) weeks. Find the zero of this function. What does this value mean in this context?

3. **FINANCE** Michael borrows $100 from his dad. The function \( v = 100 - 4.75p \) represents the outstanding balance \( v \) after \( p \) weekly payments. Find the zero of this function. Describe what this value means in this context.

4. **BAKE SALE** Ashley has $15 in the Pep Club treasury to pay for supplies for a chocolate chip cookie bake sale. The function \( d = 15 - 0.08c \) represents the dollars \( d \) left in the club treasury after making \( c \) cookies. Find the zero of this function. What does this value represent in this context?

5. **DENTAL HYGIENE** You are packing your suitcase to go away to a 14-day summer camp. The store carries three sizes of tubes of toothpaste.

<table>
<thead>
<tr>
<th>Tube</th>
<th>Size (ounces)</th>
<th>Size (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.75</td>
<td>21.26</td>
</tr>
<tr>
<td>B</td>
<td>0.9</td>
<td>25.52</td>
</tr>
<tr>
<td>C</td>
<td>3.0</td>
<td>85.04</td>
</tr>
</tbody>
</table>

Source: National Academy of Sciences

a. The function \( n = 21.26 - 0.8b \) represents the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube A. Find the zero of this function. Describe what this value means in this context.

b. The function \( n = 25.52 - 0.8b \) represents the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube B. Find the zero of this function. Describe what this value means in this context.

c. Write a function to represent the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube C. Find the zero of this function. Describe what this value means in this context.

d. If you will brush your teeth twice each day while at camp, which is the smallest tube of toothpaste you can choose? Explain your reasoning.
Composite Functions

Three things are needed to have a function—a set called the domain, a set called the range, and a rule that matches each element in the domain with only one element in the range. Here is an example.

Rule: \( f(x) = 2x + 1 \)

\[
\begin{align*}
  x & \quad f(x) \\
  1 & \quad 3 \\
  2 & \quad 5 \\
 -3 & \quad -5
\end{align*}
\]

\( f(1) = 2(1) + 1 = 2 + 1 = 3 \)
\( f(2) = 2(2) + 1 = 4 + 1 = 5 \)
\( f(-3) = 2(-3) + 1 = -6 + 1 = -5 \)

Suppose we have three sets A, B, and C and two functions described as shown below.

Rule: \( f(x) = 2x + 1 \) Rule: \( g(y) = 3y - 4 \)

\[
\begin{align*}
  A & \quad B & \quad C \\
  x & \quad f(x) & \quad g(f(x)) \\
  1 & \quad 3 & \quad 5
\end{align*}
\]

\( g(3) = 3(3) - 4 = 5 \)

Let’s find a rule that will match elements of set A with elements of set C without finding any elements in set B. In other words, let’s find a rule for the composite function \( g[f(x)] \).

Since \( f(x) = 2x + 1 \), \( g[f(x)] = g(2x + 1) \).

Since \( g(y) = 3y - 4 \), \( g(2x + 1) = 3(2x + 1) - 4 \), or \( 6x - 1 \).

Therefore, \( g[f(x)] = 6x - 1 \).

Find a rule for the composite function \( g[f(x)] \).

1. \( f(x) = 3x \) and \( g(y) = 2y + 1 \)
2. \( f(x) = x^2 + 1 \) and \( g(y) = 4y \)
3. \( f(x) = -2x \) and \( g(y) = y^2 - 3y \)
4. \( f(x) = \frac{1}{x - 3} \) and \( g(y) = y^{-1} \)

5. Is it always the case that \( g[f(x)] = f[g(x)] \)? Justify your answer.
Rate of Change and Slope

The rate of change tells, on average, how a quantity is changing over time.

Example

**POPULATION** The graph shows the population growth in China.


\[
\text{1950–1975: } \frac{\text{change in population}}{\text{change in time}} = \frac{0.93 - 0.55}{1975 - 1950} = \frac{0.38}{25} = 0.0152
\]

\[
\text{2000–2025: } \frac{\text{change in population}}{\text{change in time}} = \frac{1.45 - 1.27}{2025 - 2000} = \frac{0.18}{25} = 0.0072
\]

b. Explain the meaning of the rate of change in each case.

From 1950–1975, the growth was 0.0152 billion per year, or 15.2 million per year. From 2000–2025, the growth is expected to be 0.0072 billion per year, or 7.2 million per year.

c. How are the different rates of change shown on the graph?

There is a greater vertical change for 1950–1975 than for 2000–2025. Therefore, the section of the graph for 1950–1975 has a steeper slope.

Exercises

1. **LONGEVITY** The graph shows the predicted life expectancy for men and women born in a given year.


c. Explain the meaning of your results in Exercises 1 and 2.

d. What pattern do you see in the increase with each 25-year period?

e. Make a prediction for the life expectancy for 2050–2075. Explain how you arrived at your prediction.
Rate of Change and Slope

Find Slope The slope of a line is the ratio of change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move in the positive direction.

\[ m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } (x_1, y_1) \text{ and } (x_2, y_2) \text{ are the coordinates of any two points on a nonvertical line} \]

Example 1 Find the slope of the line that passes through \((-3, 5)\) and \((4, -2)\).

Let \((-3, 5) = (x_1, y_1)\) and \((4, -2) = (x_2, y_2)\).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{-2 - 5}{4 - (-3)} \]
\[ = \frac{-7}{7} \]
\[ = -1 \]

Example 2 Find the value of \(r\) so that the line through \((10, r)\) and \((3, 4)\) has a slope of \(-\frac{2}{7}\).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{4 - r}{3 - 10} \]
\[ = \frac{-2}{7} \]
\[ 2(-7) = 7(4 - r) \]
\[ 14 = 28 - 7r \]
\[ -14 = -7r \]
\[ 2 = r \]

Exercises

Find the slope of the line that passes through each pair of points.

1. \((4, 9), (1, 6)\)  
2. \((-4, -1), (-2, -5)\)  
3. \((-4, -1), (-4, -5)\)

4. \((2, 1), (8, 9)\)  
5. \((14, -8), (7, -6)\)  
6. \((4, -3), (8, -3)\)

7. \((1, -2), (6, 2)\)  
8. \((2, 5), (6, 2)\)  
9. \((4, 3.5), (-4, 3.5)\)

Find the value of \(r\) so the line that passes through each pair of points has the given slope.

10. \((6, 8), (r, -2), m = 1\)  
11. \((-1, -3), (7, r), m = \frac{3}{4}\)  
12. \((2, 8), (r, -4), m = -3\)

13. \((7, -5), (6, r), m = 0\)  
14. \((r, 4), (7, 1), m = \frac{3}{4}\)  
15. \((7, 5), (r, 9), m = 6\)
3-3 Skills Practice

Rate of Change and Slope

Find the slope of the line that passes through each pair of points.

1. (0, 1), (2, 5)
2. (0, 0), (3, 1)
3. (0, 1), (1, -2)

4. (2, 5), (3, 6)
5. (6, 1), (-6, 1)
6. (4, 6), (4, 8)
7. (5, 2), (5, -2)
8. (2, 5), (-3, -5)
9. (9, 8), (7, -8)
10. (-5, -8), (-8, 1)
11. (-3, 10), (-3, 7)
12. (17, 18), (18, 17)
13. (-6, -4), (4, 1)
14. (10, 0), (-2, 4)
15. (2, -1), (-8, -2)
16. (5, -9), (3, -2)
17. (12, 6), (3, -5)
18. (-4, 5), (-8, -5)
19. (-5, 6), (7, -8)

Find the value of r so the line that passes through each pair of points has the given slope.

20. (r, 3), (5, 9), m = 2
21. (5, 9), (r, -3), m = -4
22. (r, 2), (6, 3), m = \frac{1}{2}
23. (r, 4), (7, 1), m = \frac{3}{4}
24. (5, 3), (r, -5), m = 4
25. (7, r), (4, 6), m = 0
3-3 Practice

Rate of Change and Slope

Find the slope of the line that passes through each pair of points.

1. \((-2, 3), (-1, 0)\)

2. \((3, 1), (-2, -3)\)

3. \((-2, 3), (3, 3)\)

4. \((6, 3), (7, -4)\)

5. \((-9, -3), (-7, -5)\)

6. \((6, -2), (5, -4)\)

7. \((7, -4), (4, 8)\)

8. \((-7, 8), (-7, 5)\)

9. \((5, 9), (3, 9)\)

10. \((15, 2), (-6, 5)\)

11. \((3, 9), (-2, 8)\)

12. \((-2, -5), (7, 8)\)

13. \((12, 10), (12, 5)\)

14. \((0.2, -0.9), (0.5, -0.9)\)

15. \(\left(\frac{7}{3}, \frac{4}{3}\right), \left(-\frac{1}{3}, \frac{2}{3}\right)\)

Find the value of \(r\) so the line that passes through each pair of points has the given slope.

16. \((-2, r), (6, 7), m = \frac{1}{2}\)

17. \((-4, 3), (r, 5), m = \frac{1}{4}\)

18. \((-3, -4), (-5, r), m = -\frac{9}{2}\)

19. \((-5, r), (1, 3), m = \frac{7}{6}\)

20. \((1, 4), (r, 5), m\) undefined

21. \((-7, 2), (-8, r), m = -5\)

22. \((r, 7), (11, 8), m = -\frac{1}{5}\)

23. \((r, 2), (5, r), m = 0\)

24. ROOFING The pitch of a roof is the number of feet the roof rises for each 12 feet horizontally. If a roof has a pitch of 8, what is its slope expressed as a positive number?

25. SALES A daily newspaper had 12,125 subscribers when it began publication. Five years later it had 10,100 subscribers. What is the average yearly rate of change in the number of subscribers for the five-year period?
1. **HIGHWAYS** Roadway signs such as the one below are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign?

![Roadway sign with 8% grade](image)

2. **AMUSEMENT PARKS** The SheiKra roller coaster at Busch Gardens in Tampa, Florida, features a 138-foot vertical drop. What is the slope of the coaster track at this part of the ride? Explain.

3. **CENSUS** The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2009.

<table>
<thead>
<tr>
<th>Year</th>
<th>People Per Square Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>22.1</td>
</tr>
<tr>
<td>1960</td>
<td>36.4</td>
</tr>
<tr>
<td>1980</td>
<td>54.3</td>
</tr>
<tr>
<td>2000</td>
<td>79.6</td>
</tr>
<tr>
<td>2009</td>
<td>96.7</td>
</tr>
</tbody>
</table>

**Source:** Bureau of the Census, U.S. Dept. of Commerce

4. **REAL ESTATE** A realtor estimates the median price of an existing single-family home in Cedar Ridge is $221,900. Two years ago, the median price was $195,200. Find the average annual rate of change in median home price in these years.

5. **COAL EXPORTS** The graph shows the annual coal exports from U.S. mines in millions of short tons.

![Coal exports graph](image)

**Source:** Energy Information Association

a. What was the rate of change in coal exports between 2001 and 2002?

b. How does the rate of change in coal exports from 2005 to 2006 compare to that of 2001 to 2002?

c. Explain the meaning of the part of the graph with a slope of zero.
Treasure Hunt with Slopes

Using the definition of slope, draw segments with the slopes listed below in order. A correct solution will trace the route to the treasure.

1. 3
2. $\frac{1}{4}$
3. $-\frac{2}{5}$
4. 0
5. 1
6. $-1$
7. no slope
8. $\frac{2}{7}$
9. $\frac{3}{2}$
10. $\frac{1}{3}$
11. $-\frac{3}{4}$
12. 3
3-4 Study Guide and Intervention

Direct Variation

Direct Variation Equations A direct variation is described by an equation of the form \( y = kx \), where \( k \neq 0 \). We say that \( y \) varies directly as \( x \). In the equation \( y = kx \), \( k \) is the constant of variation.

Example 1 Name the constant of variation for the equation. Then find the slope of the line that passes through the pair of points.

For \( y = \frac{1}{2}x \), the constant of variation is \( \frac{1}{2} \).

\[
    m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}
\]

\[
    = \frac{1 - 0}{2 - 0} \quad (x_1, y_1) = (0, 0), (x_2, y_2) = (2, 1)
\]

\[
    = \frac{1}{2} \quad \text{Simplify.}
\]

The slope is \( \frac{1}{2} \).

Example 2 Suppose \( y \) varies directly as \( x \), and \( y = 30 \) when \( x = 5 \).

a. Write a direct variation equation that relates \( x \) and \( y \).

Find the value of \( k \).

\[
    30 = k(5) \quad \text{Direct variation equation}
\]

\[
    6 = k \quad \text{Divide each side by 5.}
\]

Therefore, the equation is \( y = 6x \).

b. Use the direct variation equation to find \( x \) when \( y = 18 \).

\[
    y = 6x \quad \text{Direct variation equation}
\]

\[
    18 = 6x \quad \text{Replace } y \text{ with } 18.
\]

\[
    3 = x \quad \text{Divide each side by 6.}
\]

Therefore, \( x = 3 \) when \( y = 18 \).

Exercises

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \( y = \frac{1}{2}x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) to \( y \). Then solve.

4. If \( y = 4 \) when \( x = 2 \), find \( y \) when \( x = 16 \).

5. If \( y = 9 \) when \( x = -3 \), find \( x \) when \( y = 6 \).

6. If \( y = -4.8 \) when \( x = -1.6 \), find \( x \) when \( y = -24 \).

7. If \( y = \frac{1}{4} \) when \( x = \frac{1}{8} \), find \( x \) when \( y = \frac{3}{16} \).
**Direct Variation**

Direct Variation Problems  The distance formula \( d = rt \) is a direct variation equation. In the formula, distance \( d \) varies directly as time \( t \), and the rate \( r \) is the constant of variation.

**Example**  TRAVEL  A family drove their car 225 miles in 5 hours.

a. Write a direct variation equation to find the distance traveled for any number of hours.

Use given values for \( d \) and \( t \) to find \( r \).

\[
d = rt  \\
225 = r(5)  \\
45 = r  \\
\]

Therefore, the direct variation equation is \( d = 45t \).

b. Graph the equation.

The graph of \( d = 45t \) passes through the origin with slope 45.

\[
m = \frac{45}{1} \quad \text{rise} = 45, \quad \text{run} = 1
\]

\( \checkmark \) CHECK  (5, 225) lies on the graph.

c. Estimate how many hours it would take the family to drive 360 miles.

\[
d = 45t  \\
360 = 45t  \\
t = 8
\]

Therefore, it will take 8 hours to drive 360 miles.

**Exercises**

1. RETAIL  The total cost \( C \) of bulk jelly beans is $4.49 times the number of pounds \( p \).

a. Write a direct variation equation that relates the variables.

b. Graph the equation on the grid at the right.

c. Find the cost of \( \frac{3}{4} \) pound of jelly beans.

2. CHEMISTRY  Charles’s Law states that, at a constant pressure, volume of a gas \( V \) varies directly as its temperature \( T \). A volume of 4 cubic feet of a certain gas has a temperature of 200 degrees Kelvin.

a. Write a direct variation equation that relates the variables.

b. Graph the equation on the grid at the right.

c. Find the volume of the same gas at 250 degrees Kelvin.
Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \((3, 1), (0, 0)\)
   \[y = \frac{1}{3}x\]

2. \((-1, 2), (0, 0)\)
   \[y = -2x\]

3. \((-2, 3), (0, 0)\)
   \[y = -\frac{3}{2}x\]

Graph each equation.

4. \(y = 3x\)

5. \(y = -\frac{3}{4}x\)

6. \(y = \frac{2}{5}x\)

Suppose \(y\) varies directly as \(x\). Write a direct variation equation that relates \(x\) and \(y\). Then solve.

7. If \(y = -8\) when \(x = -2\), find \(x\) when \(y = 32\).

8. If \(y = 45\) when \(x = 15\), find \(x\) when \(y = 15\).

9. If \(y = -4\) when \(x = 2\), find \(y\) when \(x = -6\).

10. If \(y = -9\) when \(x = 3\), find \(y\) when \(x = -5\).

11. If \(y = 4\) when \(x = 16\), find \(y\) when \(x = 6\).

12. If \(y = 12\) when \(x = 18\), find \(x\) when \(y = -16\).

Write a direct variation equation that relates the variables. Then graph the equation.

13. **TRAVEL** The total cost \(C\) of gasoline is $3.00 times the number of gallons \(g\).

   \[
   \begin{array}{c|c|c|c|c|c}
   \hline
   \text{Gallons} & 0 & 1 & 2 & 3 & 4 \\
   \hline
   \text{Cost ($)} & 0 & 3 & 6 & 9 & 12 \\
   \hline
   \end{array}
   \]

14. **SHIPPING** The number of delivered toys \(T\) is 3 times the total number of crates \(c\).

   \[
   \begin{array}{c|c|c|c|c|c}
   \hline
   \text{Crates} & 0 & 1 & 2 & 3 & 4 \\
   \hline
   \text{Toys} & 0 & 6 & 12 & 18 & 24 \\
   \hline
   \end{array}
   \]
3-4 Practice

Direct Variation

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \( (4, 3) \quad (0, 0) \)
   \[ y = \frac{3}{4} x \]

2. \( (3, 4) \quad (0, 0) \)
   \[ y = \frac{4}{3} x \]

3. \( (-2, 5) \quad (0, 0) \)
   \[ y = -\frac{5}{2} x \]

Graph each equation.

4. \( y = -2x \)

5. \( y = \frac{6}{5} x \)

6. \( y = -\frac{5}{2} x \)

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

7. If \( y = 7.5 \) when \( x = 0.5 \), find \( y \) when \( x = -0.3 \).

8. If \( y = 80 \) when \( x = 32 \), find \( x \) when \( y = 100 \).

9. If \( y = \frac{3}{4} \) when \( x = 24 \), find \( y \) when \( x = 12 \).

Write a direct variation equation that relates the variables. Then graph the equation.

10. **MEASURE** The width \( W \) of a rectangle is two thirds of the length \( \ell \).

11. **TICKETS** The total cost \( C \) of tickets is $4.50 times the number of tickets \( t \).

12. **PRODUCE** The cost of bananas varies directly with their weight. Miguel bought \( 3 \frac{1}{2} \) pounds of bananas for $1.12. Write an equation that relates the cost of the bananas to their weight. Then find the cost of \( 4 \frac{1}{4} \) pounds of bananas.
3-4 Word Problem Practice

Direct Variation

1. **ENGINES** The engine of a chainsaw requires a mixture of engine oil and gasoline. According to the directions, oil and gasoline should be mixed as shown in the graph below. What is the constant of variation for the line graphed?

![Graph of oil and gasoline mixture]

2. **RACING** In a recent year, English driver Lewis Hamilton won the United States Grand Prix at the Indianapolis Motor Speedway. His speed during the race averaged 125.145 miles per hour. Write a direct variation equation for the distance $d$ that Hamilton drove in $h$ hours at that speed.

3. **CURRENCY** The exchange rate from one currency to another varies every day. Recently the exchange rate from U.S. dollars to British pound sterling (£) was $1.58 to £1. Write and solve a direct variation equation to determine how many pounds sterling you would receive in exchange for $90 of U.S. currency.

4. **SALARY** Henry started a new job in which he is paid $9.50 an hour. Write and solve an equation to determine Henry’s gross salary for a 40-hour work week.

5. **SALES TAX** Amelia received a gift card to a local music shop for her birthday. She plans to use the gift card to buy some new CDs.
   a. Amelia chose 3 CDs that each cost $16. The sales tax on the three CDs is $3.96. Write a direct variation equation relating sales tax to the price.
   b. Graph the equation you wrote in part a.
   c. What is the sales tax rate that Amelia is paying on the CDs?
**nth Power Variation**

An equation of the form $y = kx^n$, where $k \neq 0$, describes an $n$th power variation. The variable $n$ can be replaced by 2 to indicate the second power of $x$ (the square of $x$) or by 3 to indicate the third power of $x$ (the cube of $x$).

Assume that the weight of a person of average build varies directly as the cube of that person’s height. The equation of variation has the form $w = kh^3$.

The weight that a person’s legs will support is proportional to the cross-sectional area of the leg bones. This area varies directly as the square of the person’s height. The equation of variation has the form $s = kh^2$.

**Answer each question.**

1. For a person 6 feet tall who weighs 200 pounds, find a value for $k$ in the equation $w = kh^3$.

2. Use your answer from Exercise 1 to predict the weight of a person who is 5 feet tall.

3. Find the value for $k$ in the equation $w = kh^3$ for a baby who is 20 inches long and weighs 6 pounds.

4. How does your answer to Exercise 3 demonstrate that a baby is significantly fatter in proportion to its height than an adult?

5. For a person 6 feet tall who weighs 200 pounds, find a value for $k$ in the equation $s = kh^2$.

6. For a baby who is 20 inches long and weighs 6 pounds, find an “infant value” for $k$ in the equation $s = kh^2$.

7. According to the adult equation you found (Exercise 1), how much would an imaginary giant 20 feet tall weigh?

8. According to the adult equation for weight supported (Exercise 5), how much weight could a 20-foot tall giant’s legs actually support?

9. What can you conclude from Exercises 7 and 8?
Recognize Arithmetic Sequences

A sequence is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an arithmetic sequence.

<table>
<thead>
<tr>
<th>Arithmetic Sequence</th>
<th>Terms of an Arithmetic Sequence</th>
<th>nth Term of an Arithmetic Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a numerical pattern that increases or decreases at a constant rate or value called the common difference</td>
<td>If ( a_1 ) is the first term of an arithmetic sequence with common difference ( d ), then the sequence is ( a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots ).</td>
<td>( a_n = a_1 + (n - 1)d )</td>
</tr>
</tbody>
</table>

Example 1

Determine whether the sequence 1, 3, 5, 7, 9, 11, \ldots is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since 3 − 1 = 2, 5 − 3 = 2, and so on, the common difference is 2.

Since the difference between the terms of 1, 3, 5, 7, 9, 11, \ldots is constant, this is an arithmetic sequence.

Example 2

Write an equation for the \( n \)th term of the sequence 12, 15, 18, 21, \ldots.

In this sequence, \( a_1 \) is 12. Find the common difference.

The common difference is 3.

Use the formula for the \( n \)th term to write an equation.

\[
\begin{align*}
a_n &= a_1 + (n - 1)d \\
a_n &= 12 + (n - 1)3 \\
a_n &= 12 + 3n - 3 \\
a_n &= 3n + 9
\end{align*}
\]

The equation for the \( n \)th term is \( a_n = 3n + 9 \).

Exercises

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 1, 5, 9, 13, 17, \ldots 
2. 8, 4, 0, −4, −8, \ldots 
3. 1, 3, 9, 27, 81, \ldots 

Find the next three terms of each arithmetic sequence.

4. 9, 13, 17, 21, 25, \ldots 
5. 4, 0, −4, −8, −12, \ldots 
6. 29, 35, 41, 47, \ldots 

Write an equation for the \( n \)th term of each arithmetic sequence. Then graph the first five terms of the sequence.

7. 1, 3, 5, 7, \ldots 
8. −1, −4, −7, −10, \ldots 
9. −4, −9, −14, −19, \ldots
Lesson 3-5

Study Guide and Intervention (continued)

Arithmetic Sequences as Linear Functions

Arithmetic Sequences and Functions  An arithmetic sequence is a linear function in which \( n \) is the independent variable, \( a_n \) is the dependent variable, and the common difference \( d \) is the slope. The formula can be rewritten as the function \( a_n = a_1 + (n - 1)d \), where \( n \) is a counting number.

Example  SEATING  There are 20 seats in the first row of the balcony of the auditorium. There are 22 seats in the second row, and 24 seats in the third row.

a. Write a function to represent this sequence.

The first term \( a_1 \) is 20. Find the common difference.

\[
\begin{align*}
20 & \quad 22 & \quad 24 \\
+2 & \quad +2 \\
\hline
\end{align*}
\]

The common difference is 2.

\[
a_n = a_1 + (n - 1)d \quad \text{Formula for the } n\text{th term} \\
= 20 + (n - 1)2 \quad a_1 = 20 \quad \text{and} \quad d = 2 \\
= 20 + 2n - 2 \quad \text{Distributive Property} \\
= 18 + 2n \quad \text{Simplify.}
\]

The function is \( a_n = 18 + 2n \).

b. Graph the function.

The rate of change is 2. Make a table and plot points.

\[
\begin{array}{c|c}
n & a_n \\ 
1 & 20 \\ 
2 & 22 \\ 
3 & 24 \\ 
4 & 26 \\ 
\end{array}
\]

Exercises

1. KNITTING  Sarah learns to knit from her grandmother. Two days ago, she measured the length of the scarf she is knitting to be 13 inches. Yesterday, she measured the length of the scarf to be 15.5 inches. Today it measures 18 inches. Write a function to represent the arithmetic sequence.

2. REFRESHMENTS  You agree to pour water into the cups for the Booster Club at a football game. The pitcher contains 64 ounces of water when you begin. After you have filled 8 cups, the pitcher is empty and must be refilled.

a. Write a function to represent the arithmetic sequence.

b. Graph the function.
3-5 Skills Practice

Arithmetic Sequences as Linear Functions

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 4, 7, 9, 12, . . . 2. 15, 13, 11, 9, . . .
3. 7, 10, 13, 16, . . . 4. –6, –5, –3, –1, . . .
5. –5, –3, –1, 1, . . . 6. –9, –12, –15, –18, . . .
7. 10, 15, 25, 40, . . . 8. –10, –5, 0, 5, . . .

Find the next three terms of each arithmetic sequence.

9. 3, 7, 11, 15, . . . 10. 22, 20, 18, 16, . . .
15. 2.5, 5, 7.5, 10, . . . 16. 3.1, 4.1, 5.1, 6.1, . . .

Write an equation for the \( n \)th term of each arithmetic sequence. Then graph the first five terms of the sequence.

17. 7, 13, 19, 25, . . . 18. 30, 26, 22, 18, . . . 19. –7, –4, –1, 2, . . .

20. VIDEO DOWNLOADING  Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is $1.99. The cost to download 2 episodes is $3.98. The cost to download 3 episodes is $5.97. Write a function to represent the arithmetic sequence.
Lesson 3-5

**Arithmetic Sequences as Linear Functions**

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 21, 13, 5, -3, . . .
2. -5, 12, 29, 46, . . .
3. -2.2, -1.1, 0.1, 1.3, . . .
4. 1, 4, 9, 16, . . .
5. 9, 16, 23, 30, . . .
6. -1.2, 0.6, 1.8, 3.0, . . .

Find the next three terms of each arithmetic sequence.

7. 82, 76, 70, 64, . . .
9. $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0, . . .$
10. -10, -3, 4, 11 . . .
11. 12, 10, 8, 6, . . .
12. 12, 7, 2, -3, . . .

Write an equation for the $n$th term of each arithmetic sequence. Then graph the first five terms of the sequence.

13. 9, 13, 17, 21, . . .
14. -5, -2, 1, 4, . . .
15. 19, 31, 43, 55, . . .

16. **BANKING** Chem deposited $115.00 in a savings account. Each week thereafter, he deposits $35.00 into the account.

   a. Write a function to represent the total amount Chem has deposited for any particular number of weeks after his initial deposit.

   b. How much has Chem deposited 30 weeks after his initial deposit?

17. **STORE DISPLAYS** Tamika is stacking boxes of tissue for a store display. Each row of tissues has 2 fewer boxes than the row below. The first row has 23 boxes of tissues.

   a. Write a function to represent the arithmetic sequence.

   b. How many boxes will there be in the tenth row?
3-5 Word Problem Practice

Arithmetic Sequences as Linear Functions

1. POSTAGE The price to send a large envelope first class mail is 88 cents for the first ounce and 17 cents for each additional ounce. The table below shows the cost for weights up to 5 ounces.

<table>
<thead>
<tr>
<th>Weight (ounces)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postage (dollars)</td>
<td>0.88</td>
<td>1.05</td>
<td>1.22</td>
<td>1.39</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Source: United States Postal Service

How much did a large envelope weigh that cost $2.07 to send?

2. SPORTS Wanda is the manager for the soccer team. One of her duties is to hand out cups of water at practice. Each cup of water is 4 ounces. She begins practice with a 128-ounce cooler of water. How much water is remaining after she hands out the 14th cup?

3. THEATER A theater has 20 seats in the first row, 22 in the second row, 24 in the third row, and so on for 25 rows. How many seats are in the last row?

4. NUMBER THEORY One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170–1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...  

Does this represent an arithmetic sequence? Why or why not?

5. SAVINGS Inga’s grandfather decides to start a fund for her college education. He makes an initial contribution of $3000 and each month deposits an additional $500. After one month he will have contributed $3500.

a. Write an equation for the nth term of the sequence.

b. How much money will Inga’s grandfather have contributed after 24 months?
Arithmetic Series

An arithmetic series is a series in which each term after the first may be found by adding the same number to the preceding term. Let \( S \) stand for the following series in which each term is 3 more than the preceding one.

\[
S = 2 + 5 + 8 + 11 + 14 + 17 + 20
\]

The series remains the same if we reverse the order of all the terms. So let us reverse the order of the terms and add one series to the other, term by term. This is shown at the right.

\[
S = 2 + 5 + 8 + 11 + 14 + 17 + 20
\]

\[
S = 20 + 17 + 14 + 11 + 8 + 5 + 2
\]

\[
2S = 22 + 22 + 22 + 22 + 22 + 22 + 22
\]

\[
2S = 7(22)
\]

\[
S = \frac{7(22)}{2} = 7(11) = 77
\]

Let \( a \) represent the first term of the series.

Let \( \ell \) represent the last term of the series.

Let \( n \) represent the number of terms in the series.

In the preceding example, \( a = 2, \ell = 20, \) and \( n = 7. \) Notice that when you add the two series, term by term, the sum of each pair of terms is 22. That sum can be found by adding the first and last terms, \( 2 + 20 \) or \( a + \ell. \) Notice also that there are 7, or \( n, \) such sums. Therefore, the value of \( 2S \) is \( 7(22), \) or \( n(a + \ell) \) in the general case. Since this is twice the sum of the series, you can use the formula \( S = \frac{n(a + \ell)}{2} \) to find the sum of any arithmetic series.

**Example 1**  
Find the sum: \( 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9. \)

\[
a = 1, \ell = 9, n = 9, \text{ so } S = \frac{9(1 + 9)}{2} = \frac{9 \cdot 10}{2} = 45
\]

**Example 2**  
Find the sum: \( -9 + (-5) + (-1) + 3 + 7 + 11 + 15. \)

\[
a = 29, \ell = 15, n = 7, \text{ so } S = \frac{7(-9 + 15)}{2} = \frac{7 \cdot 6}{2} = 21
\]

**Exercises**

Find the sum of each arithmetic series.

1. \( 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 \)

2. \( 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 \)

3. \( -21 + (-16) + (-11) + (-6) + (-1) + 4 + 9 + 14 \)

4. even whole numbers from 2 through 100

5. odd whole numbers between 0 and 100
3-6 Study Guide and Intervention

Proportional and Nonproportional Relationships

Proportional Relationships

If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation. If the equation passes through (0, 0) and is of the form \( y = kx \), then the relationship is proportional.

**Example**

**COMPACT DISCS**

Suppose you purchased a number of packages of blank compact discs. If each package contains 3 compact discs, you could make a chart to show the relationship between the number of packages of compact discs and the number of discs purchased. Use \( x \) for the number of packages and \( y \) for the number of compact discs.

Make a table of ordered pairs for several points of the graph.

<table>
<thead>
<tr>
<th>Number of Packages</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of CDs</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

The difference in the \( x \) values is 1, and the difference in the \( y \) values is 3. This pattern shows that \( y \) is always three times \( x \). This suggests the relation \( y = 3x \). Since the relation is also a function, we can write the equation in function notation as \( f(x) = 3x \).

The relation includes the point \((0, 0)\) because if you buy 0 packages of compact discs, you will not have any compact discs. Therefore, the relationship is proportional.

**Exercises**

1. **NATURAL GAS**

   Natural gas use is often measured in “therms.” The total amount a gas company will charge for natural gas use is based on how much natural gas a household uses. The table shows the relationship between natural gas use and the total cost.

<table>
<thead>
<tr>
<th>Gas Used (therms)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>$1.30</td>
<td>$2.60</td>
<td>$3.90</td>
<td>$5.20</td>
</tr>
</tbody>
</table>

   **a.** Graph the data. What can you deduce from the pattern about the relationship between the number of therms used and the total cost?

   **b.** Write an equation to describe this relationship.

   **c.** Use this equation to predict how much it will cost if a household uses 40 therms.
Nonproportional Relationships

If the ratio of the value of \( x \) to the value of \( y \) is different for select ordered pairs on the line, the equation is nonproportional.

Example

Write an equation in functional notation for the relation shown in the graph.

Select points from the graph and place them in a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(4)</td>
<td>(2)</td>
<td>(0)</td>
<td>(-2)</td>
<td>(-4)</td>
</tr>
</tbody>
</table>

The difference between the \( x \)-values is 1, while the difference between the \( y \)-values is \(-2\). This suggests that \( y = -2x \).

If \( x = 1 \), then \( y = -2(1) \) or \(-2\). But the \( y \)-value for \( x = 1 \) is 0.

\[
\begin{array}{c|c|c|c}
\hline
x & 1 & 2 & 3 \\
\hline
-2x & -2 & -4 & -6 \\
\hline
y & 0 & -2 & -4 \\
\hline
\end{array}
\]

\( y \) is always 2 more than \(-2x\)

This pattern shows that 2 should be added to one side of the equation. Thus, the equation is \( y = -2x + 2 \).

Exercises

Write an equation in function notation for the relation shown in the table. Then complete the table.

1. | \( x \) | \(-1\) | \(0\) | \(1\) | \(2\) | \(3\) | \(4\) |
  | \( y \) | \(-2\) | \(2\) | \(6\) |   |   |

2. | \( x \) | \(-2\) | \(-1\) | \(0\) | \(1\) | \(2\) | \(3\) |
  | \( y \) | 10   | 7   | 4   |   |   |

Write an equation in function notation for each relation.

3.

4.
3-6 Skills Practice

Proportional and Nonproportional Relationships

Write an equation in function notation for each relation.

1. [Graph of a line]

2. [Graph of a line]

3. [Graph of a line]

4. [Graph of a line]

5. [Graph of a line]

6. [Graph of a line]

7. GAMESHOWS The table shows how many points are awarded for answering consecutive questions on a gameshow.

<table>
<thead>
<tr>
<th>Question answered</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points awarded</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
</tr>
</tbody>
</table>

a. Write an equation for the data given.

b. Find the number of points awarded if 9 questions were answered.
1. **BIOLOGY** Male fireflies flash in various patterns to signal location and perhaps to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, its rate, and its shape. The table below shows the rate at which a male firefly is flashing.

<table>
<thead>
<tr>
<th>Times (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Flashes</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Write an equation in function notation for the relation.

b. How many times will the firefly flash in 20 seconds?

2. **GEOMETRY** The table shows the number of diagonals that can be drawn from one vertex in a polygon. Write an equation in function notation for the relation and find the number of diagonals that can be drawn from one vertex in a 12-sided polygon.

<table>
<thead>
<tr>
<th>Sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonals</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Write an equation in function notation for each relation.

3. 4. 5.

For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. Explain.

6. 1, 3, 5, . . .

7. 2, 7, 12, . . .

8. –3, –6, –9, . . .
Word Problem Practice

Proportional and Nonproportional Relationships

1. ONLINE SHOPPING  Ricardo is buying computer cables from an online store. If he buys 4 cables, the total cost will be $24. If he buys 5 cables, the total cost will be $29. If the total cost can be represented by a linear function, will the function be proportional or nonproportional? Explain.

2. FOOD  It takes about four pounds of grapes to produce one pound of raisins. The graph shows the relation for the number of pounds of grapes needed, \( x \), to make \( y \) pounds of raisins. Write an equation in function notation for the relation shown.

3. PARKING  Palmer Township recently installed parking meters in their municipal lot. The cost to park for \( h \) hours is represented by the equation \( C = 0.25h \).

   a. Make a table of values that represents this relationship.

   b. Describe the relationship between the time parked and the cost.

4. MUSIC  A measure of music contains the same number of beats throughout the song. The table shows the relation for the number of beats counted after a certain number of measures have been played in the six-eight time. Write an equation to describe this relationship.

<table>
<thead>
<tr>
<th>Measures Played (( x ))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Beats (( y ))</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

   Source: Sheet Music USA

5. GEOMETRY  A fractal is a pattern containing parts which are identical to the overall pattern. The following geometric pattern is a fractal.

   a. Complete the table.

<table>
<thead>
<tr>
<th>Term</th>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Smaller Triangles</td>
<td>( y )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. What are the next three numbers in the pattern?

   c. Write an equation in function notation for the pattern.
Taxicab Graphs
You have used a rectangular coordinate system to graph equations such as \( y = x - 1 \) on a coordinate plane. In a coordinate plane, the numbers in an ordered pair \((x, y)\) can be any two real numbers.

A taxicab plane is different from the usual coordinate plane. The only points allowed are those that exist along the horizontal and vertical grid lines. You may think of the points as taxicabs that must stay on the streets.

The taxicab graph shows the equations \( y = -2 \) and \( y = x - 1 \). Notice that one of the graphs is no longer a straight line. It is now a collection of separate points.

Graph these equations on the taxicab plane at the right.

1. \( y = x + 1 \)
2. \( y = -2x + 3 \)
3. \( y = 2.5 \)
4. \( x = -4 \)

Use your graphs for these problems.

5. Which of the equations has the same graph in both the usual coordinate plane and the taxicab plane?
6. Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane.

In the taxicab plane, distances are not measured diagonally, but along the streets. Write the taxi-distance between each pair of points.

7. \((0, 0)\) and \((5, 2)\)
8. \((0, 0)\) and \((-3, 2)\)
9. \((0, 0)\) and \((2, 1.5)\)

10. \((1, 2)\) and \((4, 3)\)
11. \((2, 4)\) and \((-1, 3)\)
12. \((0, 4)\) and \((-2, 0)\)

Draw these graphs on the taxicab grid at the right.

13. The set of points whose taxi-distance from \((0, 0)\) is 2 units.
14. The set of points whose taxi-distance from \((2, 1)\) is 3 units.
Multiple Choice

Read each question. Then fill in the correct answer.

1. ☐ ☐ ☐ ☐  
2. ☐ ☐ ☐ ☐  
3. ☐ ☐ ☐ ☐  
4. ☐ ☐ ☐ ☐  
5. ☐ ☐ ☐ ☐  
6. ☐ ☐ ☐ ☐  
7. ☐ ☐ ☐ ☐  
8. ☐ ☐ ☐ ☐  
9. ☐ ☐ ☐ ☐  

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

10.  
11.  
12.  
13. (grid in)  
14.  
15. (grid in)  

Extended Response

Record your answers for Question 16 on the back of this paper.
Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 16 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>For part <em>a</em>, the explanation must demonstrate an understanding that because each element of the domain is paired with exactly one element of the range, the table must represent a function. For part <em>b</em>, the explanation must demonstrate an understanding that it cannot be represented by an equation of the form ( y = kx ).</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
3 Chapter 3 Quiz 1
(Lessons 3-1 and 3-2)

1. Determine whether \( y = 2x - 1 \) is a linear equation. If so, write the equation in standard form.

2. MULTIPLE CHOICE What is the \( y \)-intercept when \( 3x - 2y = -6 \) is graphed?
   
   A. \(-3\)  
   B. \(-2\)  
   C. \(2\)  
   D. \(3\)

3. The distance \( d \) in miles that a car travels in \( t \) hours at a rate of 58 miles per hour is given by the equation \( d = 58t \). What is the best estimate of how far a car travels in 7 hours?

4. Graph \( 3x - y = 3 \).

5. Solve \( 3x - 45 = 0 \).

3 Chapter 3 Quiz 2
(Lesson 3-3)

Find the slope of the line passing through each pair of points.

1. \((5, 8)\) and \((-4, 6)\)  
2. \((9, 4)\) and \((5, -3)\)

3. MULTIPLE CHOICE Which value of \( r \) gives the line passing through \((3, 2)\) and \((r, -4)\) a slope of \(\frac{3}{2} \)?
   
   A. \(-6\)  
   B. \(-1\)  
   C. \(7\)  
   D. \(12\)

4. Determine whether the function is linear. Write linear or not linear.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-4</td>
<td>1</td>
</tr>
</tbody>
</table>

5. In 2004, there were approximately 275 students in the Delaware High School band. In 2010, that number increased to 305. Find the annual rate of change in the number of students in the band.
3 Chapter 3 Quiz 3
(Lessons 3-4 and 3-5)

1. MULTIPLE CHOICE  Suppose \( y \) varies directly as \( x \).
   If \( y = 5 \) when \( x = 8 \), find \( y \) when \( x = 64 \).

   A 25  B 40  C 61  D 102.4

2. Chris’s wages vary directly as the time she works. If her wages for 20 hours are $150, what are her wages for 38 hours?

3. Determine whether 2, 5, 9, 14, . . . is an arithmetic sequence.

4. Find the next three terms of the arithmetic sequence
   \( 5, 9, 13, 17, \ldots \).

5. Find the 15th term of the arithmetic sequence if
   \( a_1 = -3 \) and \( d = 2 \).

3 Chapter 3 Quiz 4
(Lesson 3-6)

For Questions 1–3, find the function that represents the relationship.

1. \[
\begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 3 & 6 & 9 & 12 \\
\end{array}
\]

2. \[
\begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 4 & 9 & 14 & 19 \\
\end{array}
\]

3. \[
\begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & -2 & 7 & 16 & 25 \\
\end{array}
\]

4. Is the following relationship proportional? Write yes or no.

   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 13 & 26 & 39 & 52 \\
\end{array}
\]

5. What values for \( a \) and \( b \) will make this linear relationship proportional?

   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & a & 11 & 22 & b \\
\end{array}
\]
Chapter 3 Mid-Chapter Test  
(Lessons 3-1 through 3-3)

Part I  Write the letter for the correct answer in the blank at the right of each question.

1. Find the slope of the line through (6, -7) and (4, -8).
   A $-\frac{1}{2}$  B 2  C $\frac{1}{2}$  D -2

2. Find the slope of the line through (0, 5) and (5, 5).
   F 0  G 1  H 2  J undefined

3. It is expected that 563 quadrillion thermal units of Btu (British thermal units) of energy will be consumed worldwide in 2015. In 2003, worldwide consumption was 421 quadrillion Btu. What is the expected rate of change in consumption from 2003 to 2015?
   A about 0.069 quadrillion Btu per year
   B about 11.83 quadrillion Btu per year
   C about 142 quadrillion Btu per year
   D about 0.085 years per quadrillion Btu

4. The total price of a bag of peaches varies directly with the cost per pound. If 3 pounds of peaches cost $3.60, how much would 5.5 pounds cost?
   F $1.20  G $6.60  H $6.00  J $1.96

5. A carpenter can build 8 cabinets in 3 hours. At this rate, how long will it take for the carpenter to build 26 cabinets?
   A 8 hours  B 9 hours 45 minutes  C 21 hours  D 69 hours 20 minutes

6. Solve $3x - 12 = 3x + 4$.

7. Find the value of $r$ so the line that passes through (-5, 2) and (3, $r$) has a slope of $-\frac{1}{2}$.

8. Determine whether $y = 2x - 3$ is a linear equation. If so, write the equation in standard form.

9. Graph $3y - x = 6$ by using the x- and y-intercepts.
Choose from the terms above to complete each sentence.

1. In an arithmetic sequence the constant rate of increase or decrease between successive terms is called the ___________.

2. The zero of a function is its _________________.

3. The ________________ of an equation is any value that makes the equation true.

4. An equation in the form $Ax + By = C$ is called the ________________ of a linear equation.

5. In a(n) ________________, the difference between terms is constant.

6. For a(n) ________________ $Ax + By = C$, the numbers $A$, $B$, and $C$ cannot all be zero.

7. A specific number in a sequence is called a(n) _________________.

8. The act of using a pattern to find a general rule is known as _________________.

Define each term in your own words.

9. $y$-intercept

10. rate of change
1. Where does the graph of \( y = -3x - 18 \) intersect the \( x \)-axis?
   - A \( (0, 6) \)
   - B \( (0, -6) \)
   - C \( (6, 0) \)
   - D \( (-6, 0) \)  
   1. _____

2. Tickets to see a movie cost $5 for children and $8 for adults. The equation \( 5x + 8y = 80 \) represents the number of children \( (x) \) and adults \( (y) \) who can see the movie with $80. If no adults see the movie, how many children can see the movie with $80?
   - F 6
   - G 10
   - H 13
   - J 16  
   2. _____

For Questions 3–5, find the slope of each line described.

3. the line through \( (3, 7) \) and \( (-1, 4) \)
   - A \( \frac{4}{3} \)
   - B \( \frac{3}{4} \)
   - C \( \frac{11}{2} \)
   - D \( \frac{2}{11} \)  
   3. _____

4. the line through \( (-3, 2) \) and \( (6, 2) \)
   - F \( \frac{4}{9} \)
   - G \( \frac{4}{3} \)
   - H 0
   - J undefined  
   4. _____

5. a vertical line
   - A 1
   - B 0
   - C -1
   - D undefined  
   5. _____

6. Which graph has a slope of \(-3\)?
   
   - F
   - G
   - H
   - J
   6. _____

7. COMMUNICATION In 1996, there were 171 area codes in the United States. In 2007, there were 215. Find the rate of change from 1996 to 2007.
   - A 44
   - B 4
   - C \( \frac{1}{4} \)
   - D -4  
   7. _____

For Questions 8 and 9, use the arithmetic sequence \( 12, 15, 18, 21, \ldots \).

8. Which is an equation for the \( n \)th term of the sequence?
   - F \( a_n = 3n + 9 \)
   - H \( a_n = 12n + 3 \)
   - G \( a_n = 9n + 3 \)
   - J \( a_n = n + 3 \)  
   8. _____

9. What is the 12th term in the sequence?
   - A 38
   - B 42
   - C 45
   - D 48  
   9. _____
10. Suppose \( y \) varies directly as \( x \), and \( y = 26 \) when \( x = 8 \). Find \( x \) when \( y = 65 \).

\[
F \ 3.25 \quad G \ 20 \quad H \ 47 \quad J \ 211.25
\]

11. Which line has a \( y \)-intercept of \(-2\)?

\[
A \ \ell \quad C \ t \\
B \ p \quad D \ \text{both } \ell \text{ and } t
\]

12. Which line is the graph of \( y = 2x + 4 \)?

\[
F \ \ell \quad H \ \text{the } x\text{-axis} \\
G \ p \quad J \ t
\]

13. Which arithmetic sequence has a proportional related function?

\[
A \ -4, -1, 2, \ldots \quad B \ 0, -2, -4, \ldots \quad C \ 1, 2, 3, \ldots \quad D \ -\frac{1}{2}, 0, \frac{1}{2}, \ldots
\]

14. Write \( y + 1 = -2x - 3 \) in standard form.

\[
F \ 2x + y = -4 \quad G \ y = -2x - 4 \quad H \ -2x - y = 4 \quad J \ x + \frac{1}{2}y = -2
\]

15. Find the root of \( 5x - 20 = 0 \).

\[
A \ -20 \quad B \ 0 \quad C \ 4 \quad D \ 5
\]

16. Determine which sequence is an arithmetic sequence.

\[
F \ 3, 6, 12, 24, \ldots \quad H \ -7, -3, 1, 5, \ldots \\
G \ \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots \quad J \ -10, 5, -\frac{5}{2}, \frac{5}{4}, \ldots
\]

17. Find the next three terms of the arithmetic sequence \( 5, 9, 13, 17, \ldots \)

\[
A \ 21, 23, 25 \quad B \ 21, 25, 29 \quad C \ 41, 45, 49 \quad D \ 21, 41, 61
\]

18. Find the function that represents the relationship.

\[
F \ y = 8x \quad H \ y = 14x + 8 \\
G \ y = 8x + 14 \quad J \ y = 14x + 14
\]

19. Which equation is a linear equation?

\[
A \ 4m^2 = 6 \quad C \ \frac{2}{3}xy - \frac{3}{4}y = 0 \\
B \ 3a + 5b = 3 \quad D \ x^2 + y^2 = 0
\]

20. Write an equation in function notation for the relation at the right.

\[
F \ f(x) = 2x \quad H \ f(x) = 1 - x \\
G \ f(x) = x + 1 \quad J \ f(x) = -x
\]

**Bonus** Graph \( y = x - 3 \) by using the \( x \)- and \( y \)-intercepts.

\[
B: \ _________________________
\]
Write the letter for the correct answer in the blank at the right of each question.

1. LANDSCAPING The equation $3x + 7y = 105$ represents the number of bags of sand $x$ and bags of mulch $y$ that can be bought with $105. If no bags of sand are bought, how many bags of mulch can be bought with $105?
   
   A 35  
   B 17  
   C 15  
   D 10

2. If $(a, -5)$ is a solution to the equation $3a = -2b - 7$, what is $a$?
   
   F $-1$  
   G 0  
   H 1  
   J 4

3. What is the slope of the line through $(1, 9)$ and $(-3, 16)$?
   
   A $-\frac{7}{4}$  
   B $-\frac{4}{7}$  
   C $-\frac{25}{2}$  
   D $-\frac{2}{25}$

4. Which equation is not a linear equation?
   
   F $-4v + 2w = 7$  
   G $\frac{x}{4} = y$  
   H $x = -5$  
   J $\frac{2}{x} + \frac{3}{y} = 6$

5. What is the slope of the line through $(-4, 3)$ and $(5, 3)$?
   
   A 0  
   B undefined  
   C 9  
   D 1

6. In 2005, there were 12,000 students at Beacon High. In 2010, there were 12,250. What is the rate of change in the number of students?
   
   F 250/yr  
   G 50/yr  
   H 42/yr  
   J 200/yr

7. Which is the graph of $y = \frac{2}{3}x$?
   
   A  
   B  
   C  
   D

8. If $y$ varies directly as $x$ and $y = 3$ when $x = 10$, find $x$ when $y = 8$.
   
   F $\frac{80}{3}$  
   G $\frac{12}{5}$  
   H $\frac{15}{4}$  
   J none of these

9. DRIVING A driver’s distance varies directly as the amount of time traveled. After 6 hours, a driver had traveled 390 miles. How far had the driver traveled after 4 hours?
   
   A 130 miles  
   B 220 miles  
   C 260 miles  
   D 650 miles

For Questions 10 and 11, use the following information.

The number of seats in each row of a theater form an arithmetic sequence, as shown in the table.

<table>
<thead>
<tr>
<th>Row</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seats</td>
<td>8</td>
<td>14</td>
<td>20</td>
<td>26</td>
</tr>
</tbody>
</table>

10. How many seats are in the 12th row?
   
   F 68  
   G 74  
   H 96  
   J 114

11. Which formula can be used to find the number of seats in any given row?
   
   A $a_n = 6n + 2$  
   B $a_n = 2n + 6$  
   C $a_n = n + 6$  
   D $a_n = 5n + 3$
12. Find the function that represents the relationship.

\[ F \ y = x - 3 \quad H \ y = 6x - 3 \]

\[ G \ y = 3x - 3 \quad J \ y = 6x \]

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & 0 & 1 & 2 & 3 & 4 \\
\hline
y & -3 & 3 & 9 & 15 & 21 \\
\hline
\end{array}
\]

12. _____

For Questions 13 and 14, use the relation shown in the table.

13. Which equation describes this relationship?

\[ A \ f(x) = 3x \quad C \ f(x) = x + 2 \]

\[ B \ f(x) = 4x - 1 \quad D \ f(x) = 2x + 1 \]

13. _____

14. What is the value of \( y \) when \( x = 43 \)?

\[ F \ 87 \quad H \ 49 \]

\[ G \ 85 \quad J \ 45 \]

14. _____

15. Which line shown at the right is the graph of \( x - 2y = 4 \)?

\[ A \ell \quad C \ p \]

\[ B \ m \quad D \ t \]

15. _____

16. Which equation has a graph that is a vertical line?

\[ F \ 2x = y \quad H \ 3x - 2 = 0 \]

\[ G \ y + 5 = 3 \quad J \ x - y = 0 \]

16. _____

17. What is the standard form of \( y - 7 = -\frac{2}{3}(x + 1) \)?

\[ A \ -2x + 3y = 23 \quad B \ -3x + 2y = 17 \]

\[ C \ 2x + 3y = 19 \quad D \ 3x + 2y = 11 \]

17. _____

18. Determine which sequence is not an arithmetic sequence.

\[ F \ -7, 0, 7, 14, \ldots \quad H \ 10, 6, 2, -2, \ldots \]

\[ G \ 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \quad J \ 2, 4, 8, 16, \ldots \]

18. _____

19. Which equation describes the \( n \)th term of the arithmetic sequence \( 7, 10, 13, 16, \ldots \)?

\[ A \ a_n = 3n + 4 \quad B \ a_n = 7 + 3n \]

\[ C \ a_n = -4n + 3 \quad D \ a_n = 3n - 4 \]

19. _____

20. Write an equation in function notation for the relation shown at the right.

\[ F \ f(x) = -2x \quad H \ f(x) = x - 2 \]

\[ G \ f(x) = 2x + 2 \quad J \ f(x) = -x + 2 \]

20. _____

Bonus The table shows the schedule for a theme park roller coaster.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

At what time will the roller coaster make its 15th run? B: _______________
Write the letter for the correct answer in the blank at the right of each question.

1. **PHOTOCOPIES** Black and white copies cost $0.05 each, and color copies cost $0.49 each. The equation $0.05x + 0.49y = 5$ represents the number of black and white copies $x$ and color copies $y$ that can be made with $5. If no color copies are made, how many black and white copies can be made with $5?
   A 200   B 100   C 25   D 10
   
2. If $(a, -7)$ is a solution to the equation $8a = -3b - 5$, what is $a$?
   F $-17$   G 2   H 8   J 17
   
3. What is the slope of the line through $(2, -8)$ and $(4, 1)$?
   A $\frac{-2}{9}$   B $\frac{-6}{7}$   C $\frac{-7}{6}$   D $\frac{9}{2}$
   
4. Which equation is not a linear equation?
   F $2x + 5y = 3$   G $y = -10$   H $5 = 3xy$   J $y = \frac{x}{7} + 4$
   
5. What is the slope of the line through $(-4, -6)$ and $(9, -6)$?
   A $\frac{-12}{5}$   B $\frac{-5}{12}$   C 0   D undefined
   
6. In 2005, MusicMart sold 14,550 CDs. In 2010, they sold 12,000 CDs. What is the rate of change in the number of CDs sold?
   F $-2550$ per yr   G $-510$ per yr   H $-425$ per yr   J $-2400$ per yr
   
7. Which is the graph of $y = -\frac{1}{2}x$?
   A   B   C   D
   
8. If $y$ varies directly as $x$ and $y = 5$ when $x = 8$, find $y$ when $x = 9$.
   F $\frac{72}{5}$   G $\frac{45}{8}$   H $\frac{40}{9}$   J 6
   
9. **SPRINGS** The amount a spring stretches varies directly as the weight of the object attached to it. If an 8-ounce weight stretches a spring 10 centimeters, how much weight will stretch it 15 centimeters?
   A 16 oz   B 6 oz   C 10 oz   D 12 oz
   
For Questions 10 and 11, use the following information.

The number of students seated in each row of an auditorium form an arithmetic sequence, as shown in the table.

<table>
<thead>
<tr>
<th>Row</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
</tr>
</tbody>
</table>

10. How many students are seated in the 15th row?
   F 46   G 58   H 59   J 195
   
11. Which formula can be used to find the number of students seated in any given row?
   A $a_n = 4n$   B $a_n = 4n - 1$   C $a_n = n + 3$   D $a_n = 3n + 1$
12. Find the function that represents the relationship.

\[ F \quad y = x - 5 \quad H \quad y = 7x \]
\[ G \quad y = 7x - 5 \quad J \quad y = 7x + 7 \]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>y</td>
<td></td>
<td>-5</td>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

12. _______

For Questions 13 and 14, use the relation shown in the table.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

13. _______

14. _______

15. Which line shown at the right is the graph of \( x + 2y = 6 \)?

A \( r \)  
B \( n \)  
C \( t \)  
D \( v \)

15. _______

16. Which equation has a graph that is a horizontal line?

\[ F \quad x - 7 = 0 \quad H \quad x = y \]
\[ G \quad 2y + 3 = 4 \quad J \quad x + y = 0 \]

16. _______

17. What is the standard form of \( y + 2 = \frac{1}{2}(x - 4) \)?

\[ A \quad x + 2y = 0 \quad B \quad x - 2y = 8 \quad C \quad 2x - y = 10 \quad D \quad 4x - 2y = 0 \]

17. _______

18. Determine which sequence is an arithmetic sequence.

\[ F \quad -16, -12, -8, -4, \ldots \]
\[ G \quad 4, 8, 16, 32, \ldots \]
\[ H \quad 1, 4, 2, 5, 3, \ldots \]
\[ J \quad 1, 1, 2, 3, 5, \ldots \]

18. _______

19. Which equation describes the \( n \)th term of the arithmetic sequence \(-12, -14, -16, -18, \ldots \)?

\[ A \quad a_n = -2n - 10 \quad C \quad a_n = 10n - 2 \]
\[ B \quad a_n = -12 - 2n \quad D \quad a_n = -2n + 10 \]

19. _______

20. Write an equation in function notation for the relation.

\[ F \quad f(x) = -2x \quad H \quad f(x) = -x + 2 \]
\[ G \quad f(x) = x - 2 \quad J \quad f(x) = 2x + 2 \]

20. _______

**Bonus** The table shows the arrival times for a commuter train.

<table>
<thead>
<tr>
<th>Train Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>5:15 A.M.</td>
<td>5:29 A.M.</td>
<td>5:43 A.M.</td>
<td>5:57 A.M.</td>
</tr>
</tbody>
</table>

At what time will the 12th train arrive?

B: _______
1. Tickets for a spaghetti dinner cost $4 for children and $6 for adults. The equation $4x + 6y = 36$ represents the number of children $x$ and adults $y$ who can eat at the dinner for $36. If no children are eating at the dinner, how many adults can eat for $36? 

2. If $(a, 9)$ is a solution to the equation $-4a = b - 21$, what is $a$?

3. Find the $x$-intercept of $x - 2y = 9$.

4. Solve $-3 = -2x + 1$ by graphing.

5. Find the root of $9x - 36 = 0$.

For Questions 6–8, find the slope of the line passing through each pair of points. If the slope is undefined, write undefined.

6. $(2, 5)$ and $(3, 6)$

7. $(6, -4)$ and $(-3, 7)$

8. $(-1, 3)$ and $(6, 3)$

9. In 1972, federal vehicle emission standards allowed 3.4 hydrocarbons released per mile driven. By 2007, the standards allowed only 0.8 hydrocarbons per mile driven. What was the rate of change from 1972 to 2007?

10. If a shark can swim 27 miles in 9 hours, how many miles will it swim in 12 hours?

For Questions 11 and 12, determine whether each equation is a linear equation. If so, write the equation in standard form.

11. $xy = 6$

12. $2x + 3y + 7 = 3$

13. Graph the equation $x - 4y = 2$. 
14. Graph \( y = -\frac{1}{2}x \).

15. Solve \( \frac{1}{2}x + \frac{7}{2} = 5 \) by graphing.

16. Determine whether the sequence \(-10, -7, -4, -1, \ldots\) is an arithmetic sequence. Write yes or no. If so, state the common difference.

17. Find the next three terms of the arithmetic sequence 8, 15, 22, 29, \ldots

18. Write an equation for the \( n \)th term of the sequence 12, 5, \(-2\), \(-9\), \ldots

For Questions 19 and 20, use the table below that shows the amount of gasoline a car consumes for different distances driven.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline (gal)</td>
<td>0.04</td>
<td>0.08</td>
<td>0.12</td>
<td>0.16</td>
<td>0.20</td>
</tr>
</tbody>
</table>

19. Write an equation in function notation for the relationship between distance and gasoline used.

20. How many gallons will the car consume after driving for 150 miles?

**Bonus** Graph \( x = 3 \), \( y = -1 \), and \( 2x - 2y = 0 \) on a coordinate plane. Give the vertices of the figure formed by the three lines.
1. Tickets for a school play cost $7 for students and $10 for adults. The equation $7x + 10y = 80$ represents the number of students $x$ and adults $y$ who can attend the play for $80. If no students attend, how many adults can see the play for $80? 

2. If $(a, -8)$ is a solution to the equation $-a = 4b - 7$, what is $a$?

3. Find the $x$-intercept of $3x - y = 7$.

4. Solve $-3 = -2x - 1$ by graphing.

5. Find the root of $6x - 48 = 0$.

For Questions 6–8, find the slope of the line passing through each pair of points. If the slope is undefined, write “undefined.”

6. $(4, 1)$ and $(-4, 1)$

7. $(-6, 7)$ and $(-6, -2)$

8. $(-2, 1)$ and $(3, -2)$

9. In 1995, 57.5% of students at Gardiner University graduated in 4 or fewer years of study. In 2009, that number had fallen to 52.8%. What was the rate of change for percent of students graduating within 4 years from 1995 to 2009?

10. If a snail travels 200 inches in 2 hours, how long will it take the snail to travel 50 inches?

For Questions 11 and 12, determine whether each equation is a linear equation. Write yes or no. If so, write the equation in standard form.

11. $\frac{1}{x} + \frac{1}{y} = \frac{2}{3}$

12. $4x = 2y$

13. Graph the equation $x + 4y - 3 = 0$. 

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NAME ______________________ DATE ________ PERIOD ____________

Chapter 3 Test, Form 2D
14. Graph \( y = \frac{2}{3}x \).

15. Solve \( 3 = \frac{1}{2}x + \frac{5}{2} \) by graphing.

16. Determine whether the sequence 7, 11, 15, 19, \ldots is an arithmetic sequence. Write yes or no. If so, state the common difference.

17. Find the next three terms of the arithmetic sequence 
\(-25, -22, -19, -16, \ldots\)

18. Write an equation for the \( n \)th term of the sequence 
3, 12, 21, 30, \ldots

For Questions 19 and 20, use the table below that shows the amount of gasoline a car consumes for different distances driven.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline (gal)</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
</tr>
</tbody>
</table>

19. Write an equation in function notation for the relationship between distance and gasoline used.

20. How many gallons will the car consume after driving for 140 miles?

**Bonus** Graph the points (4, \(-2\)), (4, 3), (\(-2\), 3). Find a fourth point that completes a rectangle with the given three points, then graph the rectangle on the coordinate plane.

B: ___________________
1. Determine whether $3x - 4y + 7 = 3y + 1$ is a linear equation. Write yes or no. If so, write the equation in standard form.

2. The equation $300x + 50y = 600$ represents the number of premium tickets $x$ and the number of discount tickets $y$ for a horse race that can be bought with $600. If no premium tickets are purchased, how many discount tickets can be purchased with $600?

3. If $(a, -7)$ is a solution to the equation $5a - 7b = 28$, what is $a$?

4. Find the $x$-intercept of $4x - 5y = 15$.

5. Solve $\frac{-5}{3}x + 7 = \frac{9}{2}$ by graphing.

6. Find the root of $14x + 5 = 61$.

7. Graph $\frac{2x}{3} = \frac{y}{2} - 1$.

For Questions 8 and 9, find the slope of the line passing through each pair of points. If the slope is undefined, write undefined.

8. $(-8, 7)$ and $(5, -2)$

9. $(5, 9)$ and $(5, -3)$

10. Five years ago there were approximately 35,000 people living in Lancaster. Now the population is 38,452. Find the rate of change in the population.

11. If an ostrich can run 15 kilometers in 15 minutes, how many kilometers can it run in an hour?
12. Graph \( y = -\frac{3}{4}x \).

13. Find the value of \( r \) so that the line through \((-4, 3)\) and \((r, -3)\) has a slope of \( \frac{2}{3} \).

14. Find the value of \( r \) so that the line through \((r, 5)\) and \((6, r)\) has a slope of \( \frac{5}{8} \).

15. Determine whether the sequence \( 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \) is an arithmetic sequence. If it is, state the common difference.

16. Find the value of \( y \) that makes \( 9, 4, y, -6, \ldots \) an arithmetic sequence.

17. Write an equation for the \( n \)th term of the arithmetic sequence \(-15, -11, -7, -3, \ldots\). Then graph the first five terms of the sequence.

For Questions 18 and 19, use the table below that shows the value of a vending machine over the first five years of use.

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (dollars)</td>
<td>2000</td>
<td>1810</td>
<td>1620</td>
<td>1430</td>
<td>1240</td>
<td>1050</td>
</tr>
</tbody>
</table>

18. Write an equation in function notation for the relationship between years of use \( t \) and value \( v(t) \).

19. When will the value of the vending machine reach 0?

20. Brian collects baseball cards. His father gave him 20 cards to start his collection on his tenth birthday. Each year Brian adds about 15 cards to his collection. About how many years will it take to fill his collection binder if it holds 200 cards?

**Bonus** A gym membership costs $29.99 per month. Write a direct variation equation to find the total cost \( c \) to own a gym membership for \( y \) years.

B:
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Draw a line on a coordinate plane so that you can determine at least two points on the graph.
   a. Describe how you would determine the slope of the graph and justify the slope you found.
   b. Explain how you could use the slope to write various forms of the equation of that line. Then write three forms of the equation.

2. Suppose your total cost \( C \) at a home improvement store varies directly as \( f \), the number of feet of lumber you buy.
   a. Write a direct variation equation that might model the situation.
   b. What is the meaning of the constant of variation in the equation?
   c. Suppose you also wanted to buy $10 worth of nails. Could the situation still be modeled by a direct variation equation? Why or why not?

3. a. Write a sequence that is an arithmetic sequence. State the common difference, and find \( a_6 \).
   b. Write a sequence that is not an arithmetic sequence. Determine whether the sequence has a pattern, and if so describe the pattern.
   c. Determine if the sequence 1, 1, 1, 1, \ldots is an arithmetic sequence. Explain your reasoning.

4. Draw a line on the coordinate grid below. Identify the \( x \)- and \( y \)-intercepts. Then write an equation in function notation for the line.
1. The student council is selling candy bars to earn money towards their budget for the school dance. Identify the independent and dependent variables. (Lesson 1-6)

- **A** I: student council; C I: candy bars sold; D: money earned
- **B** I: budget; D I: candy bars sold; D: school dance

2. Dion owns a delivery service. He charges his customers $15.00 for each delivery. His expenses include $7000 for the motorcycle he drives and $0.42 for gasoline per trip. Which equation could Dion use to calculate his profit $p$ for $d$ deliveries? (Lesson 1-7)

- **F** $p = 15 - 0.42d$
- **H** $p = 14.58d - 7000$
- **G** $p = 7000 + 15d$
- **J** $p = 0.42d + 7000$

3. Evaluate $60 \div 5 \cdot 6 - 3^2$. (Lesson 1-2)

- **A** -7
- **B** -4
- **C** 63
- **D** 4761

4. Jim’s new car has 150 miles on the odometer. He takes a trip and drives an average of $m$ miles each day for three weeks. Which expression represents the mileage on Jim’s car after his trip? (Lesson 2-4)

- **F** $150 + 3m$
- **G** $150 + 3m$
- **H** $150m + 21$
- **J** $150 + 21m$

5. Translate the sentence into an equation. (Lesson 2-1)

   *Five times the sum of $m$ and $t$ is as much as four times $r*.

- **A** $5m + t = 4$
- **B** $5m + t = r$
- **C** $5(m + t) = 4r$
- **D** $m + t = 5(4r)$

6. Solve $8(x - 5) = 12(4x - 1) + 12$. (Lesson 2-4)

- **F** $\frac{7}{10}$
- **G** $\frac{5}{7}$
- **H** -2
- **J** -1

7. Paul and Charlene are 420 miles apart. They start toward each other with Paul driving 16 miles per hour faster than Charlene. They meet in 5 hours. Find Charlene’s speed. (Lesson 2-9)

- **A** 34 mph
- **B** 50 mph
- **C** 40.4 mph
- **D** 68 mph

8. Determine which equation is a linear equation. (Lesson 3-1)

- **F** $x^2 + y = 4$
- **G** $x + y = 4$
- **H** $xy = 4$
- **J** $\frac{1}{x} + y = 4$

9. If $f(x) = 7 - 2x$, find $f(3) + 6$. (Lesson 1-7)

- **A** 11
- **B** 7
- **C** 14
- **D** -11

10. Chapa is beginning an exercise program that calls for 30 push-ups each day for the first week. Each week thereafter, she has to increase her push-ups by 2. Which week of her program will be the first one in which she will do 50 push-ups a day? (Lesson 3-5)

- **F** 9th week
- **G** 10th week
- **H** 11th week
- **J** 12th week
11. Which property of equality is illustrated below? (Lesson 1-3)
   If $7 + 9 = 11 + 5$ and $11 + 5 = 16$, then $7 + 9 = 16$.
   A Transitive  
   B Reflexive  
   C Substitution  
   D Symmetric  
   11.  

12. Which expression represents the missing second step of simplifying the algebraic expression? (Lesson 1-4)
   Step 1 $4(x - 3y) + 6 + 5(x + 1)$
   Step 3 $9x - 12y + 11$
   F $4x - 3y + 6 + 5x + 1$  
   H $4x - 12y + 6 + 5x + 5$  
   G $12(x - y) + 6 + x + 5$  
   J $x - 3y + 15 + x + 1$  
   12.  

13. Solve $48 = -8r$. (Lesson 2-2)
   A $r = 8$  
   B $r = 6$  
   C $r = -6$  
   D $r = -40$  
   13.  

14. Solve $4 - (-h) = 12$. (Lesson 2-2)
   F $h = 16$  
   G $h = 8$  
   H $h = -8$  
   J $h = -16$  
   14.  

For Questions 15 and 16, use the arithmetic sequence $2, 5, 8, 11, \ldots$

15. Which is an equation for the $n$th term of the sequence? (Lesson 3-5)
   A $a_n = 2n + 1$  
   B $a_n = 4n - 2$  
   C $a_n = n + 3$  
   D $a_n = 3n - 1$  
   15.  

16. What is the 20th term in the sequence? (Lesson 3-5)
   F 59  
   G 60  
   H 78  
   J 80  
   16.  

Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

17. The ratio of $a$ to $b$ is $\frac{4}{7}$. If $a$ is 16, find the value of $b$. (Lesson 2-6)

18. The equation $C = \frac{F - 32}{1.8}$ relates the temperature in degrees Fahrenheit $F$ to degrees Celsius $C$. If the temperature is $25^\circ C$, what is the temperature in degrees Fahrenheit? (Lesson 3-1)
19. Find the solution of \( y + \frac{2}{3} = \frac{22}{15} \) if the replacement set is \( \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, \frac{11}{5} \). (Lesson 1-5)

20. Simplify \( 5m + 8p + 3m + p \). (Lesson 1-3)

21. Determine the slope of the line passing through \((1, 4)\) and \((3, -1)\). (Lesson 3-3)

22. Translate the following equation into a verbal sentence.
\[
\frac{x}{4} - y = -2\left(\frac{x}{y}\right)
\] (Lesson 2-1)

23. Find the discounted price. clock: $15.00
discount: 15% (Lesson 2-7)

24. Solve \(-7x + 23 = 37\). (Lesson 2-3)

25. Use cross products to determine whether the ratios \(\frac{4}{7}\) and \(\frac{11}{15}\) form a proportion. Write yes or no. (Lesson 2-6)

26. Express the relation as a set of ordered pairs. Then determine the domain and range. (Lesson 1-6)

27. Determine whether the relation is a function. (Lesson 1-7)

28. Find the x-intercept of the graph of \(4x = 5 + y\). (Lesson 3-1)

29. Graph \(2x - 3y = 6\). (Lesson 3-1)

30. The table below shows the average amount of gas Therese’s truck uses depending on how many miles she drives.

<table>
<thead>
<tr>
<th>Gallons of Gasoline</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven</td>
<td>18</td>
<td>36</td>
<td>54</td>
<td>72</td>
<td>90</td>
</tr>
</tbody>
</table>

a. Does the table of values represent a function? Explain. (Lesson 3-6)

b. Is this a proportional relationship? Explain. (Lesson 3-6)
**3-1 Study Guide and Intervention**

**Graphing Linear Equations**

Identify Linear Equations and Intercepts. A linear equation is an equation that can be written in the form $Ax + By = C$. This is called the standard form of a linear equation.

| Standard Form of a Linear Equation | $Ax + By = C$, where $A \geq 0$, $A$ and $B$ are not both zero, and $A$, $B$, and $C$ are integers with a greatest common factor of 1 |

**Example 1**

Determine whether $y = 6 - 3x$ is a linear equation. Write the equation in standard form.

First rewrite the equation so both variables are on the same side of the equation.

$y = 6 - 3x$  
Original equation

$y + 3x = 6 - 3x + 3x$  
Add $3x$ to each side.

$3x + y = 6$  
Simplify

The equation is now in standard form, with $A = 3$, $B = 1$, and $C = 6$. This is a linear equation.

**Example 2**

Determine whether $3xy + y = 4 + 2x$ is a linear equation. Write the equation in standard form.

Since the term $3xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

**Exercises**

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. $2x = 4y$  
   yes; $2x - 4y = 0$

2. $6 + y = 8$  
   yes; $y = 2$

3. $4x - 2y = -1$  
   yes; $4x - 2y = -1$

4. $3xy + 8 = 4y$  
   no

5. $3x - 4 = 12$  
   yes; $3x = 16$

6. $y = x^2 + 7$  
   no

7. $y - 4x = 9$  
   yes; $4x - y = -9$

8. $x + 8 = 0$  
   yes; $x = -8$

9. $-2x + 3 = 4y$  
   yes; $2x + 4y = 3$

10. $2 + \frac{1}{2}x = y$  
    yes; $x - 2y = -4$

11. $\frac{1}{2}y = 12 - 4x$  
    yes; $16x + y = 48$

12. $3xy = y - 8$

13. $6x + y = 3$  
    no

14. $xy - 2 = 8$  
    no

15. $6x - 2y = 8 + y$  
    yes; $6x - 3y = 8$

16. $\frac{1}{2}x - 12y = 1$  
    yes; $x - 48y = 4$

17. $3 + x + x^2 = 0$  
    no

18. $x^2 = 2xy$  
    no
3-1 Study Guide and Intervention (continued)

Graphing Linear Equations

Graph Linear Equations

The graph of a linear equation represents all the solutions of the equation. An x-coordinate of the point at which a graph of an equation crosses the x-axis is called an x-intercept. A y-coordinate of the point at which a graph crosses the y-axis is called a y-intercept.

Example 1
Graph $3x + 2y = 6$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$ and solve for $x$. The x-intercept is 2. The graph intersects the x-axis at (2, 0).

To find the y-intercept, let $x = 0$ and solve for $y$.

The y-intercept is 3. The graph intersects the y-axis at (0, 3).

Plot the points (2, 0) and (0, 3) and draw the line through them.

Example 2
Graph $y - 2x = 1$ by making a table.

Solve the equation for $y$.

$y - 2x + 2x = 1 + 2x$

$y = 2x + 1$

Select five values for the domain and make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2x + 1$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$2(-2) + 1 = -3$</td>
<td>$2(-2) - 3$</td>
<td>$(-2, -3)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2(1) + 1 = 3$</td>
<td>$2(1) + 1$</td>
<td>$(1, 3)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2(2) + 1 = 5$</td>
<td>$2(2) + 1$</td>
<td>$(2, 5)$</td>
</tr>
</tbody>
</table>

Graph each equation by making a table.

1. $2x + y = -2$
2. $3x - 6y = -3$
3. $-2x + y = -2$
4. $y = 2x$
5. $x - y = -1$
6. $x + 2y = 4$

Graph each equation by using the x- and y-intercepts.

7. $x - y = 3$
8. $10x = -5y$
9. $4x = 2y + 6$
3-1 Practice

Graphing Linear Equations

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form and determine the x- and y-intercepts.

1. 4x + 2y = 9  
   yes; 4x = y; 2; x = \frac{1}{2}, y = -2

2. 8x - 3y = 6 - 4x  
   yes; 4x - y = 2; x = \frac{1}{2}, y = -2

3. 7x + y + 3 = y  
   yes; 7x = -3; x = -\frac{3}{7}, y = none

4. 5 - 2y = 3x  
   yes; 3x + 2y = 5; x = \frac{1}{2}, y = \frac{1}{2}

Graph each equation.

7. \frac{1}{2}x - y = 2

8. 5x - 2y = 7

9. 1.5x + 3y = 9

10. COMMUNICATIONS A telephone company charges $4.95 per month for long distance calls plus $0.05 per minute. The monthly cost c of long distance calls can be described by the equation c = 0.05m + 4.95, where m is the number of minutes.
   a. Find the y-intercept of the graph of the equation.
      (0, 4.95)
   b. Graph the equation.
   c. If you talk 140 minutes, what is the monthly cost?
      $11.95

11. MARINE BIOLOGY Killer whales usually swim at a rate of 3.2–9.7 kilometers per hour, though they can travel up to 48.4 kilometers per hour. Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance d of the whale has traveled in t hours can be predicted by the equation d = 4.5t.

   a. Graph the equation.
   b. Use the graph to predict the time it takes the killer whale to travel 30 kilometers. between 6 h and 7 h

3-1 Word Problem Practice

Graphing Linear Equations

1. FOOTBALL One football season, the Carolina Panthers won 4 more games than they lost. This can be represented by y = x + 4, where x is the number of games lost and y is the number of games won. Write this linear equation in standard form. x - y = -4

2. TOWING Pick-M-Up Towing Company charges $40 to hook a car and $1.70 for each mile that it is towed. The equation y = 1.7x + 40 represents the total cost y for x miles towed. Determine the y-intercept. Describe what the value means in this context.
   The y-intercept is 40, which is the fee to hook the car.

3. SHIPPING The OOCL Shenzhen, one of the world’s largest container ships, carries 8063 TEUs (1280 cubic feet containers). Workers can unload a ship at a rate of 3 TEUs every minute. Using this rate, write and graph an equation to determine how many hours it will take the workers to unload half of the containers from the Shenzhen.
   \( y = 8063 - 60x; \) about 674 hours, or 67 hours and 21.5 minutes

4. BUSINESS The equation \( y = 1000x - 5000 \) represents the monthly profits of a start-up dry cleaning company. Time in months is x and profit in dollars is y. The first date of operation is when time is zero. However, preparation for opening the business began 3 months earlier with the purchase of equipment and supplies. Graph the linear function for x-values from -3 to 8.

5. BONE GROWTH The height of a woman can be predicted by the equation \( y = 81.2 + 3.34x \), where \( y \) is her height in centimeters and \( x \) is the length of her radius bone in centimeters.
   a. Is this a linear function? Explain. yes; the equation can be written in standard form for where \( A = 1 \), \( B = -3.34 \), and \( C = -81.2 \).
   b. What are the r- and h-intercepts of the equation? Do they make sense in the situation? Explain. 
      \( y \)-intercept = 81.2; \( x \)-intercept = -24.3; no, we would expect a woman 81.2 cm tall to have arms, and a negative radius length has no real meaning.
   c. Use the function to find the approximate height of a woman whose radius bone is 20 centimeters long. 165 cm
Translating Linear Graphs

Linear graphs can be translated on the coordinate plane. This means that the graph moves up, down, right, or left without changing its direction. Translating the graph up or down affects the $y$-coordinate for a given $x$ value. Translating the graph right or left affects the $x$-coordinate for a given $y$-value.

**Example**

Translate the graph of $y = 2x + 2$, 3 units up.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Exercises

Graph the function and the translation on the same coordinate plane.

1. $y = x + 4$, 3 units down
2. $y = 2x - 2$, 2 units left
3. $y = -2x + 1$, 1 unit right
4. $y = -x - 3$, 2 units up

**Exercises**

1. A photo printer offers a subscription for digital photo finishing. The subscription costs $4.99 per month. Each standard size photo a subscriber prints costs $0.19. Use a spreadsheet to graph the equation $y = 4.99 + 0.19x$, where $x$ is the number of photos printed and $y$ is the total monthly cost. See students’ work.

2. A long distance service plan includes a $8.95 per month fee plus $0.05 per minute of calls. Use a spreadsheet to graph the equation $y = 8.95 + 0.05x$, where $x$ is the number of minutes of calls and $y$ is the total monthly cost. See students’ work.

3-1 Spreadsheet Activity

Linear Equations

In addition to organizing data, a spreadsheet can be used to represent data graphically.

**Example**

An internet retailer charges $1.99 per order plus $0.99 per item to ship books and CDs. Graph the equation $y = 1.99 + 0.99x$, where $x$ is the number of items ordered and $y$ is the shipping cost.

Step 1 Use column A for the numbers of items and column B for the shipping costs.

Step 2 Create a graph from the data. Select the data in columns A and B and select Chart from the Insert menu. Select an XY (Scatter) chart to show the data points connected with line segments.
3-2 Study Guide and Intervention

Solving Linear Equations by Graphing

Solve by Graphing You can solve an equation by graphing the related function. The solution of the equation is the x-intercept of the function.

Example

Solve the equation $2x - 2 = -4$ by graphing.

First set the equation equal to 0. Then replace 0 with $f(x)$. Make a table of ordered pairs. Graph the function and locate the x-intercept.

\[
\begin{align*}
2x - 2 &= -4 & \text{Original equation} \\
2x - 2 + 4 &= -4 + 4 & \text{Add 4 to each side.} \\
2x + 2 &= 0 & \text{Simplify.} \\
f(x) &= 2x + 2 & \text{Replace 0 with } f(x).
\end{align*}
\]

To graph the function, make a table. Graph the ordered pairs.

\[
\begin{array}{c|c|c|c}
\hline
x & f(x) = 2x + 2 & f(x) & (x, f(x)) \\
\hline
1 & f(1) = 2(1) + 2 & 4 & (1, 4) \\
-1 & f(-1) = 2(-1) + 2 & 0 & (-1, 0) \\
-2 & f(-2) = 2(-2) + 2 & -2 & (-2, -2) \\
\hline
\end{array}
\]

The graph intersects the x-axis at $(-1, 0)$. The solution to the equation is $x = -1$.

Exercises

Solve each equation.

1. $3x - 3 = 0$ 
2. $-2x + 1 = 5 - 2x$ 
3. $-x + 4 = 0$ 
4. $0 = 4x - 1$ 
5. $5x - 1 = 5x$ 
6. $-3x + 1 = 0$

3-2 Study Guide and Intervention (continued)

Solving Linear Equations by Graphing

Estimate Solutions by Graphing Sometimes graphing does not provide an exact solution, but only an estimate. In these cases, solve algebraically to find the exact solution.

Example

WALKING You and your cousin decide to walk the 7-mile trail at the state park to the ranger station. The function $d = 7 - 3.2t$ represents your distance $d$ from the ranger station after $t$ hours. Find the zero of this function. Describe what this value means in this context.

Make a table of values to graph the function.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$d = 7 - 3.2t$</th>
<th>$d$</th>
<th>($t, d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$7 - 3.2(0)$</td>
<td>7</td>
<td>(0, 7)</td>
</tr>
<tr>
<td>1</td>
<td>$7 - 3.2(1)$</td>
<td>3.8</td>
<td>(1, 3.8)</td>
</tr>
<tr>
<td>2</td>
<td>$7 - 3.2(2)$</td>
<td>0.6</td>
<td>(2, 0.6)</td>
</tr>
</tbody>
</table>

The graph intersects the $t$-axis between $t = 2$ and $t = 3$, but closer to $t = 2$. It will take you and your cousin just over two hours to reach the ranger station.

You can check your estimate by solving the equation algebraically.

Exercises

1. MUSIC Jessica wants to record her favorite songs to one CD. The function $C = 80 - 3.22n$ represents the recording time $C$ available after $n$ songs are recorded. Find the zero of this function. Describe what this value means in this context.

   Just under 25; only 24 songs can be recorded on one CD

2. GIFT CARDS Enrique uses a gift card to buy coffee at a coffee shop. The initial value of the gift card is $20. The function $n = 20 - 2.75c$ represents the amount of money still left on the gift card $n$ after purchasing $c$ cups of coffee. Find the zero of this function. Describe what this value means in this context.

   Just over 7; Enrique can buy 7 cups of coffee with the gift card
**3-2 Skills Practice**

**Solving Linear Equations by Graphing**

Solve each equation.

1. \(2x - 5 = -3 + 2x\)
2. \(-3x + 2 = 0\)
3. \(3x + 2 = 3x - 1\)
4. \(4x - 1 = 4x + 2\)
5. \(4x - 1 = 0\)
6. \(0 = 5x + 3\)
7. \(0 = -2x + 4\)
8. \(-3x + 8 = 5 - 3x\)
9. \(-x + 1 = 0\)

10. **GIFT CARDS** You receive a gift card for trading cards from a local store. The function \(d = 20 - 1.95c\) represents the remaining dollars \(d\) on the gift card after obtaining \(c\) packages of cards. Find the zero of this function. Describe what this value means in this context.

\[c \approx 10.26; \text{ you can purchase 10 packages of trading cards with the gift card.}\]

**3-2 Practice**

**Solving Linear Equations by Graphing**

Solve each equation.

1. \(\frac{1}{2}x - 2 = 0\)
2. \(-3x + 2 = -1\)
3. \(4x - 2 = -2\)
4. \(\frac{1}{3}x + 2 = \frac{1}{3}x - 1\)
5. \(\frac{5}{3}x + 4 = 3\)
6. \(\frac{3}{2}x + 1 = \frac{3}{2}x - 7\)

Solve each equation by graphing. Verify your answer algebraically.

7. \(13x + 2 = 11x - 1\)
8. \(-3x - 3 = -4x - 3\)
9. \(-\frac{1}{2}x + 2 = \frac{3}{2}x - 1\)

10. **DISTANCE** A bus is driving at 60 miles per hour toward a bus station that is 250 miles away. The function \(d = 250 - 60t\) represents the distance \(d\) from the bus station the bus is \(t\) hours after it has started driving. Find the zero of this function. Describe what this value means in this context.

\[t \approx 4.17 \text{ hr; the bus will arrive at the station in approximately 4.17 hours.}\]
3-2 Word Problem Practice

Solving Linear Equations by Graphing

1. **PET CARE** You buy a 6.3-pound bag of dry cat food for your cat. The function \( c = 6.3 - 0.25p \) represents the amount of cat food \( c \) remaining in the bag when the cat is fed the same amount each day for \( p \) days. Find the zero of this function. Describe what this value means in this context.

2. **SAVINGS** Jessica is saving for college using a direct deposit from her paycheck into a savings account. The function \( m = 3045 - 52.5t \) represents the amount of money \( m \) still needed after \( t \) weeks. Find the zero of this function. Describe what this value means in this context.

3. **FINANCE** Michael borrows $100 from his dad. The function \( v = 100 - 4.75d \) represents the outstanding balance \( v \) after \( d \) weekly payments. Find the zero of this function. Describe what this value means in this context.

4. **BAKE SALE** Ashley has $15 in the Pep Club treasury to pay for supplies for a chocolate chip cookie bake sale. The function \( \frac{d}{d} = 15 - 0.06c \) represents the dollars \( d \) left in the club treasury after making \( c \) cookies. Find the zero of this function. Describe what this value represents in this context.

5. **DENTAL HYGIENE** You are packing your suitcase to go away to a 14-day summer camp. The store carries three sizes of tubes of toothpaste.

<table>
<thead>
<tr>
<th>Tube</th>
<th>Size (ounces)</th>
<th>Size (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.75</td>
<td>21.26</td>
</tr>
<tr>
<td>B</td>
<td>0.9</td>
<td>25.52</td>
</tr>
<tr>
<td>C</td>
<td>3.0</td>
<td>85.04</td>
</tr>
</tbody>
</table>

Source: National Academy of Sciences

a. The function \( n = 21.26 - 0.8b \) represents the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube A. Find the zero of this function. Describe what this value means in this context.

b. The function \( n = 25.52 - 0.8b \) represents the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube B. Find the zero of this function. Describe what this value means in this context.

c. Write a function to represent the number of remaining brushings \( n \) using \( b \) grams per brushing using Tube C. Find the zero of this function. Describe what this value means in this context.

d. If you will brush your teeth twice each day while at camp, which is the smallest tube of toothpaste you can choose? Explain your reasoning.

Tube B: You need 28 brushings. Tube A is not enough and Tube C is too much.

3-2 Enrichment

**Composite Functions**

Three things are needed to have a function—a set called the domain, a set called the range, and a rule that matches each element in the domain with only one element in the range. Here is an example.

Rule: \( f(x) = 2x + 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ f(-3) = 2(-3) + 1 = 58 + 1 = 59 \]

Suppose we have three sets \( A, B, \) and \( C \) and two functions described as shown below.

Rule: \( f(x) = 2x + 1 \) Rule: \( g(y) = 3y - 4 \)

\[ g(3) = 3(3) - 4 = 9 - 4 = 5 \]

Let’s find a rule that will match elements of set \( A \) with elements of set \( C \) without finding any elements in set \( B \). In other words, let’s find a rule for the composite function \( g(f(x)) \).

\[ g(f(x)) = g(2x + 1) = 3(2x + 1) = 6x + 3 \]

Therefore, \( g(f(x)) = 6x + 3 \).

Find a rule for the composite function \( g(f(x)) \).

1. \( f(x) = 3x \) and \( g(y) = 2y + 1 \)
2. \( f(x) = x^2 + 1 \) and \( g(y) = 4y \)
3. \( f(x) = -2x \) and \( g(y) = y^3 - 3y \)
4. \( f(x) = \frac{1}{x - 3} \) and \( g(y) = y^{-1} \)

\[ g(f(x)) = 4x^2 + 6x \]

5. Is it always the case that \( g(f(x)) = f(g(x)) \)? Justify your answer.

No. For example, in Exercise 1,

\[ f(g(x)) = f(2x + 1) = 3(2x + 1) = 6x + 3, \text{ not } 6x + 1. \]
3-3 Study Guide and Intervention

Rate of Change and Slope

Rate of Change  The rate of change tells, on average, how a quantity is changing over time.

Example

Population The graph shows the population growth in China.


1950–1975: change in population = 0.38 or 0.0152
change in time = 1975 – 1950

2000–2025: change in population = 1.45 – 1.27
change in time = 2025 – 2000

b. Explain the meaning of the rate of change in each case.

From 1950–1975, the growth was 0.0152 billion per year, or 15.2 million per year.
From 2000–2025, the growth is expected to be 0.0072 billion per year, or 7.2 million per year.

c. How are the different rates of change shown on the graph?

There is a greater vertical change for 1950–1975 than for 2000–2025. Therefore, the section of the graph for 1950–1975 has a steeper slope.

Exercises

1. LONGEVITY The graph shows the predicted life expectancy for men and women born in a given year.

a. Find the rates of change for women from 2000–2025 and 2025–2050. 0.16/yr, 0.12/yr
b. Find the rates of change for men from 2000–2025 and 2025–2050. 0.16/yr, 0.12/yr
c. Explain the meaning of your results in Exercices 1 and 2. Both men and women increased their life expectancy at the same rates.

d. What pattern do you see in the increase with each 25-year period? While life expectancy increases, it does not increase at a constant rate.

e. Make a prediction for the life expectancy for 2050–2075. Explain how you arrived at your prediction. Sample answer: 89 for women and 83 for men; the decrease in rate from 2000–2025 to 2025–2050 is 0.04/yr. If the decrease in the rate remains the same, the change of rate for 2050–2075 might be 0.08/yr and 25(0.08) = 2 years of increase over the 25-year span.

Find the value of $r$ so that the line through $(18, r)$ and $(3, 4)$ has a slope of $-\frac{5}{7}$.

$\frac{5}{7} = \frac{4 - r}{18 - 3}$

Cross multiply.

$14 = 28 - 7r$

Distribute Property

$14 + 7r = 28$

Subtract 14 from each side.

$7r = 14$

Divide each side by 7.

$\frac{7r}{7} = \frac{14}{7}$

$r = 2$
3-3 Skills Practice  
**Rate of Change and Slope**

Find the slope of the line that passes through each pair of points.

1. \((2, 5), (3, 6)\)
2. \((0, 1), (-6, 1)\)
3. undefined
4. \((2, 5), (3, 6)\)
5. \((6, 1), (-6, 1)\)
6. \((4, 6), (4, 8)\)
7. \((5, 2), (5, -2)\)
8. \((2, 5), (-3, -5)\)
9. \((9, 8), (7, -8)\)
10. \((-5, -8), (-8, 1)\)
11. \((-3, 10), (-3, 7)\)
12. \((17, 18), (18, 17)\)
13. \((-6, -4), (4, 1)\)
14. \((10, 0), (-2, 4)\)
15. \((2, -1), (-8, -2)\)
16. \((5, -9), (3, -2)\)
17. \((12, 6), (3, -5)\)
18. \((-4, 5), (-8, -5)\)

Find the value of \(r\) so the line that passes through each pair of points has the given slope.

19. \((-5, 6), (7, -8)\)
20. \((r, 3), (5, 9)\), \(m = 2\)
21. \((5, 9), (r, -3)\), \(m = -4\)
22. \((r, 2), (6, 3)\), \(m = \frac{1}{2}\)
23. \((r, 4), (7, 1)\), \(m = \frac{3}{4}\)
24. \((5, 3), (r, -5)\), \(m = 4\)
25. \((7, r), (4, 6)\), \(m = 0\)

Answers (Lesson 3-3)

1. undefined
2. undefined
3. undefined
4. \(-1\)
5. 0
6. undefined
7. undefined
8. undefined
9. 8
10. undefined
11. undefined
12. undefined
13. undefined
14. 0
15. undefined
16. undefined
17. 4
18. 5
19. undefined
20. undefined
21. undefined
22. 2
23. undefined
24. undefined
25. undefined
1. **HIGHWAYS** Roadway signs such as the one below are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign?

\[ \frac{2}{25} \]

2. **AMUSEMENT PARKS** The SheiKra roller coaster at Busch Gardens in Tampa, Florida, features a 138-foot vertical drop. What is the slope of the coaster track at this part of the ride? Explain.

The slope is undefined because the drop is vertical.

3. **CENSUS** The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2009.

- Increased about 1.9 people per square mile

<table>
<thead>
<tr>
<th>Year</th>
<th>People Per Square Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>22.1</td>
</tr>
<tr>
<td>1960</td>
<td>36.4</td>
</tr>
<tr>
<td>1980</td>
<td>54.3</td>
</tr>
<tr>
<td>2000</td>
<td>79.6</td>
</tr>
<tr>
<td>2009</td>
<td>96.7</td>
</tr>
</tbody>
</table>

Source: Bureau of the Census, U.S. Dept. of Commerce

4. **REAL ESTATE** A realtor estimates the median price of an existing single-family home in Cedar Ridge is $221,900. Two years ago, the median price was $195,200. Find the average annual rate of change in median home price in these years.

$13,350

5. **COAL EXPORTS** The graph shows the annual coal exports from U.S. mines in millions of short tons.

Source: Energy Information Association

a. What was the rate of change in coal exports between 2001 and 2002?

\[ -9 \text{ million tons per year or } -\frac{9}{1} \]

b. How does the rate of change in coal exports from 2005 to 2006 compare to that of 2001 to 2002?

In 2005–2006, the rate was 0 compared to \( -\frac{9}{1} \) in 2001–2002.

c. Explain the meaning of the part of the graph with a slope of zero.

The slope indicates that there was no change in the amount of coal exported between 2005 and 2006.

Enrichment: Treasure Hunt with Slopes

Using the definition of slope, draw segments with the slopes listed below in order. A correct solution will trace the route to the treasure.

1. \( \frac{3}{2} \)
2. \( \frac{1}{4} \)
3. \( \frac{2}{5} \)
4. 0
5. 1
6. \( -\frac{1}{3} \)
7. no slope
8. \( -\frac{3}{2} \)
9. \( -\frac{3}{4} \)
10. \( \frac{1}{3} \)
11. \( -\frac{3}{4} \)
12. 3
3-4 Study Guide and Intervention

Direct Variation

A direct variation is described by an equation of the form \( y = kx \), where \( k \neq 0 \). We say that \( y \) varies directly as \( x \). In the equation \( y = kx \), \( k \) is the constant of variation.

\textbf{Example 1}  
Name the constant of variation for the equation. Then find the slope of the line that passes through each pair of points.

\[ \text{For } y = \frac{1}{2}x, \text{ the constant of variation is } \frac{1}{2}. \]

\[ m = \frac{y_2 - y_1}{x_2 - x_1}; \text{ Slope formula} \]

\[ = \frac{\frac{3}{2} - \frac{1}{2}}{2 - 1}; \text{ Simplify} \]

\[ = \frac{1}{2}. \]

The slope is \( \frac{1}{2} \).

\textbf{Example 2}  
Suppose \( y \) varies directly as \( x \), and \( y = 30 \) when \( x = 5 \).

\begin{enumerate}
  \item Write a direct variation equation that relates \( x \) and \( y \).
  \begin{align*}
  y &= kx \\
  30 &= k(5) \\
  6 &= k \\
  \text{Divide each side by 5.}
  \end{align*}

  Therefore, the equation is \( y = 6x \).

  \item Use the direct variation equation to find \( x \) when \( y = 18 \).
  \begin{align*}
  y &= 6x \\
  18 &= 6x \\
  3 &= x \\
  \text{Divide each side by 6.}
  \end{align*}

  Therefore, \( x = 3 \) when \( y = 18 \).
\end{enumerate}

\textbf{Exercises}  
Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

\begin{enumerate}
  \item [1.] \begin{tabular}{|c|c|}
    \hline
    \( x \) & \( y \) \\
    \hline
    1 & 3 \\
    4 & 12 \\
    \hline
  \end{tabular}

  \text{Suppose } y \text{ varies directly as } x. \text{ Write a direct variation equation that relates } x \text{ to } y. \text{ Then solve.}

  \begin{enumerate}
    \item If \( y = 4 \) when \( x = 2 \), find \( y \) when \( x = 16 \). \( y = 2x; 32 \)
    \item If \( y = 9 \) when \( x = -3 \), find \( x \) when \( y = 6 \). \( y = -3x; -2 \)
    \item If \( y = -4.8 \) when \( x = -1.6 \), find \( x \) when \( y = -24 \). \( y = 3x; -8 \)
    \item If \( y = \frac{1}{4} \) when \( x = \frac{1}{8} \), find \( x \) when \( y = \frac{3}{16} \). \( y = 2x; \frac{3}{32} \)
  \end{enumerate}
\end{enumerate}
Skills Practice

Direct Variation

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

1. \( y = 3x \)
2. \( y = -\frac{3}{4}x \)
3. \( y = \frac{5}{3}x \)

Graph each equation.

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

7. If \( y = 8 \) when \( x = 2 \), find \( x \) when \( y = 45 \) when \( x = 15 \), find \( x \) when \( y = -4 \) when \( x = 2 \), find \( y \) when \( x = -6 \). \( y = -2x \); 12
8. If \( y = 15 \), find \( x \) when \( y = -9 \) when \( x = 3 \), find \( y \) when \( x = -5 \). \( y = -3x \); 15
9. If \( y = 4 \) when \( x = 16 \), find \( y \) when \( x = 6 \). \( y = \frac{1}{4}x \); \( \frac{3}{2} \)
10. If \( y = 12 \) when \( x = 18 \), find \( x \) when \( y = -16 \). \( y = \frac{3}{2}x \); -24

Write a direct variation equation that relates the variables. Then graph the equation.

11. GASOLINE The total cost \( C \) of gasoline is \$3.00\( g \) times the number of gallons \( g \).

12. TRAVEL The total cost \( C \) of gasoline is \$3.00\( g \) times the number of gallons \( g \).

13. TOYS The number of delivered toys \( T \) is 3 times the total number of crates \( c \).

14. SHIPPING The number of delivered toys \( T \) is 3 times the total number of crates \( c \).

### Graph each equation.

### Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

7. If \( y = 7.5 \) when \( x = 0.5 \), find \( x \) when \( y = -0.3 \). \( y = 15x \); -4.5
8. If \( y = 80 \) when \( x = 32 \), find \( x \) when \( y = 100 \). \( y = 2.5x \); 40
9. If \( y = \frac{3}{4} \) when \( x = 24 \), find \( y \) when \( x = 12 \). \( y = \frac{1}{32}x \); \( \frac{3}{8} \)

Write a direct variation equation that relates the variables. Then graph the equation.

10. MEASURE The width \( W \) of a rectangle is two thirds of the length \( l \).

11. TICKETS The total cost \( C \) of tickets is \$4.50\( t \) times the number of tickets \( t \).

12. PRODUCE The cost of bananas varies directly with their weight. Miguel bought 3\( \frac{1}{2} \) pounds of bananas for \$1.12. Write an equation that relates the cost of the bananas to their weight. Then find the cost of \( 4 \frac{1}{2} \) pounds of bananas. \( C = 0.32p \); \$1.36
3-4 Word Problem Practice

Direct Variation

1. ENGINES The engine of a chainsaw requires a mixture of engine oil and gasoline. According to the directions, oil and gasoline should be mixed as shown in the graph below. What is the constant of variation for the line graphed? 2.5

2. RACING In a recent year, English driver Lewis Hamilton won the United States Grand Prix at the Indianapolis Motor Speedway. His speed during the race averaged 125.145 miles per hour. Write a direct variation equation for the distance d that Hamilton drove in t hours at that speed. d = 125.145t

3. CURRENCY The exchange rate from one currency to another varies every day. Recently the exchange rate from U.S. dollars to British pound sterling (£) was $1.58 to £1. Write and solve a direct variation equation to determine how many pounds sterling you would receive in exchange for $90 of U.S. currency.

4. SALARY Henry started a new job in which he is paid $9.50 an hour. Write and solve an equation to determine Henry's gross salary for a 40-hour work week. p = 9.5h; $380

5. SALES TAX Amelia received a gift card to a local music shop for her birthday. She plans to use the gift card to buy some new CDs.
   a. Amelia chose 3 CDs that each cost $16. The sales tax on the three CDs is $3.96. Write a direct variation equation relating sales tax to the price.
      \[ T = 0.0625P \]
   b. Graph the equation you wrote in part a.

   c. What is the sales tax rate that Amelia is paying on the CDs? 8.25%

3-4 Enrichment

nth Power Variation

An equation of the form \( y = kx^n \), where \( k \neq 0 \), describes an nth power variation. The variable \( n \) can be replaced by 2 to indicate the second power of \( x \) (the square of \( x \)) or by 3 to indicate the third power of \( x \) (the cube of \( x \)).

Assume that the weight of a person of average build varies directly as the cube of that person's height. The equation of variation has the form \( w = kh^3 \).

The weight that a person's legs will support is proportional to the cross-sectional area of the leg bones. This area varies directly as the square of the person's height. The equation of variation has the form \( s = kh^2 \).

Answer each question.

1. For a person 6 feet tall who weighs 200 pounds, find a value for \( k \) in the equation \( w = kh^3 \).
   \[ k = 0.93 \]

2. Use your answer from Exercise 1 to predict the weight of a person who is 5 feet tall.
   about 116 pounds

3. Find the value for \( k \) in the equation \( w = kh^3 \) for a baby who is 20 inches long and weighs 6 pounds.
   \[ k = 1.296 \text{ for } h = \frac{5}{3} \text{ ft} \]

4. How does your answer to Exercise 3 demonstrate that a baby is significantly fatter in proportion to its height than an adult?
   \( k \) has a greater value.

5. For a person 6 feet tall who weighs 200 pounds, find a value for \( k \) in the equation \( s = kh^2 \).
   \[ k = 5.56 \]

6. For a baby who is 20 inches long and weighs 6 pounds, find an “infant value” for \( k \) in the equation \( s = kh^2 \).
   \[ k = 2.16 \text{ for } h = \frac{5}{3} \text{ ft} \]

7. According to the adult equation you found (Exercise 1), how much would an imaginary giant 20 feet tall weigh?
   7440 pounds

8. According to the adult equation for weight supported (Exercise 5), how much weight could a 20-foot tall giant's legs actually support?
   only 2224 pounds

9. What can you conclude from Exercises 7 and 8?
   Answers will vary. For example, bone strength limits the size humans can attain.
3-5 Study Guide and Intervention

Arithmetic Sequences as Linear Functions

Recognize Arithmetic Sequences. A sequence is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an arithmetic sequence.

<table>
<thead>
<tr>
<th>Arithmetic Sequence</th>
<th>a numerical pattern that increases or decreases at a constant rate or value called the common difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms of an Arithmetic Sequence</td>
<td>If $a_1$ is the first term of an arithmetic sequence with common difference $d$, then the sequence is $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$</td>
</tr>
<tr>
<td>$n^{th}$ Term of an Arithmetic Sequence</td>
<td>$a_n = a_1 + (n-1)d$</td>
</tr>
</tbody>
</table>

Example 1 Determine whether the sequence 1, 3, 5, 7, 9, 11, \ldots is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since $3 - 1 = 2$, $5 - 3 = 2$, and so on, the common difference is 2.

Since the difference between the terms of 1, 3, 5, 7, 9, 11, \ldots is constant, this is an arithmetic sequence.

Example 2 Write an equation for the $n^{th}$ term of the sequence 12, 15, 18, 21, \ldots.

In this sequence, $a_1 = 12$. Find the common difference.

$\begin{align*}
12 & \quad 15 \\
+3 & \quad +3 \\
\hline
15 & \quad 18 \\
+3 & \quad +3 \\
\hline
21 & \\
\end{align*}$

The common difference is 3.

Use the formula for the $n^{th}$ term to write an equation.

$a_n = a_1 + (n-1)d$  

Formula for the $n^{th}$ term

$a_n = 12 + (n-1)3$  

$a_n = 12 + 3n - 3$  

Distributive Property

$a_n = 3n + 9$  

Simplify

The equation for the $n^{th}$ term is $a_n = 3n + 9$.

Exercises

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 1, 5, 9, 13, 17, \ldots

2. 8, 4, 0, -4, -8, \ldots

3. 1, 3, 9, 27, 81, \ldots

yes; $d = 4$  

yes; $d = -4$  

no; no common difference

Find the next three terms of each arithmetic sequence.

4. 4, 9, 13, 17, 21, 25, \ldots

5. 5, 0, -5, -10, -15, \ldots

6. 29, 35, 41, 47, \ldots

$29, 33, 37$  

$-16, -20, -24$  

$53, 59, 65$

Write an equation for the $n^{th}$ term of each arithmetic sequence. Then graph the first five terms of the sequence.

7. $7, 1, 5, 9, \ldots$

$a_n = 2n - 1$

8. $8, -1, -4, -7, \ldots$

$a_n = -3n + 2$

9. $9, -4, -9, -14, -19, \ldots$

$a_n = -5n + 1$

Exercises (continued)

Arithmetic Sequences as Functions

An arithmetic sequence is a linear function in which $n$ is the independent variable, $a_1$ is the dependent variable, and the common difference $d$ is the slope. The formula can be rewritten as the function $a_n = a_1 + (n - 1)d$, where $n$ is a counting number.

Example SEATING There are 20 seats in the first row of the balcony of the auditorium. There are 22 seats in the second row, and 24 seats in the third row.

a. Write a function to represent this sequence.

The first term $a_1$ is 20. Find the common difference.

$\begin{align*}
20 & \quad 22 \\
+2 & \quad +2 \\
\hline
22 & \quad 24 \\
+2 & \quad +2 \\
\hline
26 & \\
\end{align*}$

The common difference is 2.

$a_n = a_1 + (n - 1)d$  

Formula for the $n^{th}$ term

$a_n = 20 + (n - 1)2$  

$a_n = 20 + 2n - 2$  

Distributive Property

$a_n = 18 + 2n$  

Simplify

The function is $a_n = 18 + 2n$.

Exercises

1. KNITTING Sarah learns to knit from her grandmother. Two days ago, she measured the length of the scarf she is knitting to be 13 inches. Yesterday, she measured the length of the scarf to be 15.5 inches. Today it measures 18 inches. Write a function to represent the arithmetic sequence. $a_n = 13 + 2.5n$

2. REFRESHMENTS You agree to pour water into the cups for the Booster Club at a football game. The pitcher contains 64 ounces of water when you begin. After you have filled 8 cups, the pitcher is empty and must be refilled.

a. Write a function to represent the arithmetic sequence.

$a_n = -8n$

b. Graph the function.
**3-5** Skills Practice

**Arithmetic Sequences as Linear Functions**

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 4, 7, 9, 12, . . .  no  
2. 15, 13, 11, 9, . . .  yes; -2
3. 7, 10, 13, 16, . . .  yes; 3
4. -6, -5, -3, -1, . . .  no
5. -5, -3, -1, . . .  yes; 2
6. -9, -12, -15, -18, . . .  yes; -3
7. 10, 15, 20, 40, . . .  no
8. -10, -5, 0, 5, . . .  yes; 5

Find the next three terms of each arithmetic sequence.

9. 9, 3, 11, 15, . . .  19, 23, 27
10. 14, 12, 10
11. -13, -11, -9, -7, . . .  -5, -3, -1
12. -14, -17, -20
13. 19, 24, 29, 34, . . .  39, 44, 49
14. 14, 16, 17, -2, -11, . . .  -20, -29, -38
15. 12.5, 7.5, 10, . . .  12.5, 15, 17.5
16. 10.3, 4.1, 1.5, 1.6, . . .  7.1, 8.1, 9.1

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms of the sequence.

17. 7, 13, 19, 25, . . .  \(a_n = 6n + 1\)
18. 30, 26, 22, 18, . . .  \(a_n = -4n + 34\)
19. -7, -4, -1, 2, . . .  \(a_n = 3n - 10\)

20. **VIDEO DOWNLOADING** Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is $1.99. The cost to download 2 episodes is $3.98. The cost to download 3 episodes is $5.97. Write a function to represent the arithmetic sequence.

\[a_n = 1.99n\]

**3-5** Practice

**Arithmetic Sequences as Linear Functions**

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

1. 21, 13, 5, . . .  yes; \(d = -8\)
2. -5, 12, 29, 46, . . .  yes; \(d = 17\)
3. -2, -1.1, 0.1, 1.3, . . .  no; no common difference
4. 1, 4, 9, 16, . . .  yes; \(d = 7\)
5. 9, 16, 23, 30, . . .  no; no common difference
6. -12, 0, 8, 16, 24, . . .  no; no common difference

Find the next three terms of each arithmetic sequence.

7. 82, 76, 70, 64, . . .  88, 82, 76
8. -49, -35, -21, -7, . . .  3, 11, 19
9. 3, 4, 5, . . .  4, 5, 6
10. 58, 52, 46, . . .  72, 76, 80
11. 7, 21, 35, . . .  1, 5, 9
12. 10, -3, 4, 11, . . .  -10, 13, 16
13. 18, 25, 32, . . .  4, 2, 0
14. -10, -17, -24, . . .  -17, -22, -27
15. 12, 7, 2, -3, . . .  -8, -13, -18
16. 19, 24, 29, 34, . . .  39, 44, 49
17. 14, -5, 2, 14, 26, . . .  29, 31, 33
18. 15, 19, 23, 27, . . .  29, 33, 37
19. 15, 20, 25, 30, . . .  35, 40, 45
20. 5, 9, 13, . . .  17, 21

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms of the sequence.

10. 1, 3, 5, 7, 9, . . .  \(a_n = 2n - 1\)
11. 2, 4, 6, 8, 10, . . .  \(a_n = 2n\)
12. -2, -4, -6, -8, -10, . . .  \(a_n = -2n\)

21. **BANKING** Chem deposited $115.00 in a savings account. Each week thereafter, he deposits $35.00 into the account.

a. Write a function to represent the total amount Chem has deposited for any particular number of weeks after his initial deposit. \(a_n = 35n + 115\)

b. How much has Chem deposited 30 weeks after his initial deposit? $1165

22. **STORE DISPLAYS** Tamika is stacking boxes of tissue for a store display. Each row of tissues has 2 fewer boxes than the row below. The first row has 23 boxes of tissues.

a. Write a function to represent the arithmetic sequence. \(a_n = -2n + 25\)

b. How many boxes will there be in the tenth row? 5
3-5 Word Problem Practice

Arithmetic Sequences as Linear Functions

1. POSTAGE The price to send a large envelope first class mail is 88 cents for the first ounce and 17 cents for each additional ounce. The table below shows the cost for weights up to 5 ounces.

<table>
<thead>
<tr>
<th>Weight (ounces)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postage (dollars)</td>
<td>0.88</td>
<td>1.05</td>
<td>1.22</td>
<td>1.39</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Source: United States Postal Service

How much did a large envelope weigh that cost $2.07 to send? 8 ounces

2. SPORTS Wanda is the manager for the soccer team. One of her duties is to hand out cups of water at practice. Each cup of water is 4 ounces. She begins practice with a 128-ounce cooler of water. How much water is remaining after she hands out the 14th cup? 72 ounces

3. THEATER A theater has 20 seats in the first row, 22 in the second row, 24 in the third row, and so on for 25 rows. How many seats are in the last row? 68 seats

4. NUMBER THEORY One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170-1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

Does this represent an arithmetic sequence? Why or why not?

No, because the difference between terms is not constant.

b. How much money will Inga's grandfather have contributed after 24 months? $15,000

5. SAVINGS Inga's grandfather decides to start a fund for her college education. He makes an initial contribution of $3000 and each month deposits an additional $500. After one month he will have contributed $3300.

a. Write an equation for the nth term of the sequence. \( a_n = 3000 + 500n \)

b. How much money will Inga’s grandfather have contributed after 24 months? $15,000

3-5 Enrichment

Arithmetic Series

An arithmetic series is a series in which each term after the first may be found by adding the same number to the preceding term. Let \( S \) stand for the following series in which each term is 3 more than the preceding one.

\[
S = 2 + 5 + 8 + 11 + 14 + 17 + 20
\]

The series remains the same if we reverse the order of all the terms. So let us reverse the order of the terms and add one series to the other, term by term. This is shown at the right.

\[
S = \frac{22 + 22}{2} = 22
\]

\[
S = 7(22)
\]

Let \( a \) represent the first term of the series.

Let \( \ell \) represent the last term of the series.

Let \( n \) represent the number of terms in the series.

In the preceding example, \( a = 2, \ell = 20, \) and \( n = 7. \) Notice that when you add the two series, term by term, the sum of each pair of terms is 22. That sum can be found by adding the first and last terms, \( 2 + 20 = 22. \) Notice also that there are 7, or \( n, \) such sums. Therefore, the value of \( 2S \) is \( 7(22), \) or \( n(a + \ell) \) in the general case. Since this is twice the sum of the series, you can use the formula \( S = \frac{n(a + \ell)}{2} \) to find the sum of any arithmetic series.

Example 1

Find the sum: \( 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9. \)

\( a = 1, \ell = 9, \) and \( n = 9, \) so \( S = \frac{9(1 + 9)}{2} = \frac{10(9)}{2} = 45. \)

Example 2

Find the sum: \( -9 + (-5) + (-1) + 3 + 7 + 11 + 15. \)

\( a = 29, \ell = 15, \) and \( n = 7, \) so \( S = \frac{7(-9 + 15)}{2} = \frac{7(6)}{2} = 21. \)

Exercises

Find the sum of each arithmetic series.

1. \( 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 \)
2. \( 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 \)
3. \( -21 + (-16) + (-11) + (-6) + (-1) + 4 + 9 + 14 \)
4. even whole numbers from 2 through 100
5. odd whole numbers between 0 and 100
### Study Guide and Intervention (continued)

#### Proportional and Nonproportional Relationships

**Nonproportional Relationships** If the ratio of the value of \( x \) to the value of \( y \) is different for select ordered pairs on the line, the equation is nonproportional.

**Example** Write an equation in function notation for the relation shown in the graph.

Select points from the graph and place them in a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-2)</td>
<td>(-4)</td>
<td>(-6)</td>
<td>(0)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

The difference between the \( x \)-values is 1, while the difference between the \( y \)-values is \(-2\). This suggests that \( y = -2x \).

If \( x = 1 \), then \( y = -2(1) \) or \(-2\). But the \( y \)-value for \( x = 1 \) is 0.

\[ y \text{ is always } 2 \text{ more than } -2x \]

This pattern shows that 2 should be added to one side of the equation. Thus, the equation is \( y = -2x + 2 \).

**Exercises**

1. **NATURAL GAS** Natural gas use is often measured in “therms.” The total amount a gas company will charge for natural gas use is based on how much natural gas a household uses. The table shows the relationship between natural gas use and the total cost.

<table>
<thead>
<tr>
<th>Gas Used (therms)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>$1.30</td>
<td>$2.60</td>
<td>$3.90</td>
<td>$5.20</td>
<td></td>
</tr>
</tbody>
</table>

   a. Graph the data. What can you deduce from the pattern about the relationship between the number of therms used and the total cost?
   
   **The relationship is proportional.**

   b. Write an equation to describe this relationship. \( y = 1.30x \)

   c. Use this equation to predict how much it will cost if a household uses 40 therms.

   \( \$52.00 \)

   **Exercises**

1. **Graph** the data. What can you deduce from the pattern about the relationship between the number of therms used and the total cost?

   **The relationship is proportional.**

2. Write an equation to describe this relationship. \( y = 1.30x \)

3. Use this equation to predict how much it will cost if a household uses 40 therms.

   \( \$52.00 \)
Chapter 3

3-6 Skills Practice
Proportional and Nonproportional Relationships

Write an equation in function notation for each relation.

1. \( f(x) = -2x \)
2. \( f(x) = x - 2 \)
3. \( f(x) = 1 - x \)
4. \( f(x) = x + 6 \)
5. \( f(x) = 5 - x \)
6. \( f(x) = 2x - 1 \)

7. GAMESHOWS  The table shows how many points are awarded for answering consecutive questions on a gameshow.

<table>
<thead>
<tr>
<th>Question answered</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points awarded</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
</tr>
</tbody>
</table>

a. Write an equation for the data given. \( y = 200x \)
b. Find the number of points awarded if 9 questions were answered. 1800

3-6 Practice
Proportional and Nonproportional Relationships

1. BIOLOGY  Male fireflies flash in various patterns to signal location and perhaps to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, its rate, and its shape. The table below shows the rate at which a male firefly is flashing.

<table>
<thead>
<tr>
<th>Times (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Flashes</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Write an equation in function notation for the relation. \( f(t) = 2t \), where \( t \) is the time in seconds and \( f(t) \) is the number of flashes
b. How many times will the firefly flash in 20 seconds? 40

2. GEOMETRY  The table shows the number of diagonals that can be drawn from one vertex in a polygon. Write an equation in function notation for the relation and find the number of diagonals that can be drawn from one vertex in a 12-sided polygon. \( f(s) = s - 3 \), where \( s \) is the number of sides and \( f(s) \) is the number of diagonals; 9

Write an equation in function notation for each relation.

3. \( f(x) = -\frac{1}{2}x \)
4. \( f(x) = 3x - 6 \)
5. \( f(x) = 2x + 4 \)

For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. Explain.

6. 1, 3, 5, . . .  \( a(n) = 2n - 1; \)  \( \text{nonproportional; not of form } y = kx \)
7. 2, 7, 12, . . .  \( a(n) = 5n - 3; \)  \( \text{nonproportional; not of form } y = kx \)
8. -3, -6, -9, . . .  \( a(n) = -3n; \)  \( \text{proportional; of form } y = kx \)
Chapter 3

3-6 Word Problem Practice

Proportional and Nonproportional Relationships

1. **ONLINE SHOPPING** Ricardo is buying computer cables from an online store. If he buys 4 cables, the total cost will be $24. If he buys 5 cables, the total cost will be $30. If the total cost can be represented by a linear function, will the function be proportional or nonproportional? Explain.

2. **FOOD** It takes about four pounds of grapes to produce one pound of raisins. The graph shows the relation for the number of pounds of grapes needed, \( x \), to make \( y \) pounds of raisins. Write an equation to describe this relationship.

3. **PARKING** Palmer Township recently installed parking meters in their municipal lot. The cost to park for \( h \) hours is represented by the equation \( C = 0.25 \times 500 \). Make a table of values that represents this relationship.

4. **MUSIC** A measure of music contains the same number of beats throughout the song. The table shows the relation for the number of beats counted after a certain number of measures have been played in the six-eight time. Write an equation to describe this relationship.

<table>
<thead>
<tr>
<th>Measures Played (x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Beats (y)</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

   Source: Sheet Music USA

5. **GEOMETRY** A fractal is a pattern containing parts which are identical to the overall pattern. The following geometric pattern is a fractal.

   ![Fractal Pattern](image)

   a. Complete the table.

<table>
<thead>
<tr>
<th>Term</th>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Smaller Triangles</td>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

   b. What are the next three numbers in the pattern? 25, 36, 49

   c. Write an equation in function notation for the pattern. \( f(x) = x^3 \)

3-6 Enrichment

**Taxicab Graphs**

You have used a rectangular coordinate system to graph equations such as \( y = x - 1 \) on a coordinate plane. In a coordinate plane, the numbers in an ordered pair \((x, y)\) can be any two real numbers.

A **taxicab plane** is different from the usual coordinate plane. The only points allowed are those that exist along the horizontal and vertical grid lines. You may think of the points as taxicabs that must stay on the streets.

The taxicab graph shows the equations \( y = -2 \) and \( y = x - 1 \).

Notice that one of the graphs is no longer a straight line. It is now a collection of separate points.

Graph these equations on the taxicab plane at the right.

1. \( y = x + 1 \)
2. \( y = -2x + 3 \)
3. \( y = 2x \)
4. \( x = -4 \)

Use your graphs for these problems.

5. Which of the equations has the same graph in both the usual coordinate plane and the taxicab plane? \( x = -4 \)

6. Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane.

   \( x = A \) and \( y = B \), where \( A \) and \( B \) are integers

In the taxicab plane, distances are not measured diagonally, but along the streets. Write the taxic-distance between each pair of points.

7. \((0, 0)\) and \((5, 2)\)
8. \((0, 0)\) and \((-3, 2)\)
9. \((0, 0)\) and \((2, 1.5)\)
10. \((1, 2)\) and \((4, 3)\)
11. \((2, 4)\) and \((-1, 3)\)
12. \((0, 4)\) and \((-2, 0)\)

   7 units
   5 units
   3.5 units
   4 units
   4 units
   6 units

Draw these graphs on the taxicab grid at the right.

13. The set of points whose taxic-distance from \((0, 0)\) is 2 units. **indicated by \( x \)**
14. The set of points whose taxic-distance from \((2, 1)\) is 3 units. **indicated by dots**
## Chapter 3 Assessment Answer Key

### Quiz 1 (Lessons 3-1 and 3-2)

1. **yes; 2x – y = 1**

2. **D**

3. **about 400 miles**

4. ![Graph](image)

5. **x = 15**

### Quiz 2 (Lessons 3-3)

1. **\( \frac{2}{9} \)**

2. **\( \frac{7}{4} \)**

3. **B**

4. **not linear**

5. **increases by 5/yr**

### Quiz 3 (Lessons 3-4 and 3-5)

1. **B**

2. **$285**

3. **no**

4. **21, 25, 29**

5. **25**

### Quiz 4 (Lesson 3-6)

1. **\( y = 3x + 3 \)**

2. **\( y = 5x + 4 \)**

3. **\( y = 9x - 2 \)**

4. **no**

5. **increases by 5/yr**

6. **no solution**

7. **-2**

8. **yes; 2x – y = 3**

9. ![Graph](image)

### Mid-Chapter Test

1. **C**

2. **F**

3. **B**

4. **G**

5. **B**

6. **no solution**

7. **-2**

8. **yes; 2x – y = 3**

9. ![Graph](image)
# Chapter 3 Assessment Answer Key

Vocabulary Test Form 1
Page 48

<table>
<thead>
<tr>
<th></th>
<th>Form 1</th>
<th>Page 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>common difference</strong></td>
<td><strong>G</strong></td>
</tr>
<tr>
<td>2</td>
<td><strong>x-intercept</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>3</td>
<td><strong>root</strong></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td><strong>standard form</strong></td>
<td><strong>F</strong></td>
</tr>
<tr>
<td>5</td>
<td><strong>arithmetic sequence</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>6</td>
<td><strong>linear equation</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>7</td>
<td><strong>term</strong></td>
<td><strong>C</strong></td>
</tr>
<tr>
<td>8</td>
<td><strong>inductive reasoning</strong></td>
<td><strong>H</strong></td>
</tr>
<tr>
<td>9</td>
<td><strong>Sample answer:</strong> the y-coordinate of the point at which the graph crosses the y-axis</td>
<td><strong>J</strong></td>
</tr>
<tr>
<td>10</td>
<td><strong>Sample answer:</strong> Rate of change is a ratio that describes, on average, how much a quantity changes with respect to a change in another quantity.</td>
<td><strong>F</strong></td>
</tr>
</tbody>
</table>

**See students’ work;**  
x: 3, y: -3

**B:**
Chapter 3 Assessment Answer Key

Form 2A
Page 51

1. C
2. H
3. A
4. J
5. A
6. G
7. D
8. F
9. C
10. G
11. A

B: 11:08 A.M.

Form 2B
Page 52

12. H
13. D
14. F
15. C
16. G
17. B
18. F
19. A
20. H

10. F
11. D

B: 7:49 A.M.
Chapter 3 Assessment Answer Key

Form 2C
Page 55

1. 6
2. 3
3. 9
4. \( x = 2 \)
5. 4
6. 1
7. \(-\frac{11}{9}\)
8. 0
9. about \(-0.07\) yr
10. 36 mi
11. no
12. yes; \(2x + 3y = -4\)
13. \( (3, -1), (-1, -1), (3, 3) \)

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14. \( y = \frac{1}{2}x + 3 \)
15. \( x = 3 \)
16. yes; 3
17. 36, 43, 50
18. \( a_n = -7n + 19 \)
19. \( f(d) = 0.04d \)
20. 6
Chapter 3 Assessment Answer Key

Form 2D
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1. ______ 8 _______
2. ______ 39 _______
3. ______ 7/3 _______

4. 

5. ______ 8 _______

6. ______ 0 _______
7. ______ undefined _______
8. ______ -3/5 _______

9. ______ about -1%/yr _______
10. ______ 1/2 h _______

11. ______ no _______
12. ______ yes; 4x - 2y = 0 _______
13. 

14. ______ f(d) = 0.05d _______
15. 

16. ______ yes; 4 _______
17. ______ -13, -10, -7 _______
18. ______ a_n = 9n - 6 _______
19. 

20. ______ 7 _______

B: ______ (-2, -2) _______
Chapter 3 Assessment Answer Key

Form 3
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1. yes; $3x - 7y = -6$

2. 12

3. $-4.2$

4. 3.75

5. $y$
   $x = 1.5$
   $O$

6. 4

7. $y$
   $O$

8. $-\frac{9}{13}$

9. undefined

10. 690.4/yr

11. 60

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12. $y$
   $O$

13. $-13$

14. $\frac{70}{13}$

15. yes; $\frac{1}{2}$

16. $y = -1$

17. $a_n = 4n - 19$

18. $v(t) = 2000 - 190t$
   about $10\frac{1}{2}$ yrs

19. 12 yrs

20. $c = 359.88y$
## Chapter 3 Assessment Answer Key

### Page 61, Extended-Response Assessment

**Scoring Rubric**

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
| 4     | **Superior** A correct solution that is supported by well-developed, accurate explanations | - Shows thorough understanding of the concepts of points on the coordinate plane, transformations, relations, functions, linear equations, arithmetic sequences, and patterns.  
- Uses appropriate strategies to solve problems.  
- Computations are correct.  
- Written explanations are exemplary.  
- Graphs are accurate and appropriate.  
- Goes beyond requirements of some or all problems. |
| 3     | **Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation | - Shows an understanding of the concepts of points on the coordinate plane, transformations, relations, functions, linear equations, arithmetic sequences, and patterns.  
- Uses appropriate strategies to solve problems.  
- Computations are mostly correct.  
- Written explanations are effective.  
- Graphs are mostly accurate and appropriate.  
- Satisfies all requirements of problems. |
| 2     | **Nearly Satisfactory** A partially correct interpretation and/or solution to the problem | - Shows an understanding of most of the concepts of points on the coordinate plane, transformations, relations, functions, linear equations, arithmetic sequences, and patterns.  
- May not use appropriate strategies to solve problems.  
- Computations are mostly correct.  
- Written explanations are satisfactory.  
- Graphs are mostly accurate.  
- Satisfies the requirements of most of the problems. |
| 1     | **Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation | - Final computation is correct.  
- No written explanations or work is shown to substantiate the final computation.  
- Graphs may be accurate but lack detail or explanation.  
- Satisfies minimal requirements of some of the problems. |
| 0     | **Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given | - Shows little or no understanding of most of the concepts of points on the coordinate plane, transformations, relations, functions, linear equations, arithmetic sequences, and patterns.  
- Does not use appropriate strategies to solve problems.  
- Computations are incorrect.  
- Written explanations are unsatisfactory.  
- Graphs are inaccurate or inappropriate.  
- Does not satisfy requirements of problems.  
- No answer may be given. |
Chapter 3 Assessment Answer Key
Page 61, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A26, the following sample answers may be used as guidance in evaluating assessment items.

1a. After drawing a graph, students should explain that they can use the two points on the graph to determine the slope. This can be done by counting squares for the rise and run of the line or by using the coordinates of the points in the slope formula.

1b. The slope is the value of \( m \) in the slope-intercept form \( y = mx + b \). By substituting the value of \( m \) and the coordinates of one of the points for \( x \) and \( y \), the value of \( b \) can be found and an equation written using \( m \) and \( b \). The slope and either ordered pair can be used to write the point-slope form of an equation. The standard form of an equation is an algebraic manipulation of either of the other two forms of equations. See students’ equations for the line drawn.

2a. The student writes an equation in the form \( C = kf \). Sample answer: \( C = 2f \)

2b. The student should explain that the constant of variation represents the price per foot of the lumber.

2c. The student explains the new function is in the form \( C = kf + 10 \), which is not a direct variation.

3a. Sample answer: 2, 5, 8, 11, … ; The common difference is 3; \( a_6 = 17 \)

3b. Sample answer: 5, 3, 8, 6, 11, 9, 14, … ; The pattern is to subtract 2 from the first term to find the second term, then add 5 to the second term to find the third term. Repeat the process of subtracting 2 then adding 5.

3c. The sequence 1, 1, 1, 1, … is a set of numbers whose difference between successive terms is the constant number 0. Thus, this sequence is an arithmetic sequence by the definition.

4. The student should draw a line on the coordinate grid and then identify the \( x \)- and \( y \)-coordinates. The student should write a linear equation in function notation for the line they drew.
1. ⬜ ⬜ ● ●
2. ⬜ ⬜ ● ●
3. ⬜ ⬜ ● ●
4. ⬜ ⬜ ● ●
5. ⬜ ⬜ ● ●
6. ⬜ ⬜ ● ●
7. ● ● ● ●
8. ● ● ● ●
9. ● ● ● ●
10. ⬜ ⬜ ● ●

11. ● ● ● ●
12. ○ ○ ● ●
13. ○ ○ ● ●
14. ○ ● ○ ○
15. ○ ○ ○ ●
16. ● ● ○ ○

17.  

18.  

Chapter 3 Assessment Answer Key
Standardized Test Practice
19. \( \frac{4}{5} \)

20. \( 8m + 9p \)

21. \( -\frac{5}{2} \)

22. yes, because each element of the domain is paired with exactly one element of the range.

23. \$12.75

24. \(-2\)

25. no

26. \(
\{(-4, 3), (-4, -1), 
(-2, 2), (0, -1), (2, 3), 
(3, -3), (4, 1)\};
\)
domain:
\(
\{-4, -2, 0, 2, 3, 4\};
\)
range: \(
\{-3, -1, 1, 2, 3\}
\)

27. not a function

28. 1.25

29. [Graph of a linear function]

Yes, because each element of the domain is paired with exactly one element of the range.

30a. yes, because it can be represented by an equation in the form

30b. \( y = 18x \).