CONSUMABLE WORKBOOKS  Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-660292-3</td>
<td>978-0-07-660292-6</td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660291-5</td>
<td>978-0-07-660291-9</td>
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<tr>
<td>Spanish Version</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660294-X</td>
<td>978-0-07-660294-0</td>
</tr>
</tbody>
</table>

Answers For Workbooks  The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED  All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

Spanish Assessment Masters  (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 6 Test Form 2A and Form 2C.
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Teacher’s Guide to Using the
Chapter 6 Resource Masters

The Chapter 6 Resource Masters includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 6-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Guided Practice exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the Chapter 6 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 9 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests
- Form 1 contains multiple-choice questions and is intended for use with below grade level students.
- Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers
- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.
# 6 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>consistent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dependent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>elimination</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inconsistent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>independent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>substitution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>system of equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>system of inequalities</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Anticipation Guide

## Solving Systems of Linear Equations

### Step 1

**Before you begin Chapter 6**
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A solution of a system of equations is any ordered pair that satisfies one of the equations</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>A system of equations of parallel lines will have no solutions.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>A system of equations of two perpendicular lines will have infinitely many solutions.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>It is not possible to have exactly two solutions to a system of linear equations</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>The most accurate way to solve a system of equations is to graph the equations to see where they intersect.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>To solve a system of equations, such as ( 2x - y = 21 ) and ( 3y = 2x - 6 ), by substitution, solve one of the equations for one variable and substitute the result into the other equation.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>When solving a system of equations, a result that is a true statement, such as (-5 = -5), means the equations do not share a common solution.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Adding the equations ( 3x - 4y = 8 ) and ( 2x + 4y = 7 ) results in a 0 coefficient for ( y ).</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>The equation ( 7x - 2y = 12 ) can be multiplied by 2 so that the coefficient of ( y ) is (-4).</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>The result of multiplying (-7x - 3y = 11) by (-3) is (-1x + 9y = 11).</td>
<td></td>
</tr>
</tbody>
</table>

### Step 2

**After you complete Chapter 6**
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
# Ejercicios preparatorios

## Resuelve sistemas de ecuaciones lineales

### Paso 1

**Antes de comenzar el Capítulo 6**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Una solución de un sistema de ecuaciones es cualquier par ordenado que satisface una de las ecuaciones</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Un sistema de ecuaciones de rectas paralelas no tendrá soluciones.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Un sistema de ecuaciones de dos rectas perpendiculares tendrá un número infinito de soluciones.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>No es posible tener exactamente dos soluciones para un sistema de ecuaciones lineales.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>La forma más precisa de resolver un sistema de ecuaciones es graficar las ecuaciones y ver dónde se intersecan.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Para resolver un sistema de ecuaciones, como $2x - y = 21$ y $3y = 2x - 6$, por sustitución, despeja una variable en una de la ecuaciones y reemplaza el resultado en la otra ecuación.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Cuando se resuelve un sistema de ecuaciones, un resultado que es un enunciado verdadero, como $-5 = -5$, significa que las ecuaciones no comparten una solución.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>El sumar las ecuaciones $3x - 4y = 8$ y $2x + 4y = 7$ resulta en un coeficiente de 0 para $y$.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>La ecuación $7x - 2y = 12$ se puede multiplicar por 2, de modo que el coeficiente de $y$ es $-4$.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>El resultado de multiplicar $-7x - 3y = 11$ por $-3$ es $-1x + 9y = 11$.</td>
<td></td>
</tr>
</tbody>
</table>

### Paso 2

**Después de completar el Capítulo 6**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
**6-1 Study Guide and Intervention**

**Graphing Systems of Equations**

**Possible Number of Solutions** Two or more linear equations involving the same variables form a **system of equations**. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. The table below summarizes information about systems of linear equations.

<table>
<thead>
<tr>
<th>Graph of a System</th>
<th>intersecting lines</th>
<th>same line</th>
<th>parallel lines</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[Diagram]</td>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
<tr>
<td>Number of Solutions</td>
<td>exactly one solution</td>
<td>infinitely many solutions</td>
<td>no solution</td>
</tr>
<tr>
<td>Terminology</td>
<td>consistent and independent</td>
<td>consistent and dependent</td>
<td>inconsistent</td>
</tr>
</tbody>
</table>

---

**Example** Use the graph at the right to determine whether each system is **consistent** or **inconsistent** and if it is **independent** or **dependent**.

a. \(y = -x + 2\)
   \(y = x + 1\)
   Since the graphs of \(y = -x + 2\) and \(y = x + 1\) intersect, there is one solution. Therefore, the system is consistent and independent.

b. \(y = -x + 2\)
   \(3x + 3y = -3\)
   Since the graphs of \(y = -x + 2\) and \(3x + 3y = -3\) are parallel, there are no solutions. Therefore, the system is inconsistent.

c. \(3x + 3y = -3\)
   \(y = -x - 1\)
   Since the graphs of \(3x + 3y = -3\) and \(y = -x - 1\) coincide, there are infinitely many solutions. Therefore, the system is consistent and dependent.

---

**Exercises**

Use the graph at the right to determine whether each system is **consistent** or **inconsistent** and if it is **independent** or **dependent**.

1. \(y = -x - 3\)
   \(y = x - 1\)

2. \(2x + 2y = -6\)
   \(y = -x - 3\)

3. \(y = -x - 3\)
   \(2x + 2y = 4\)

4. \(2x + 2y = -6\)
   \(3x + y = 3\)
Solve by Graphing  One method of solving a system of equations is to graph the equations on the same coordinate plane.

Example  Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. \( x + y = 2 \)
\( x - y = 4 \)

The graphs intersect. Therefore, there is one solution. The point \((3, -1)\) seems to lie on both lines. Check this estimate by replacing \(x\) with 3 and \(y\) with \(-1\) in each equation.

\[
\begin{align*}
x + y &= 2 \\
3 + (-1) &= 2 \checkmark
\end{align*}
\]

\[
\begin{align*}
x - y &= 4 \\
3 - (-1) &= 3 + 1 \text{ or } 4 \checkmark
\end{align*}
\]

The solution is \((3, -1)\).

b. \( y = 2x + 1 \)
\( 2y = 4x + 2 \)

The graphs coincide. Therefore there are infinitely many solutions.

Exercises
Graph each system and determine the number of solutions it has. If it has one solution, name it.

1. \( y = -2 \)
\( 3x - y = -1 \)

2. \( x = 2 \)
\( 2x + y = 1 \)

3. \( y = \frac{1}{2}x \)
\( x + y = 3 \)

4. \( 2x + y = 6 \)
\( 2x - y = -2 \)

5. \( 3x + 2y = 6 \)
\( 3x + 2y = -4 \)

6. \( 2y = -4x + 4 \)
\( y = -2x + 2 \)
Lesson 6-1

Skills Practice

Graphing Systems of Equations

Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.

1. \(y = x - 1\)  \(\quad y = -x + 1\)
2. \(x - y = -4\)  \(\quad y = x + 4\)
3. \(y = x + 4\)  \(\quad 2x - 2y = 2\)
4. \(y = 2x - 3\)  \(\quad 2x - 2y = 2\)

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

5. \(2x - y = 1\) \(\quad y = -3\)
6. \(x = 1\) \(\quad 2x + y = 4\)
7. \(3x + y = -3\) \(\quad 3x + y = 3\)
8. \(y = x + 2\) \(\quad x - y = -2\)
9. \(x + 3y = -3\) \(\quad x - 3y = -3\)
10. \(y - x = -1\) \(\quad x + y = 3\)
11. \(x - y = 3\) \(\quad x - 2y = 3\)
12. \(x + 2y = 4\) \(\quad y = -\frac{1}{2}x + 2\)
13. \(y = 2x + 3\) \(\quad 3y = 6x - 6\)
6-1 Practice

Graphing Systems of Equations

Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.

1. \(x + y = 3\)  \(x + y = -3\)
2. \(2x - y = -3\)  \(4x - 2y = -6\)
3. \(x + 3y = 3\)  \(x + y = -3\)
4. \(x + 3y = 3\)  \(2x - y = -3\)

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

5. \(3x - y = -2\)  \(3x - y = 0\)
6. \(y = 2x - 3\)  \(4x = 2y + 6\)
7. \(x + 2y = 3\)  \(3x - y = -5\)

8. BUSINESS Nick plans to start a home-based business producing and selling gourmet dog treats. He figures it will cost $20 in operating costs per week plus $0.50 to produce each treat. He plans to sell each treat for $1.50.

   a. Graph the system of equations \(y = 0.5x + 20\) and \(y = 1.5x\) to represent the situation.

   b. How many treats does Nick need to sell per week to break even?

9. SALES A used book store also started selling used CDs and videos. In the first week, the store sold 40 used CDs and videos, at $4.00 per CD and $6.00 per video. The sales for both CDs and videos totaled $180.00.

   a. Write a system of equations to represent the situation.

   b. Graph the system of equations.

   c. How many CDs and videos did the store sell in the first week?
1. **BUSINESS** The widget factory will sell a total of \( y \) widgets after \( x \) days according to the equation \( y = 200x + 300 \). The gadget factory will sell \( y \) gadgets after \( x \) days according to the equation \( y = 200x + 100 \). Look at the graph of the system of equations and determine whether it has no solution, one solution, or infinitely many solutions.

2. **ARCHITECTURE** An office building has two elevators. One elevator starts out on the 4th floor, 35 feet above the ground, and is descending at a rate of 2.2 feet per second. The other elevator starts out at ground level and is rising at a rate of 1.7 feet per second. Write a system of equations to represent the situation.

3. **FITNESS** Olivia and her brother William had a bicycle race. Olivia rode at a speed of 20 feet per second while William rode at a speed of 15 feet per second. To be fair, Olivia decided to give William a 150-foot head start. The race ended in a tie. How far away was the finish line from where Olivia started?

4. **AVIATION** Two planes are in flight near a local airport. One plane is at an altitude of 1000 meters and is ascending at a rate of 400 meters per minute. The second plane is at an altitude of 5900 meters and is descending at a rate of 300 meters per minute.

   a. Write a system of equations that represents the progress of each plane.

   b. Make a graph that represents the progress of each plane.
Chapter 6

6-1 Enrichment

Graphing a Trip

The distance formula, \( d = rt \), is used to solve many types of problems. If you graph an equation such as \( d = 50t \), the graph is a model for a car going at 50 mph. The time the car travels is \( t \); the distance in miles the car covers is \( d \). The slope of the line is the speed.

Suppose you drive to a nearby town and return. You average 50 mph on the trip out but only 25 mph on the trip home. The round trip takes 5 hours. How far away is the town?

The graph at the right represents your trip. Notice that the return trip is shown with a negative slope because you are driving in the opposite direction.

Solve each problem.

1. Estimate the answer to the problem in the above example. About how far away is the town?

Graph each trip and solve the problem.

2. An airplane has enough fuel for 3 hours of safe flying. On the trip out the pilot averages 200 mph flying against a headwind. On the trip back, the pilot averages 250 mph. How long a trip out can the pilot make?

3. You drive to a town 100 miles away. On the trip out you average 25 mph. On the trip back you average 50 mph. How many hours do you spend driving?

4. You drive at an average speed of 50 mph to a discount shopping plaza, spend 2 hours shopping, and then return at an average speed of 25 mph. The entire trip takes 8 hours. How far away is the shopping plaza?
Graphing Calculator Activity

Solution to a System of Linear Equations

A graphing calculator can be used to solve a system of linear equations graphically. The solution of a system of linear equations can be found by using the TRACE feature or by using the intersect command under the CALC menu.

Example

Solve each system of linear equations.

a. \( x + y = 0 \)
   \[ x - y = -4 \]
   Using TRACE: Solve each equation for \( y \) and enter each equation into \( Y= \). Then graph using Zoom 8: ZInteger. Use TRACE to find the solution.
   Keystrokes: \[ Y= (-) X,T,0,n \text{ ENTER} X,T,0,n \text{ ENTER} 4 \text{ ZOOM 6} \]
   The solution is \((-2, 2)\).

b. \( 2x + y = 4 \)
   \[ 4x + 3y = 3 \]
   Using CALC: Solve each equation for \( y \), enter each into the calculator, and graph. Use CALC to determine the solution.
   Keystrokes: \[ Y= (-) 2 \text{ ENTER} (\quad -4 \div 3) \text{ ENTER} (\quad 1 \text{ ZOOM 6 2nd [CALC] 5 ENTER ENTER ENTER} \text{ ENTER} \]
   To change the x-value to a fraction, press \[ \text{2nd [QUIT] X,T,0,n MATH ENTER ENTER} \]
   The solution is \((4.5, -5)\) or \((\frac{9}{2}, -5)\).

Exercises

Solve each system of linear equations.

1. \( y = 2 \)
   \[ 5x + 4y = 18 \]

2. \( y = -x + 3 \)
   \[ y = x + 1 \]

3. \( x + y = -1 \)
   \[ 2x - y = -8 \]

4. \( -3x + y = 10 \)
   \[ -x + 2y = 0 \]

5. \( -4x + 3y = 10 \)
   \[ 7x + y = 20 \]

6. \( 5x + 3y = 11 \)
   \[ x - 5y = 5 \]

7. \( 3x - 2y = -4 \)
   \[ -4x + 3y = 5 \]

8. \( 3x + 2y = 4 \)
   \[ -6x - 4y = -8 \]

9. \( 4x - 5y = 0 \)
   \[ 6x - 5y = 10 \]
**Example 1** Use substitution to solve the system of equations.

\[ \begin{align*}
y &= 2x \\
4x - y &= -4
\end{align*} \]

Substitute \(2x\) for \(y\) in the second equation.

\[ \begin{align*}
4x - y &= -4 & \text{Second equation} \\
4x - 2x &= -4 & y = 2x \\
2x &= -4 & \text{Combine like terms.} \\
x &= -2 & \text{Divide each side by 2 and simplify.}
\end{align*} \]

Use \(y = 2x\) to find the value of \(y\).

\[ \begin{align*}
y &= 2x & \text{First equation} \\
y &= 2(-2) & x = -2 \\
y &= -4 & \text{Simplify.}
\end{align*} \]

The solution is \((-2, -4)\).

**Example 2** Solve for one variable, then substitute.

\[ \begin{align*}
x + 3y &= 7 \\
2x - 4y &= -6
\end{align*} \]

Solve the first equation for \(x\) since the coefficient of \(x\) is 1.

\[ \begin{align*}
x + 3y &= 7 & \text{First equation} \\
x + 3y - 3y &= 7 - 3y & \text{Subtract 3y from each side.} \\
x &= 7 - 3y & \text{Simplify.}
\end{align*} \]

Find the value of \(y\) by substituting \(7 - 3y\) for \(x\) in the second equation.

\[ \begin{align*}
2x - 4y &= -6 & \text{Second equation} \\
2(7 - 3y) - 4y &= -6 & x = 7 - 3y \\
14 - 6y - 4y &= -6 & \text{Distributive Property} \\
14 - 10y &= -6 & \text{Combine like terms.} \\
14 - 10y - 14 &= -6 - 14 & \text{Subtract 14 from each side.} \\
-10y &= -20 & \text{Simplify.} \\
y &= 2 & \text{Divide each side by -10 and simplify.}
\end{align*} \]

Use \(y = 2\) to find the value of \(x\).

\[ \begin{align*}
x &= 7 - 3y \\
x &= 7 - 3(2) \\
x &= 1
\end{align*} \]

The solution is \((1, 2)\).

**Exercises**

Use substitution to solve each system of equations.

1. \(y = 4x\) 
   \[3x - y = 1\]
2. \(x = 2y\) 
   \[y = x - 2\]
3. \(x = 2y - 3\) 
   \[x = 2y + 4\]
4. \(x - 2y = -1\) 
   \[3y = x + 4\]
5. \(x - 4y = 1\) 
   \[2x - 8y = 2\]
6. \(x + 2y = 0\) 
   \[3x + 4y = 4\]
7. \(2b = 6a - 14\) 
   \[3a - b = 7\]
8. \(x + y = 16\) 
   \[2y = -2x + 2\]
9. \(y = -x + 3\) 
   \[2y + 2x = 4\]
10. \(x = 2y\) 
    \[0.25x + 0.5y = 10\]
11. \(x - 2y = -5\) 
    \[x + 2y = -1\]
12. \(-0.2x + y = 0.5\) 
    \[0.4x + y = 1.1\]
6-2  Study Guide and Intervention  (continued)

Substitution

Solve Real-World Problems  Substitution can also be used to solve real-world problems involving systems of equations. It may be helpful to use tables, charts, diagrams, or graphs to help you organize data.

Example  CHEMISTRY  How much of a 10% saline solution should be mixed with a 20% saline solution to obtain 1000 milliliters of a 12% saline solution?

Let \( s \) = the number of milliliters of 10% saline solution.
Let \( t \) = the number of milliliters of 20% saline solution.
Use a table to organize the information.

<table>
<thead>
<tr>
<th></th>
<th>10% saline</th>
<th>20% saline</th>
<th>12% saline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total milliliters</td>
<td>( s )</td>
<td>( t )</td>
<td>1000</td>
</tr>
<tr>
<td>Milliliters of saline</td>
<td>0.10s</td>
<td>0.20t</td>
<td>0.12(1000)</td>
</tr>
</tbody>
</table>

Write a system of equations.
\[
s + t = 1000 \\
0.10s + 0.20t = 0.12(1000)
\]

Use substitution to solve this system.
\[
s + t = 1000 \\
s = 1000 - t \\
0.10s + 0.20t = 0.12(1000) \\
0.10(1000 - t) + 0.20t = 0.12(1000) \\
100 - 0.10t + 0.20t = 0.12(1000) \\
100 + 0.10t = 0.12(1000) \\
0.10t = 20 \\
\frac{0.10t}{0.10} = \frac{20}{0.10} \\
t = 200
\]

800 milliliters of 10% solution and 200 milliliters of 20% solution should be used.

Exercises

1. SPORTS  At the end of the 2007–2008 football season, 38 Super Bowl games had been played with the current two football leagues, the American Football Conference (AFC) and the National Football Conference (NFC). The NFC won two more games than the AFC. How many games did each conference win?

2. CHEMISTRY  A lab needs to make 100 gallons of an 18% acid solution by mixing a 12% acid solution with a 20% solution. How many gallons of each solution are needed?

3. GEOMETRY  The perimeter of a triangle is 24 inches. The longest side is 4 inches longer than the shortest side, and the shortest side is three-fourths the length of the middle side. Find the length of each side of the triangle.
6-2 Skills Practice

Substitution

Use substitution to solve each system of equations.

1. \( y = 4x \)
   \( x + y = 5 \)

2. \( y = 2x \)
   \( x + 3y = -14 \)

3. \( y = 3x \)
   \( 2x + y = 15 \)

4. \( x = -4y \)
   \( 3x + 2y = 20 \)

5. \( y = x - 1 \)
   \( x + y = 3 \)

6. \( x = y - 7 \)
   \( x + 8y = 2 \)

7. \( y = 4x - 1 \)
   \( y = 2x - 5 \)

8. \( y = 3x + 8 \)
   \( 5x + 2y = 5 \)

9. \( 2x - 3y = 21 \)
   \( y = 3 - x \)

10. \( y = 5x - 8 \)
    \( 4x + 3y = 33 \)

11. \( x + 2y = 13 \)
    \( 3x - 5y = 6 \)

12. \( x + 5y = 4 \)
    \( 3x + 15y = -1 \)

13. \( 3x - y = 4 \)
    \( 2x - 3y = -9 \)

14. \( x + 4y = 8 \)
    \( 2x - 5y = 29 \)

15. \( x - 5y = 10 \)
    \( 2x - 10y = 20 \)

16. \( 5x - 2y = 14 \)
    \( 2x - y = 5 \)

17. \( 2x + 5y = 38 \)
    \( x - 3y = -3 \)

18. \( x - 4y = 27 \)
    \( 3x + y = -23 \)

19. \( 2x + 2y = 7 \)
    \( x - 2y = -1 \)

20. \( 2.5x + y = -2 \)
    \( 3x + 2y = 0 \)
Use substitution to solve each system of equations.

1. \( y = 6x \\
   2x + 3y = -20 \)

2. \( x = 3y \\
   3x - 5y = 12 \)

3. \( x = 2y + 7 \\
   x = y + 4 \)

4. \( y = 2x - 2 \\
   y = x + 2 \)

5. \( y = 2x + 6 \\
   2x - y = 2 \)

6. \( 3x + y = 12 \\
   y = -x - 2 \)

7. \( x + 2y = 13 \\
   -2x - 3y = -18 \)

8. \( x - 2y = 3 \\
   4x - 8y = 12 \)

9. \( x - 5y = 36 \\
   2x + y = -16 \)

10. \( 2x - 3y = -24 \\
    x + 6y = 18 \)

11. \( x + 14y = 84 \\
    2x - 7y = -7 \)

12. \( 0.3x - 0.2y = 0.5 \\
    x - 2y = -5 \)

13. \( 0.5x + 4y = -1 \\
    x + 2.5y = 3.5 \)

14. \( 3x - 2y = 11 \\
    x - \frac{1}{2}y = 4 \)

15. \( \frac{1}{2}x + 2y = 12 \)

16. \( \frac{1}{3}x - y = 3 \\
    2x + y = 25 \)

17. \( 4x - 5y = -7 \\
    y = 5x \)

18. \( x + 3y = -4 \\
    2x + 6y = 5 \)

19. **EMPLOYMENT** Kenisha sells athletic shoes part-time at a department store. She can earn either $500 per month plus a 4% commission on her total sales, or $400 per month plus a 5% commission on total sales.
   
   a. Write a system of equations to represent the situation.

   b. What is the total price of the athletic shoes Kenisha needs to sell to earn the same income from each pay scale?

   c. Which is the better offer?

20. **MOVIE TICKETS** Tickets to a movie cost $7.25 for adults and $5.50 for students. A group of friends purchased 8 tickets for $52.75.
   
   a. Write a system of equations to represent the situation.

   b. How many adult tickets and student tickets were purchased?
1. **BUSINESS** Mr. Randolph finds that the supply and demand for gasoline at his station are generally given by the following equations.

\[
\begin{align*}
    x - y &= -2 \\
    x + y &= 10
\end{align*}
\]

Use substitution to find the equilibrium point where the supply and demand lines intersect.

2. **GEOMETRY** The measures of complementary angles have a sum of 90 degrees. Angle A and angle B are complementary, and their measures have a difference of 20°. What are the measures of the angles?

3. **MONEY** Harvey has some $1 bills and some $5 bills. In all, he has 6 bills worth $22. Let \( x \) be the number of $1 bills and let \( y \) be the number of $5 bills. Write a system of equations to represent the information and use substitution to determine how many bills of each denomination Harvey has.

4. **POPULATION** Sanjay is researching population trends in South America. He found that the population of Ecuador to increased by 1,000,000 and the population of Chile to increased by 600,000 from 2004 to 2009. The table displays the information he found.

<table>
<thead>
<tr>
<th>Country</th>
<th>2004 Population</th>
<th>5-Year Population Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ecuador</td>
<td>13,000,000</td>
<td>+1,000,000</td>
</tr>
<tr>
<td>Chile</td>
<td>16,000,000</td>
<td>+600,000</td>
</tr>
</tbody>
</table>

*Source: World Almanac*

If the population growth for each country continues at the same rate, in what year are the populations of Ecuador and Chile predicted to be equal?

5. **CHEMISTRY** Shelby and Calvin are doing a chemistry experiment. They need 5 ounces of a solution that is 65% acid and 35% distilled water. There is no undiluted acid in the chemistry lab, but they do have two flasks of diluted acid: Flask A contains 70% acid and 30% distilled water. Flask B contains 20% acid and 80% distilled water.

a. Write a system of equations that Shelby and Calvin could use to determine how many ounces they need to pour from each flask to make their solution.

b. Solve your system of equations. How many ounces from each flask do Shelby and Calvin need?
Intersection of Two Parabolas

Substitution can be used to find the intersection of two parabolas. Replace the
y-value in one of the equations with the y-value in terms of x from the other equation.

Example
Find the intersection of the two parabolas.
\[ y = x^2 + 5x + 6 \]
\[ y = x^2 + 4x + 3 \]

Graph the equations.

From the graph, notice that the two graphs intersect in one point.

Use substitution to solve for the point of intersection.
\[ x^2 + 5x + 6 = x^2 + 4x + 3 \]
\[ 5x + 6 = 4x + 3 \] Subtract \( x^2 \) from each side.
\[ x + 6 = 3 \] Subtract 4x from each side.
\[ x = -3 \] Subtract 6 from each side.

The graphs intersect at \( x = -3 \).

Replace \( x \) with \(-3\) in either equation to find the y-value.
\[ y = x^2 + 5x + 6 \] Original equation
\[ y = (-3)^2 + 5(-3) + 6 \] Replace \( x \) with \(-3\).
\[ y = 9 - 15 + 6 \text{ or } 0 \] Simplify.

The point of intersection is \((-3, 0)\).

Exercises

Use substitution to find the point of intersection of the graphs of each pair of equations.

1. \[ y = x^2 + 8x + 7 \]
   \[ y = x^2 + 2x + 1 \]

2. \[ y = x^2 + 6x + 8 \]
   \[ y = x^2 + 4x + 4 \]

3. \[ y = x^2 + 5x + 6 \]
   \[ y = x^2 + 7x + 6 \]
Elimination Using Addition Using Addition and Subtraction

Elimination Using Addition In systems of equations in which the coefficients of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called elimination.

**Example 1** Use elimination to solve the system of equations.

\[ x - 3y = 7 \]
\[ 3x + 3y = 9 \]

Write the equations in column form and add to eliminate y.

\[
\begin{align*}
4x &= 16 \\
x &= 4
\end{align*}
\]

Solve for x.

\[
\frac{4x}{4} = \frac{16}{4}
\]

\[ x = 4 \]

Substitute 4 for x in either equation and solve for y.

\[
\begin{align*}
4 - 3y &= 7 \\
4 - 3y - 4 &= 7 - 4 \\
-3y &= 3 \\
\frac{-3y}{-3} &= \frac{3}{-3} \\
y &= -1
\end{align*}
\]

The solution is (4, -1).

**Example 2** The sum of two numbers is 70 and their difference is 24. Find the numbers.

Let x represent one number and y represent the other number.

\[
\begin{align*}
x + y &= 70 \\
(+) x - y &= 24
\end{align*}
\]

\[
\begin{align*}
2x &= 94 \\
\frac{2x}{2} &= \frac{94}{2} \\
x &= 47
\end{align*}
\]

Substitute 47 for x in either equation.

\[
\begin{align*}
47 + y &= 70 \\
47 + y - 47 &= 70 - 47 \\
y &= 23
\end{align*}
\]

The numbers are 47 and 23.

**Exercises**

Use elimination to solve each system of equations.

1. \[ x + y = -4 \]
   \[ x - y = 2 \]
2. \[ 2x - 3y = 14 \]
   \[ x + 3y = -11 \]
3. \[ 3x - y = -9 \]
   \[ -3x - 2y = 0 \]
4. \[ -3x - 4y = -1 \]
   \[ 3x - y = -4 \]
5. \[ 3x + y = 4 \]
   \[ 2x - y = 6 \]
6. \[ -2x + 2y = 9 \]
   \[ 2x - y = -6 \]
7. \[ 2x + 2y = -2 \]
   \[ 3x - 2y = 12 \]
8. \[ 4x - 2y = -1 \]
   \[ -4x + 4y = -2 \]
9. \[ x - y = 2 \]
   \[ x + y = -3 \]
10. \[ 2x - 3y = 12 \]
    \[ 4x + 3y = 24 \]
11. \[ -0.2x + y = 0.5 \]
    \[ 0.2x + 2y = 1.6 \]
12. \[ 0.1x + 0.3y = 0.9 \]
    \[ 0.1x - 0.3y = 0.2 \]

13. Rema is older than Ken. The difference of their ages is 12 and the sum of their ages is 50. Find the age of each.

14. The sum of the digits of a two-digit number is 12. The difference of the digits is 2. Find the number if the units digit is larger than the tens digit.
Elimination Using Subtraction  In systems of equations where the coefficients of the $x$ or $y$ terms are the same, solve the system by subtracting the equations.

**Example**  Use elimination to solve the system of equations.

\[
\begin{align*}
2x - 3y &= 11 \\
5x - 3y &= 14
\end{align*}
\]

Write the equations in column form and subtract.

\[
\begin{align*}
2x - 3y &= 11 \\
(-5x + 3y) &= 14 \\
-3x &= 3
\end{align*}
\]

Subtract the two equations. $y$ is eliminated.

\[
\begin{align*}
-3x &= 3 \\
x &= 1
\end{align*}
\]

Divide each side by $-3$.

\[
\begin{align*}
3y &= 9 \\
y &= -3
\end{align*}
\]

Divide each side by $3$.

The solution is $(1, -3)$.

**Exercises**

Use elimination to solve each system of equations.

1. \[
\begin{align*}
6x + 5y &= 4 \\
6x - 7y &= -20
\end{align*}
\]

2. \[
\begin{align*}
3m - 4n &= -14 \\
3m + 2n &= -2
\end{align*}
\]

3. \[
\begin{align*}
3a + b &= 1 \\
a + b &= 3
\end{align*}
\]

4. \[
\begin{align*}
-3x - 4y &= -23 \\
-3x + y &= 2
\end{align*}
\]

5. \[
\begin{align*}
x - 3y &= 11 \\
2x - 3y &= 16
\end{align*}
\]

6. \[
\begin{align*}
x - 2y &= 6 \\
x + y &= 3
\end{align*}
\]

7. \[
\begin{align*}
2a - 3b &= -13 \\
2a + 2b &= 7
\end{align*}
\]

8. \[
\begin{align*}
4x + 2y &= 6 \\
4x + 4y &= 10
\end{align*}
\]

9. \[
\begin{align*}
5x - y &= 6 \\
5x + 2y &= 3
\end{align*}
\]

10. \[
\begin{align*}
6x - 3y &= 12 \\
4x - 3y &= 24
\end{align*}
\]

11. \[
\begin{align*}
x + 2y &= 3.5 \\
x - 3y &= -9
\end{align*}
\]

12. \[
\begin{align*}
0.2x + y &= 0.7 \\
0.2x + 2y &= 1.2
\end{align*}
\]

13. The sum of two numbers is 70. One number is ten more than twice the other number. Find the numbers.

14. **GEOMETRY** Two angles are supplementary. The measure of one angle is $10^\circ$ more than three times the other. Find the measure of each angle.
Use elimination to solve each system of equations.

1. \(x - y = 1\)  
   \(x + y = 3\)

2. \(-x + y = 1\)  
   \(x + y = 11\)

3. \(x + 4y = 11\)  
   \(x - 6y = 11\)

4. \(-x + 3y = 6\)  
   \(x + 3y = 18\)

5. \(3x + 4y = 19\)  
   \(3x + 6y = 33\)

6. \(x + 4y = -8\)  
   \(x - 4y = -8\)

7. \(3x + 4y = 2\)  
   \(4x - 4y = 12\)

8. \(3x - y = -1\)  
   \(-3x - y = 5\)

9. \(2x - 3y = 9\)  
   \(-5x - 3y = 30\)

10. \(x - y = 4\)  
    \(2x + y = -4\)

11. \(3x - y = 26\)  
    \(-2x - y = -24\)

12. \(5x - y = -6\)  
    \(-x + y = 2\)

13. \(6x - 2y = 32\)  
    \(4x - 2y = 18\)

14. \(3x + 2y = -19\)  
    \(-3x - 5y = 25\)

15. \(7x + 4y = 2\)  
    \(7x + 2y = 8\)

16. \(2x - 5y = -28\)  
    \(4x + 5y = 4\)

17. The sum of two numbers is 28 and their difference is 4. What are the numbers?

18. Find the two numbers whose sum is 29 and whose difference is 15.

19. The sum of two numbers is 24 and their difference is 2. What are the numbers?

20. Find the two numbers whose sum is 54 and whose difference is 4.

21. Two times a number added to another number is 25. Three times the first number minus the other number is 20. Find the numbers.
6-3 Practice

Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

1. \[ x - y = 1 \] \[ x + y = -9 \]
2. \[ p + q = -2 \] \[ p - q = 8 \]
3. \[ 4x + y = 23 \] \[ 3x - y = 12 \]
4. \[ 2x + 5y = -3 \] \[ 2x + 2y = 6 \]
5. \[ 3x + 2y = -1 \] \[ 4x + 2y = -6 \]
6. \[ 5x + 3y = 22 \] \[ 5x - 2y = 2 \]
7. \[ 5x + 2y = 7 \] \[ -2x + 2y = -14 \]
8. \[ 3x - 9y = -12 \] \[ 3x - 15y = -6 \]
9. \[ -4c - 2d = -2 \] \[ 2c - 2d = -14 \]
10. \[ 2x - 6y = 6 \] \[ 2x + 3y = 24 \]
11. \[ 7x + 2y = 2 \] \[ 7x - 2y = -30 \]
12. \[ 4.25x - 1.28y = -9.2 \] \[ x + 1.28y = 17.6 \]
13. \[ 2x + 4y = 10 \] \[ x - 4y = -2.5 \]
14. \[ 2.5x + y = 10.7 \] \[ 2.5x + 2y = 12.9 \]
15. \[ 6m - 8n = 3 \] \[ 2m - 8n = -3 \]
16. \[ 4a + b = 2 \] \[ 4a + 3b = 10 \]
17. \[ -\frac{1}{3}x - \frac{4}{3}y = -2 \] \[ \frac{1}{3}x - \frac{2}{3}y = 4 \]
18. \[ \frac{3}{4}x - \frac{1}{2}y = 8 \] \[ \frac{3}{2}x + \frac{1}{2}y = 19 \]

19. The sum of two numbers is 41 and their difference is 5. What are the numbers?
20. Four times one number added to another number is 36. Three times the first number minus the other number is 20. Find the numbers.
21. One number added to three times another number is 24. Five times the first number added to three times the other number is 36. Find the numbers.

22. LANGUAGES  English is spoken as the first or primary language in 78 more countries than Farsi is spoken as the first language. Together, English and Farsi are spoken as a first language in 130 countries. In how many countries is English spoken as the first language? In how many countries is Farsi spoken as the first language?

23. DISCOUNTS At a sale on winter clothing, Cody bought two pairs of gloves and four hats for $43.00. Tori bought two pairs of gloves and two hats for $30.00. What were the prices for the gloves and hats?
6-3 Word Problem Practice

Elimination Using Addition and Subtraction

1. NUMBER FUN Ms. Simms, the sixth grade math teacher, gave her students this challenge problem.
   Twice a number added to another number is 15. The sum of the two numbers is 11.
   Lorenzo, an algebra student who was Ms. Simms aide, realized he could solve the problem by writing the following system of equations.
   \[2x + y = 15\]
   \[x + y = 11\]
   Use the elimination method to solve the system and find the two numbers.

2. GOVERNMENT The Texas State Legislature is comprised of state senators and state representatives. The sum of the number of senators and representatives is 181. There are 119 more representatives than senators. How many senators and how many representatives make up the Texas State Legislature?

3. RESEARCH Melissa wondered how much it would cost to send a letter by mail in 1990, so she asked her father. Rather than answer directly, Melissa’s father gave her the following information. It would have cost $3.70 to send 13 postcards and 7 letters, and it would have cost $2.65 to send 6 postcards and 7 letters. Use a system of equations and elimination to find how much it cost to send a letter in 1990.

4. SPORTS As of 2010, the New York Yankees had won more World Series Championships than any other team. In fact, the Yankees had won 3 fewer than 3 times the number of World Series championships won by the second most-winning team, the St. Louis Cardinals. The sum of the two teams’ World Series championships is 37. How many times has each team won the World Series?

5. BASKETBALL In 2005, the average ticket prices for Dallas Mavericks games and Boston Celtics games are shown in the table below. The change in price is from the 2004 season to the 2005 season.

<table>
<thead>
<tr>
<th>Team</th>
<th>Average Ticket Price</th>
<th>Change in Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dallas</td>
<td>$53.60</td>
<td>$0.53</td>
</tr>
<tr>
<td>Boston</td>
<td>$55.93</td>
<td>$1.08</td>
</tr>
</tbody>
</table>

**Source:** Team Marketing Report

a. Assume that tickets continue to change at the same rate each year after 2005. Let \(x\) be the number of years after 2005, and \(y\) be the price of an average ticket. Write a system of equations to represent the information in the table.

b. In how many years will the average ticket price for Dallas approximately equal that of Boston?
Systems of equations can involve more than 2 equations and 2 variables. It is possible to solve a system of 3 equations and 3 variables using elimination.

### Example

Solve the following system.

\[
\begin{align*}
  x + y + z &= 6 \\
  3x - y + z &= 8 \\
  x - z &= 2
\end{align*}
\]

**Step 1:** Use elimination to get rid of the \( y \) in the first two equations.

\[
\begin{align*}
  x + y + z &= 6 \\
  3x - y + z &= 8 \\
  \hline
  4x + 2z &= 14
\end{align*}
\]

**Step 2:** Use the equation you found in step 1 and the third equation to eliminate the \( z \).

\[
\begin{align*}
  4x + 2z &= 14 \\
  x - z &= 2 \\
  \hline
  6x &= 18
\end{align*}
\]

So, \( x = 3 \).

**Step 3:** Replace \( x \) with 3 in the third original equation to determine \( z \).

\[
3 - z = 2, \text{ so } z = 1.
\]

**Step 4:** Replace \( x \) with 3 and \( z \) with 1 in either of the first two original equation to determine the value of \( y \).

\[
3 + y + 1 = 6 \text{ or } 4 + y = 6. \quad \text{So, } y = 2.
\]

So, the solution to the system of equations is \((3, 2, 1)\).

### Exercises

Solve each system of equations.

1. \[
\begin{align*}
  3x + 2y + z &= 42 \\
  2y + z + 12 &= 3x \\
  x - 3y &= 0
\end{align*}
\]

2. \[
\begin{align*}
  x + y + z &= -3 \\
  2x + 3y + 5z &= -4 \\
  2y - z &= 4
\end{align*}
\]

3. \[
\begin{align*}
  x + y + z &= 7 \\
  x + 2y + z &= 10 \\
  2y + z &= 5
\end{align*}
\]
**6-4 Study Guide and Intervention**

**Elimination Using Multiplication**

Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

### Example 1

Use elimination to solve the system of equations.

\[
\begin{align*}
x + 10y &= 3 \\
4x + 5y &= 5
\end{align*}
\]

If you multiply the second equation by \(-2\), you can eliminate the \(y\) terms.

\[
\begin{align*}
x + 10y &= 3 \\
(+) \quad -8x - 10y &= -10 \\
\hline
-7x &= -7 \\
\hline
x &= 1
\end{align*}
\]

Substitute 1 for \(x\) in either equation.

\[
\begin{align*}
1 + 10y &= 3 \\
10y &= 2 \\
y &= \frac{1}{5}
\end{align*}
\]

The solution is \((1, \frac{1}{5})\).

### Example 2

Use elimination to solve the system of equations.

\[
\begin{align*}
3x - 2y &= -7 \\
2x - 5y &= 10
\end{align*}
\]

If you multiply the first equation by 2 and the second equation by \(-3\), you can eliminate the \(x\) terms.

\[
\begin{align*}
6x - 4y &= -14 \\
(+) \quad -6x + 15y &= -30 \\
\hline
11y &= -44 \\
\hline
y &= -4
\end{align*}
\]

Substitute \(-4\) for \(y\) in either equation.

\[
\begin{align*}
3x - 2(-4) &= -7 \\
3x + 8 &= -7 \\
3x + 8 - 8 &= -7 - 8 \\
x &= -5
\end{align*}
\]

The solution is \((-5, -4)\).

### Exercises

Use elimination to solve each system of equations.

1. \(2x + 3y = 6\)  
   \(x + 2y = 5\)
2. \(2m + 3n = 4\)  
   \(-m + 2n = 5\)
3. \(3a - b = 2\)  
   \(a + 2b = 3\)
4. \(4x + 5y = 6\)  
   \(6x - 7y = -20\)
5. \(4x - 3y = 22\)  
   \(2x - y = 10\)
6. \(3x - 4y = -4\)  
   \(x + 3y = -10\)
7. \(4x - y = 9\)  
   \(5x + 2y = 8\)
8. \(4a - 3b = -8\)  
   \(2a + 2b = 3\)
9. \(2x + 2y = 5\)  
   \(4x - 4y = 10\)
10. \(6x - 4y = -8\)  
    \(4x + 2y = -3\)
11. \(4x + 2y = -5\)  
    \(-2x - 4y = 1\)
12. \(2x + y = 3.5\)  
    \(-x + 2y = 2.5\)

13. **GARDENING** The length of Sally’s garden is 4 meters greater than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally’s garden?
6-4 Study Guide and Intervention (continued)

Elimination Using Multiplication

Solve Real-World Problems Sometimes it is necessary to use multiplication before elimination in real-world problems.

Example CANOEING During a canoeing trip, it takes Raymond 4 hours to paddle 12 miles upstream. It takes him 3 hours to make the return trip paddling downstream. Find the speed of the canoe in still water.

Read You are asked to find the speed of the canoe in still water.

Solve Let \( c \) = the rate of the canoe in still water.
Let \( w \) = the rate of the water current.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d )</th>
<th>( r \times t = d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Against the Current</td>
<td>( c - w )</td>
<td>4</td>
<td>12</td>
<td>( (c - w)4 = 12 )</td>
</tr>
<tr>
<td>With the Current</td>
<td>( c + w )</td>
<td>3</td>
<td>12</td>
<td>( (c + w)3 = 12 )</td>
</tr>
</tbody>
</table>

So, our two equations are \( 4c - 4w = 12 \) and \( 3c + 3w = 12 \).

Use elimination with multiplication to solve the system. Since the problem asks for \( c \), eliminate \( w \).

\[
\begin{align*}
4c - 4w &= 12 & \Rightarrow & \text{Multiply by 3} & 12c - 12w &= 36 \\
3c + 3w &= 12 & \Rightarrow & \text{Multiply by 4} & 12c + 12w &= 48
\end{align*}
\]

\[
\frac{24c}{24} = \frac{84}{24} \quad \text{Divide each side by 24.}
\]
\[
c = 3.5
\]

So, the rate of the canoe in still water is 3.5 miles per hour.

Exercises

1. FLIGHT An airplane traveling with the wind flies 450 miles in 2 hours. On the return trip, the plane takes 3 hours to travel the same distance. Find the speed of the airplane if the wind is still.

2. FUNDRAISING Benji and Joel are raising money for their class trip by selling gift wrapping paper. Benji raises $39 by selling 5 rolls of red wrapping paper and 2 rolls of foil wrapping paper. Joel raises $57 by selling 3 rolls of red wrapping paper and 6 rolls of foil wrapping paper. For how much are Benji and Joel selling each roll of red and foil wrapping paper?
6-4  Skills Practice

Elimination Using Multiplication

Use elimination to solve each system of equations.

1. \(x + y = -9\)
   \(5x - 2y = 32\)

2. \(3x + 2y = -9\)
   \(x - y = -13\)

3. \(2x + 5y = 3\)
   \(-x + 3y = -7\)

4. \(2x + y = 3\)
   \(-4x - 4y = -8\)

5. \(4x - 2y = -14\)
   \(3x - y = -8\)

6. \(2x + y = 0\)
   \(5x + 3y = 2\)

7. \(5x + 3y = -10\)
   \(3x + 5y = -6\)

8. \(2x + 3y = 14\)
   \(3x - 4y = 4\)

9. \(2x - 3y = 21\)
   \(5x - 2y = 25\)

10. \(3x + 2y = -26\)
    \(4x - 5y = -4\)

11. \(3x - 6y = -3\)
    \(2x + 4y = 30\)

12. \(5x + 2y = -3\)
    \(3x + 3y = 9\)

13. Two times a number plus three times another number equals 13. The sum of the two numbers is 7. What are the numbers?

14. Four times a number minus twice another number is \(-16\). The sum of the two numbers is \(-1\). Find the numbers.

15. FUNDRAISING Trisha and Byron are washing and vacuuming cars to raise money for a class trip. Trisha raised $38 washing 5 cars and vacuuming 4 cars. Byron raised $28 by washing 4 cars and vacuuming 2 cars. Find the amount they charged to wash a car and vacuum a car.
Elimination Using Multiplication

Use elimination to solve each system of equations.

1. \(2x - y = -1\)
   \(3x - 2y = 1\)

2. \(5x - 2y = -10\)
   \(3x + 6y = 66\)

3. \(7x + 4y = -4\)
   \(5x + 8y = 28\)

4. \(2x - 4y = -22\)
   \(3x + 3y = 30\)

5. \(3x + 2y = -9\)
   \(5x - 3y = 4\)

6. \(4x - 2y = 32\)
   \(-3x - 5y = -11\)

7. \(3x + 4y = 27\)
   \(5x - 3y = 16\)

8. \(0.5x + 0.5y = -2\)
   \(x - 0.25y = 6\)

9. \(2x - \frac{3}{4}y = -7\)
   \(x + \frac{1}{2}y = 0\)

10. \(6x - 3y = 21\)
    \(2x + 2y = 22\)

11. \(3x + 2y = 11\)
    \(2x + 6y = -2\)

12. \(-3x + 2y = -15\)
    \(2x - 4y = 26\)

13. Eight times a number plus five times another number is \(-13\). The sum of the two numbers is 1. What are the numbers?

14. Two times a number plus three times another number equals 4. Three times the first number plus four times the other number is 7. Find the numbers.

15. **FINANCE** Gunther invested $10,000 in two mutual funds. One of the funds rose 6% in one year, and the other rose 9% in one year. If Gunther’s investment rose a total of $684 in one year, how much did he invest in each mutual fund?

16. **CANOEING** Laura and Brent paddled a canoe 6 miles upstream in four hours. The return trip took three hours. Find the rate at which Laura and Brent paddled the canoe in still water.

17. **NUMBER THEORY** The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 more than the original number. Find the number.
1. **SOCCER** Suppose a youth soccer field has a perimeter of 320 yards and its length measures 40 yards more than its width. Ms. Hughey asks her players to determine the length and width of their field. She gives them the following system of equations to represent the situation. Use elimination to solve the system to find the length and width of the field.

\[
2L + 2W = 320 \\
L - W = 40
\]

2. **SPORTS** The Fan Cost Index (FCI) tracks the average costs for attending sporting events, including tickets, drinks, food, parking, programs, and souvenirs. According to the FCI, a family of four would spend a total of $592.30 to attend two Major League Baseball (MLB) games and one National Basketball Association (NBA) game. The family would spend $691.31 to attend one MLB and two NBA games. Write and solve a system of equations to find the family's costs for each kind of game according to the FCI.

3. **ART** Mr. Santos, the curator of the children’s museum, recently made two purchases of clay and wood for a visiting artist to sculpt. Use the table to find the cost of each product per kilogram.

<table>
<thead>
<tr>
<th>Clay (kg)</th>
<th>Wood (kg)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>$35.50</td>
</tr>
<tr>
<td>3.5</td>
<td>6</td>
<td>$50.45</td>
</tr>
</tbody>
</table>

4. **TRAVEL** Antonio flies from Houston to Philadelphia, a distance of about 1340 miles. His plane travels with the wind and takes 2 hours and 20 minutes. At the same time, Paul is on a plane from Philadelphia to Houston. Since his plane is heading against the wind, Paul's flight takes 2 hours and 50 minutes. What was the speed of the wind in miles per hour?

5. **BUSINESS** Suppose you start a business assembling and selling motorized scooters. It costs you $1500 for tools and equipment to get started, and the materials cost $200 for each scooter. Your scooters sell for $300 each.

   a. Write and solve a system of equations representing the total costs and revenue of your business.

   b. Describe what the solution means in terms of the situation.

   c. Give an example of a reasonable number of scooters you could assemble and sell in order to make a profit, and find the profit you would make for that number of scooters.
George Washington Carver and Percy Julian

In 1990, George Washington Carver and Percy Julian became the first African Americans elected to the National Inventors Hall of Fame. Carver (1864–1943) was an agricultural scientist known worldwide for developing hundreds of uses for the peanut and the sweet potato. His work revitalized the economy of the southern United States because it was no longer dependent solely upon cotton. Julian (1898–1975) was a research chemist who became famous for inventing a method of making a synthetic cortisone from soybeans. His discovery has had many medical applications, particularly in the treatment of arthritis.

There are dozens of other African American inventors whose accomplishments are not as well known. Their inventions range from common household items like the ironing board to complex devices that have revolutionized manufacturing. The exercises that follow will help you identify just a few of these inventors and their inventions.

Match the inventors with their inventions by matching each system with its solution. (Not all the solutions will be used.)

1. Sara Boone  
   \[ x + y = 2 \]
   \[ x - y = 10 \]
   A. (1, 4) automatic traffic signal

2. Sarah Goode  
   \[ x = 2 - y \]
   \[ 2y + x = 9 \]
   B. (4, -2) eggbeater

3. Frederick M. Jones  
   \[ y = 2x + 6 \]
   \[ y = -x - 3 \]
   C. (-2, 3) fire extinguisher

4. J. L. Love  
   \[ 2x + 3y = 8 \]
   \[ 2x - y = -8 \]
   D. (-5, 7) folding cabinet bed

5. T. J. Marshall  
   \[ y - 3x = 9 \]
   \[ 2y + x = 4 \]
   E. (6, -4) ironing board

6. Jan Matzeliger  
   \[ y + 4 = 2x \]
   \[ 6x - 3y = 12 \]
   F. (-2, 4) pencil sharpener

7. Garrett A. Morgan  
   \[ 3x - 2y = -5 \]
   \[ 3y - 4x = 8 \]
   G. (-3, 0) portable x-ray machine

8. Norbert Rillieux  
   \[ 3x - y = 12 \]
   \[ y - 3x = 15 \]
   H. (2, -3) player piano

I. no solution evaporating pan for refining sugar

J. infinitely many solutions lasting (shaping) machine for manufacturing shoes

Enrichment
6-5 Study Guide and Intervention
Applying Systems of Linear Equations

Determine The Best Method  You have learned five methods for solving systems of linear equations: graphing, substitution, elimination using addition, elimination using subtraction, and elimination using multiplication. For an exact solution, an algebraic method is best.

Example  At a baseball game, Henry bought 3 hotdogs and a bag of chips for $14. Scott bought 2 hotdogs and a bag of chips for $10. Each of the boys paid the same price for their hotdogs, and the same price for their chips. The following system of equations can be used to represent the situation. Determine the best method to solve the system of equations. Then solve the system.

\[
\begin{align*}
3x + y &= 14 \\
2x + y &= 10
\end{align*}
\]

- Since neither the coefficients of \( x \) nor the coefficients of \( y \) are additive inverses, you cannot use elimination using addition.
- Since the coefficient of \( y \) in both equations is 1, you can use elimination using subtraction. You could also use the substitution method or elimination using multiplication.

The following solution uses elimination by subtraction to solve this system.

\[
\begin{align*}
3x + y &= 14 \\
2x + y &= 10
\end{align*}
\]

\[
\begin{align*}
(\times) \quad 2x + (-) y &= (-)10 \\
x &= 4
\end{align*}
\]

The variable \( y \) is eliminated.

\[
\begin{align*}
3(4) + y &= 14 \\
y &= 2
\end{align*}
\]

Solve for \( y \).

This means that hot dogs cost $4 each and a bag of chips costs $2.

Exercises
Determine the best method to solve each system of equations. Then solve the system.

1. \( 5x + 3y = 16 \)
\( 3x - 5y = -4 \)
2. \( 3x - 5y = 7 \)
\( 2x + 5y = 13 \)

3. \( y + 3x = 24 \)
\( 5x - y = 8 \)
4. \( -11x - 10y = 17 \)
\( 5x - 7y = 50 \)
Apply Systems Of Linear Equations

When applying systems of linear equations to problem situations, it is important to analyze each solution in the context of the situation.

Example  BUSINESS  A T-shirt printing company sells T-shirts for $15 each. The company has a fixed cost for the machine used to print the T-shirts and an additional cost per T-shirt. Use the table to estimate the number of T-shirts the company must sell in order for the income to equal expenses.

<table>
<thead>
<tr>
<th>T-shirt Printing Cost</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>printing machine</td>
<td>$3000.00</td>
</tr>
<tr>
<td>blank T-shirt</td>
<td>$5.00</td>
</tr>
</tbody>
</table>

Understand  You know the initial income and the initial expense and the rates of change of each quantity with each T-shirt sold.

Plan  Write an equation to represent the income and the expenses. Then solve to find how many T-shirts need to be sold for both values to be equal.

Solve  Let \(x\) = the number of T-shirts sold and let \(y\) = the total amount.

\[
\begin{align*}
\text{income} & \quad y = 0 + 15x \\
\text{expenses} & \quad y = 3000 + 5x
\end{align*}
\]

You can use substitution to solve this system.

\[
\begin{align*}
y & = 15x \\
15x & = 3000 + 5x \\
10x & = 3000 \\
x & = 300
\end{align*}
\]

This means that if 300 T-shirts are sold, the income and expenses of the T-shirt company are equal.

Check  Does this solution make sense in the context of the problem? After selling 300 T-shirts, the income would be about \(300 \times 15\) or \$4500. The costs would be about \(3000 + 300 \times 5\) or \$4500.

Exercises

Refer to the example above. If the costs of the T-shirt company change to the given values and the selling price remains the same, determine the number of T-shirts the company must sell in order for income to equal expenses.

1. printing machine: $5000.00; T-shirt: $10.00 each
2. printing machine: $2100.00; T-shirt: $8.00 each
3. printing machine: $8800.00; T-shirt: $4.00 each
4. printing machine: $1200.00; T-shirt: $12.00 each
Determine the best method to solve each system of equations. Then solve the system.

1. \[5x + 3y = 16\]
   \[3x - 5y = -4\]

2. \[3x - 5y = 7\]
   \[2x + 5y = 13\]

3. \[y = 3x - 24\]
   \[5x - y = 8\]

4. \[-11x - 10y = 17\]
   \[5x - 7y = 50\]

5. \[4x + y = 24\]
   \[5x - y = 12\]

6. \[6x - y = -145\]
   \[x = 4 - 2y\]

7. **VEGETABLE STAND** A roadside vegetable stand sells pumpkins for $5 each and squashes for $3 each. One day they sold 6 more squash than pumpkins, and their sales totaled $98. Write and solve a system of equations to find how many pumpkins and squash they sold?

8. **INCOME** Ramiro earns $20 per hour during the week and $30 per hour for overtime on the weekends. One week Ramiro earned a total of $650. He worked 5 times as many hours during the week as he did on the weekend. Write and solve a system of equations to determine how many hours of overtime Ramiro worked on the weekend.

9. **BASKETBALL** Anya makes 14 baskets during her game. Some of these baskets were worth 2-points and others were worth 3-points. In total, she scored 30 points. Write and solve a system of equations to find how 2-points baskets she made.
### 6-5 Practice

**Applying Systems of Linear Equations**

Determine the best method to solve each system of equations. Then solve the system.

1. \(1.5x - 1.9y = -29\) 
   \(x - 0.9y = 4.5\)

2. \(1.2x - 0.8y = -6\)
   \(4.8x + 2.4y = 60\)

3. \(18x - 16y = -312\)
   \(78x - 16y = 408\)

4. \(14x + 7y = 217\)
   \(14x + 3y = 189\)

5. \(x = 3.6y + 0.7\)
   \(2x + 0.2y = 38.4\)

6. \(5.3x - 4y = 43.5\)
   \(x + 7y = 78\)

7. **BOOKS** A library contains 2000 books. There are 3 times as many non-fiction books as fiction books. Write and solve a system of equations to determine the number of non-fiction and fiction books.

8. **SCHOOL CLUBS** The chess club has 16 members and gains a new member every month. The film club has 4 members and gains 4 new members every month. Write and solve a system of equations to find when the number of members in both clubs will be equal.

9. Tia and Ken each sold snack bars and magazine subscriptions for a school fund-raiser, as shown in the table. Tia earned $132 and Ken earned $190.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>snack bars</td>
<td>Tia</td>
</tr>
<tr>
<td>magazine subscriptions</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

**a.** Define variable and formulate a system of linear equation from this situation.

**b.** What was the price per snack bar? Determine the reasonableness of your solution.
6-5 Word Problem Practice

Applying Systems of Linear Equations

1. **MONEY** Veronica has been saving dimes and quarters. She has 94 coins in all, and the total value is $19.30. How many dimes and how many quarters does she have?

2. **CHEMISTRY** How many liters of 15% acid and 33% acid should be mixed to make 40 liters of 21% acid solution?

<table>
<thead>
<tr>
<th>Concentration of Solution</th>
<th>Amount of Solution (L)</th>
<th>Amount of Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>33%</td>
<td>y</td>
<td></td>
</tr>
<tr>
<td>21%</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

3. **BUILDINGS** The Sears Tower in Chicago is the tallest building in North America. The total height of the tower $t$ and the antenna that stands on top of it $a$ is 1729 feet. The difference in heights between the building and the antenna is 279 feet. How tall is the Sears Tower?

4. **PRODUCE** Roger and Trevor went shopping for produce on the same day. They each bought some apples and some potatoes. The amount they bought and the total price they paid are listed in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Apples (lb)</th>
<th>Potatoes (lb)</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roger</td>
<td>8</td>
<td>7</td>
<td>18.85</td>
</tr>
<tr>
<td>Trevor</td>
<td>2</td>
<td>10</td>
<td>12.88</td>
</tr>
</tbody>
</table>

What was the price of apples and potatoes per pound?

5. **SHOPPING** Two stores are having a sale on T-shirts that normally sell for $20. Store S is advertising an $s$ percent discount, and Store T is advertising a $t$ dollar discount. Rose spends $63 for three T-shirts from Store S and one from Store T. Manny spends $140 on five T-shirts from Store S and four from Store T. Find the discount at each store.

6. **TRANSPORTATION** A Speedy River barge bound for New Orleans leaves Baton Rouge, Louisiana, at 9:00 A.M. and travels at a speed of 10 miles per hour. A Rail Transport freight train also bound for New Orleans leaves Baton Rouge at 1:30 P.M. the same day. The train travels at 25 miles per hour, and the river barge travels at 10 miles per hour. Both the barge and the train will travel 100 miles to reach New Orleans.

a. How far will the train travel before catching up to the barge?

b. Which shipment will reach New Orleans first? At what time?

c. If both shipments take an hour to unload before heading back to Baton Rouge, what is the earliest time that either one of the companies can begin to load grain to ship in Baton Rouge?
Cramer’s Rule

Cramer’s Rule is a method for solving a system of equations. To use Cramer’s Rule, set up a matrix to represent the equations. A matrix is a way of organizing data.

**Example**

Solve the following system of equations using Cramer’s Rule.

\[
\begin{align*}
2x + 3y &= 13 \\
x + y &= 5
\end{align*}
\]

**Step 1:** Set up a matrix representing the coefficients of \(x\) and \(y\).

\[
A = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}
\]

**Step 2:** Find the determinant of matrix \(A\).

If a matrix \(A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\), then the determinant, \(\text{det}(A) = ad - bc\).

\[
\text{det}(A) = 2(1) - 1(3) = -1
\]

**Step 3:** Replace the first column in \(A\) with 13 and 5 and find the determinant of the new matrix.

\[
A_1 = \begin{vmatrix} 13 & 3 \\ 5 & 1 \end{vmatrix}; \quad \text{det} (A_1) = 13(1) - 5(3) = -2
\]

**Step 4:** To find the value of \(x\) in the solution to the system of equations, determine the value of \(\frac{\text{det}(A_1)}{\text{det}(A)}\).

\[
\frac{\text{det}(A_1)}{\text{det}(A)} = \frac{-2}{-1} = 2
\]

**Step 5:** Repeat the process to find the value of \(y\). This time, replace the second column with 13 and 5 and find the determinant.

\[
A_2 = \begin{vmatrix} 2 & 13 \\ 1 & 5 \end{vmatrix}; \quad \text{det} (A_2) = 2(5) - 1(13) = -3 \quad \text{and} \quad \frac{\text{det}(A_2)}{\text{det}(A)} = \frac{-3}{-1} = 3
\]

So, the solution to the system of equations is \((2, 3)\).

**Exercises**

Use Cramer’s Rule to solve each system of equations.

1. \(2x + y = 1\) \quad 3. \(x - y = 4\)
   \(3x + 5y = 5\) \quad \(3x - 5y = 8\)

2. \(x + y = 4\) \quad 4. \(4x - y = 3\)
   \(2x - 3y = -2\) \quad \(x + y = 7\)

5. \(3x - 2y = 7\) \quad 6. \(6x - 5y = 1\)
   \(2x + y = 14\) \quad \(3x + 2y = 5\)
6-6 Study Guide and Intervention

Systems of Inequalities

Systems of Inequalities  The solution of a system of inequalities is the set of all ordered pairs that satisfy both inequalities. If you graph the inequalities in the same coordinate plane, the solution is the region where the graphs overlap.

Example 1  Solve the system of inequalities by graphing.

\[ y > x + 2 \]
\[ y \leq -2x - 1 \]

The solution includes the ordered pairs in the intersection of the graphs. This region is shaded at the right. The graphs of \( y = x + 2 \) and \( y = -2x - 1 \) are boundaries of this region. The graph of \( y = x + 2 \) is dashed and is not included in the graph of \( y > x + 2 \).

Example 2  Solve the system of inequalities by graphing.

\[ x + y > 4 \]
\[ x + y < -1 \]

The graphs of \( x + y = 4 \) and \( x + y = -1 \) are parallel. Because the two regions have no points in common, the system of inequalities has no solution.

Exercises

Solve each system of inequalities by graphing.

1. \[ y > -1 \]
   \[ x < 0 \]

2. \[ y > -2x + 2 \]
   \[ y \leq x + 1 \]

3. \[ y < x + 1 \]
   \[ 3x + 4y \geq 12 \]

4. \[ 2x + y \geq 1 \]
   \[ x - y \geq -2 \]

5. \[ y \leq 2x + 3 \]
   \[ y \geq -1 + 2x \]

6. \[ 5x - 2y < 6 \]
   \[ y > -x + 1 \]
Apply Systems of Inequalities In real-world problems, sometimes only whole numbers make sense for the solution, and often only positive values of $x$ and $y$ make sense.

**Example**  BUSINESS AAA Gem Company produces necklaces and bracelets. In a 40-hour week, the company has 400 gems to use. A necklace requires 40 gems and a bracelet requires 10 gems. It takes 2 hours to produce a necklace and a bracelet requires one hour. How many of each type can be produced in a week?

Let $n =$ the number of necklaces that will be produced and $b =$ the number of bracelets that will be produced. Neither $n$ or $b$ can be a negative number, so the following system of inequalities represents the conditions of the problems.

\[
\begin{align*}
  n &\geq 0 \\
  b &\geq 0 \\
  b + 2n &\leq 40 \\
  10b + 40n &\leq 400
\end{align*}
\]

The solution is the set ordered pairs in the intersection of the graphs. This region is shaded at the right. Only whole-number solutions, such as $(5, 20)$ make sense in this problem.

**Exercises**

For each exercise, graph the solution set. List three possible solutions to the problem.

1. **HEALTH**  Mr. Flowers is on a restricted diet that allows him to have between 1600 and 2000 Calories per day. His daily fat intake is restricted to between 45 and 55 grams. What daily Calorie and fat intakes are acceptable?

2. **RECREATION**  Maria had $150 in gift certificates to use at a record store. She bought fewer than 20 recordings. Each tape cost $5.95 and each CD cost $8.95. How many of each type of recording might she have bought?
6-6 Skills Practice

Systems of Inequalities

Solve each system of inequalities by graphing.

1. \(x > -1\)
   \(y \leq -3\)

2. \(y > 2\)
   \(x < -2\)

3. \(y > x + 3\)
   \(y \leq -1\)

4. \(x < 2\)
   \(y - x \leq 2\)

5. \(x + y \leq -1\)
   \(x + y \geq 3\)

6. \(-x > 4\)
   \(x + y > 2\)

7. \(y > x + 1\)
   \(y \geq -x + 1\)

8. \(y \geq -x + 2\)
   \(y < 2x - 2\)

9. \(y < 2x + 4\)
   \(y \geq x + 1\)

Write a system of inequalities for each graph.

10.

11.

12.
6-6 Practice

Systems of Inequalities

Solve each system of inequalities by graphing.

1. \( y > x - 2 \)
   \( y \leq x \)

2. \( y \geq x + 2 \)
   \( y > 2x + 3 \)

3. \( x + y \geq 1 \)
   \( x + 2y > 1 \)

4. \( y < 2x - 1 \)
   \( y > 2 - x \)

5. \( y > x - 4 \)
   \( 2x + y \leq 2 \)

6. \( 2x - y \geq 2 \)
   \( x - 2y \geq 2 \)

7. FITNESS Diego started an exercise program in which each week he works out at the gym between 4.5 and 6 hours and walks between 9 and 12 miles.
   a. Make a graph to show the number of hours Diego works out at the gym and the number of miles he walks per week.
   b. List three possible combinations of working out and walking that meet Diego’s goals.

8. SOUVENIRS Emily wants to buy turquoise stones on her trip to New Mexico to give to at least 4 of her friends. The gift shop sells stones for either $4 or $6 per stone. Emily has no more than $30 to spend.
   a. Make a graph showing the numbers of each price of stone Emily can purchase.
   b. List three possible solutions.
6-6 Word Problem Practice

Systems of Inequalities

1. **PETS** Renée’s Pet Store never has more than a combined total of 20 cats and dogs and never more than 8 cats. This is represented by the inequalities \(x \leq 8\) and \(x + y \leq 20\). Solve the system of inequalities by graphing.

2. **WAGES** The minimum wage for one group of workers in Texas is $7.25 per hour effective Sept. 1, 2008. The graph below shows the possible weekly wages for a person who makes at least minimum wages and works at most 40 hours. Write the system of inequalities for the graph.

3. **FUND RAISING** The Camp Courage Club plans to sell tins of popcorn and peanuts as a fundraiser. The Club members have $900 to spend on products to sell and want to order up to 200 tins in all. They also want to order at least as many tins of popcorn as tins of peanuts. Each tin of popcorn costs $3 and each tin of peanuts costs $4. Write a system of equations to represent the conditions of this problem.

4. **BUSINESS** For maximum efficiency, a factory must have at least 100 workers, but no more than 200 workers on a shift. The factory also must manufacture at least 30 units per worker.
   a. Let \(x\) be the number of workers and let \(y\) be the number of units. Write four inequalities expressing the conditions in the problem given above.
   b. Graph the systems of inequalities.
   c. List at least three possible solutions.
Describing Regions

The shaded region inside the triangle can be described with a system of three inequalities.

\[ y < -x + 1 \]
\[ y > \frac{1}{3}x - 3 \]
\[ y > -9x - 31 \]

Write systems of inequalities to describe each region. You may first need to divide a region into triangles or quadrilaterals.

1.

2.

3.
Shading Absolute Value Inequalities

Absolute value inequalities of the form \( y \leq a |x - h| + k \) or \( y \geq a |x - h| + k \) can be graphed using the SHADE command or by using the shading feature in the Y= screen.

**Example 1** Graph \( y \leq 2 |x - 3| + 2 \).

**Method 1 SHADE command**

Enter the boundary equation \( y = 2 |x - 3| + 2 \) into \( Y1 \). From the home screen, use SHADE to shade the region under the boundary equation because the inequality uses \(<\). Use \( \text{Ymin} \) as the lower boundary and the boundary equation, \( Y1 \), as the upper boundary.

Keystrokes: \( \text{2nd} \ [\text{DRAW}] \ 7 \ \text{VARS} \ \text{ENTER} \ 4 \ \text{ ENTER} \ \text{VARS} \ \text{ENTER} \ \text{ENTER} \).

**Method 2 Y=**

On the Y= screen, move the cursor to the far left and repeatedly press \( \text{ENTER} \) to toggle through the graph choices. Be sure to select the option to shade below \( \leq \). Then enter the boundary equation and graph.

The same techniques can be used to solve a system of absolute value inequalities.

**Example 2** Solve the system \( y \geq |x + 5| - 4 \) and \( y \leq 3 |x - 6| + 1 \).

Enter \( y = |x + 5| - 4 \) in \( Y1 \) and set the graph to shade above. Then enter \( y = 3 |x - 6| + 1 \) in \( Y2 \) and set the graph to shade below. Graph the inequalities. Notice that a different pattern is used for the second inequality. The region where the two patterns overlap is the solution to the system.

**Exercises**

Solve each system of inequalities by graphing.

1. \( y \leq 2 |x - 4| + 1 \) \( y > |x + 3| + 6 \)
2. \( y < \frac{1}{2} |x - 3| + 2 \)
3. \( y \leq 4 |x - 4| + 3 \) \( y \leq |x + 1| + 4 \)
4. \( y \leq \frac{1}{4} |x + 1| + 1 \) \( y \geq 3 |x - 4| - 2 \) \( y \geq 2 |x + 3| + 5 \)
Spreadsheet Activity

Systems of Inequalities

Example

TopSport Shoe Company has a total of 9600 minutes of machine time each week to cut the materials for the two types of athletic shoes they make. There are a total of 28,000 minutes of worker time per week for assembly. It takes 3 minutes to cut and 12 minutes to assemble a pair of Runners and 2 minutes to cut and 10 minutes to assemble a pair of Flyers. Is it possible for the company to make 1200 pairs of Runners and 1400 pairs of Flyers in a week?

Step 1  Represent the situation using a system of inequalities. Let \( r \) represent the number of Runners and \( f \) represent the number of Flyers.

\[
3r + 2f \leq 9600 \\
12r + 10f \leq 28,000
\]

Step 2  Columns A and B contain the values of \( r \) and \( f \). Columns C and D contain the formulas for the inequalities. The formulas will return TRUE or FALSE. If both inequalities are true for an ordered pair, then the ordered pair is a solution to the system of inequalities.

Since one of the inequalities is false for (1200, 1400), the ordered pair is not part of the solution set. The company cannot make 1200 pairs of Runners and 1400 pairs of Flyers in a week.

Exercises

Use the spreadsheet to determine whether TopSport can make the following combinations of shoes.

1. 1000 Runners, 1100 Flyers
2. 1200 Runners, 1000 Flyers
3. 2000 Runners, 500 Flyers
4. 300 Runners, 1500 Flyers
5. 950 Runners, 1400 Flyers
6. If TopSport can either buy another cutting machine or hire more assemblers, which would make more combinations of shoe production possible? Explain.
Use this recording sheet with pages 386–387 of the Student Edition.

**Multiple Choice**

Read each question. Then fill in the correct answer.

1. ○ ○ ○ ○
2. ○ ○ ○ ○
3. ○ ○ ○ ○
4. ○ ○ ○ ○
5. ○ ○ ○ ○
6. ○ ○ ○ ○

**Short Response/Gridded Response**

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

9. ___________ (grid in)
10. ___________
11. ___________ (grid in)
12. ___________
13. ___________
14. ___________ (grid in)
15. ___________
16. ___________
17. ___________

**Extended Response**

Record your answers for Question 18 on the back of this paper.
6 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 18 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The equation and calculations written in part a correctly represent the information provided. They show that 250 canned goods were collected on Day 1 and that after 5 days, 5(250) or 1250 cans will be collected. In part b, students should note that the total number of cans collected each day will vary and that the total after Day 5 is highly unlikely to be exactly 1250.</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

1. \( y = \frac{3}{2}x \) \( \quad \) \( \) \( y = -x + 5 \)
2. \( x - 2y = -2 \) \( \quad \) \( \) \( x - 2y = 3 \)

For Questions 3 and 4, use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solutions or infinitely many solutions.

3. \( 3x - 2y = -7 \) \( \quad \) \( \) \( y = x + 4 \)
4. \( -6x - 2y = -20 \) \( \quad \) \( \) \( y = -3x + 10 \)

5. MULTIPLE CHOICE In order for José and Marty to compete against each other during the wrestling season next year they need to be in the same weight category. José weighs 180 pounds and plans to gain 2 pounds per week. Marty weighs 249 pounds and plans to lose 1 pound per week. In how many weeks will they weigh the same?

A 23 \( \quad \) B 34.5 \( \quad \) C 69 \( \quad \) D 226

For Questions 1–4, use elimination to solve each system of equations.

1. \( x + y = 4 \) \( \quad \) \( \) \( x - y = 7 \)
2. \( -2x + y = 5 \) \( \quad \) \( \) \( 2x + 3y = 3 \)
3. \( 4x + 6y = -10 \) \( \quad \) \( \) \( 8x - 3y = 25 \)
4. \( 2x + 3y = 1 \) \( \quad \) \( \) \( 5x - 4y = 14 \)

5. MULTIPLE CHOICE If \( 5x - 3y = 7 \) and \( -3x - 5y = 23 \), what is the value of \( x \)?

A \((-1, -4)\) \( \quad \) B \((-4, -1)\) \( \quad \) C \(-4\) \( \quad \) D \(-1\)
Determine the best method to solve each system of equations. Then solve the system.

1. \(2y + 1 = x\)
   \(3x + y = 17\)
2. \(7x + 5y = 29\)
   \(21x - 25y = -33\)
3. \(-2x + 5y = 14\)
   \(-2x - 3y = -2\)
4. \(4x + 2y = 1\)
   \(-4x + 4y = 5\)

5. **MULTIPLE CHOICE** Timothy and his brothers are selling lemonade and cookies to raise funds for a camping trip. Paul bought two cups of lemonade and two cookies for $3 and Nancy bought three cookies and two cups of lemonade for $4.00. What is the cost of a cup of lemonade?

   A $2  
   B $1  
   C $0.75  
   D $0.50

---

1. Solve the system of inequalities by graphing.
   \(y < -3x + 2\)
   \(y - 5x \leq 3\)

2. Solve the system of inequalities by graphing.
   \(y < \frac{1}{2}x + 2\)
   \(y \geq -3x - 1\)

3. **MULTIPLE CHOICE** LaShawn designs websites for local businesses. He charges $25 an hour to build a website, and charges $15 an hour to update websites once he builds them. He wants to earn at least $100 every week, but he does not want to work more than 6 hours each week. What is a possible solution to describe how many hours LaShawn can spend building a website \((x)\) and updating a website \((y)\) in a week?

   A \(1, 4\)  
   B \(1, 6\)  
   C \(2, 3\)  
   D \(3, 3\)
Chapter 6 Mid-Chapter Test
(Lessons 6-1 through 6-4)

**Part I** Write the letter for the correct answer in the blank at the right of each question.

Use the graph for Questions 1 and 2. For Questions 1 and 2, determine how many solutions exist for each system of equations.

<table>
<thead>
<tr>
<th>A</th>
<th>no solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>one solution</td>
</tr>
<tr>
<td>C</td>
<td>infinitely many solutions</td>
</tr>
<tr>
<td>D</td>
<td>cannot be determined</td>
</tr>
</tbody>
</table>

1. \(x + y = 3\)  
   \(x - y = 3\)

2. \(x + y = 3\)  
   \(x + y = -2\)

3. The solution to which system of equations has a positive \(y\) value?
   - F \(x + y = 3\)
   - G \(x + y = 3\)
   - H \(x - 3y = -2\)
   - J \(x + y = 3\)

   \(x - 3y = -2\)  
   \(x + y = -2\)  
   \(x + y = -2\)  
   \(x - y = 3\)

4. If \(y = 5x - 3\) and \(3x - y = -1\), what is the value of \(y\)?
   - A 2
   - B -1
   - C 7
   - D -8

5. Use elimination to solve the system of equations for \(y\).
   \(x - 5y = -6\)  
   \(x + 2y = 8\)

   - F \(\frac{2}{3}\)
   - G 2
   - H 4
   - J \(4\frac{2}{3}\)

6. How many solutions does the system \(2y = 10x - 14\) and \(5x - y = 7\) have?
   - A one
   - B two
   - C none
   - D infinitely many

**Part II**

7. Graph the system of equations. Then determine whether it has no solution, one solution, or infinitely many solutions. If the system has one solution name it.
   \(x - 3y = -3\)
   \(x + 3y = 9\)

   For Questions 9–11, use elimination to solve each system of equations.

8. Use substitution to solve the system of equations.
   \(4x + y = 0\)
   \(2y + x = -7\)

9. \(x - 4y = 11\)
   \(5x - 7y = -10\)

10. \(2r + 3t = 9\)
    \(3r + 2t = 12\)

11. \(9x + 2y = -17\)
    \(-11x + 2y = 3\)

12. At the end of a recent WNBA regular season, the Phoenix Mercury had 12 more victories than losses. The number of victories they had was one more than two times the number of losses. How many regular season games did the Phoenix Mercury play during that season?
Chapter 6 Vocabulary Test

Choose from the terms above to complete each sentence.

1. Two equations, such as \( y = 3x - 6 \) and \( y = 12 + 4x \), together are called a(n) _____.

2. If the graphs of the equations of a system intersect or coincide, the system of equations is _____.

3. If a system of equations has an infinite number of solutions, the system is _____.

4. Sometimes adding two equations of a system will give an equation with only one variable. This is helpful when you are solving the system by _____.

Define the terms in your own words.

5. substitution

6. independent
Write the letter for the correct answer in the blank at the right of each question.

Use the graph for Questions 1–4.

1. The ordered pair (3, 2) is the solution of which system of equations?
   - A \( y = \frac{1}{3}x + 3 \)
   - B \( y = \frac{1}{3}x + 3 \)
   - C \( y = \frac{1}{3}x + 3 \)
   - D \( y = \frac{1}{3}x + 3 \)

   1. _____

For Questions 2–4, find how many solutions exist for each system of equations.

2. \( y = 2x \)
   - F no solution
   - G infinitely many solutions

3. \( y = 2x \)
   - H one solution
   - J cannot be determined

4. \( y = -3x + 2 \)
   - K no solution
   - L infinitely many solutions

5. When solving the system of equations, \( r = 4 - t \)
   - M no solution
   - N one solution

   which expression could be substituted for \( r \) in the second equation?
   - A \( 4 - t \)
   - B \( 4 - r \)
   - C \( t - 4 \)
   - D \( \frac{4}{t} \)

6. If \( x = 2 \) and \( 3x + y = 5 \), what is the value of \( y \)?
   - F 0
   - G -1
   - H 11
   - J 10

7. Use substitution to solve the system of equations.
   - \( n = 3m - 11 \)
   - \( 2m + 3n = 0 \)

   A \((-2, 3)\)
   - B \((-3, 2)\)
   - C \((3, -2)\)
   - D \((2, -3)\)

8. Use elimination to solve the system of equations.
   - \( x - y = 5 \)
   - \( x + y = 3 \)

   A \((4, 1)\)
   - B \((4, -1)\)
   - C \((-4, 1)\)
   - D \((-4, -1)\)

9. Use elimination to solve the system of equations.
   - \( x + 6y = 10 \)
   - \( x + 5y = 9 \)

   A \((1, 4)\)
   - B \((4, 1)\)
   - C \((-1, -4)\)
   - D \((-4, -1)\)

10. Use elimination to find the value of \( x \) in the solution for the system of equations.
    - \( 2x + 2y = 10 \)
    - \( 2x - 3y = 5 \)

    A 1
    - B 10
    - C 4
    - D -2

11. To eliminate the variable \( y \) in the system of equations, \( 6x + 4y = 22 \)
    multiply the second equation by which number?
    - A 3
    - B 9
    - C 22
    - D 4

   11. _____
For Questions 12 and 13, determine the best method to solve the system of equations.

A substitution  C elimination using subtraction
B elimination using addition  D elimination using multiplication

12. $5x - 2y = 4$  
   $2x + 2y = 8$  

13. $y = 3x + 12$  
   $2x + y = 16$

14. The length of a rectangle is three times the width. The sum of the length and the width is 24 inches. What is the length of the rectangle?
F 3 inches  G 6 inches  H 9 inches  J 18 inches

15. An airport shuttle company owns sedans that have a maximum capacity of 3 passengers and vans that have a maximum capacity of 8 passengers. Their 12 vehicles have a combined maximum capacity of 61 passengers. How many vans does the company own?
A 5  B 8  C 12  D 7

For Questions 16 and 17, use the graph.
16. What is a solution to the system of inequalities?
A (0, 2)  C (1, 1)
B (0, 0)  D (2, 4)

17. What system of inequalities is represented?
F $y < x + 2$  G $y > x + 2$  H $y > x + 2$
   $y \leq -x + 1$  $y \leq -x + 1$  $y \geq -x + 1$
   $y \geq -x + 1$

B: Determine the best method to solve the system of equations. Then solve the system.

$x - 3y = 0$
$2x - 7y = 0$
**Chapter 6 Test, Form 2A**

**Write the letter for the correct answer in the blank at the right of each question.**

Use the graph for Questions 1–4.

For Questions 1 and 2, determine how many solutions exist for each system of equations.

A no solution  
B one solution  
C infinitely many solutions  
D cannot be determined

1. \( y = 3x + 3 \)  
   \( 3x - y = 2 \)  
2. \( x + 2y = -1 \)  
   \( 2x + 3y = 0 \)  

3. The solution to which system of equations has an \( x \) value of 3?
   
   F \( x + 2y = -1 \)  
   G \( 3x - y = 2 \)  
   H \( y = 3x + 3 \)  
   J \( 2x + 3y = 0 \)  

4. The solution to which system of equations has a \( y \) value of 0?
   
   A \( x + 2y = -1 \)  
   B \( 3x - y = 2 \)  
   C \( y = 3x + 3 \)  
   D \( 2x + 3y = 0 \)  

5. When solving the system of equations, \( x + 2y = 15 \)  
   which expression could be substituted for \( x \) in the second equation?
   
   F \( 15 - 2y \)  
   G \( 21 - 5x \)  
   H \( \frac{15 - x}{2} \)  
   J \( \frac{21 - 2y}{5} \)  

6. If \( x = 2y + 3 \) and \( 4x - 5y = 9 \), what is the value of \( y \)?
   
   A 2  
   B 1  
   C \( -1 \)  
   D \( -2 \)  

7. Use elimination to solve the system \( x + 7y = 16 \) and \( 3x - 7y = 4 \) for \( x \).
   
   F 3  
   G 4  
   H 5  
   J \( -6 \)  

8. Use elimination to solve the system \( x - 5y = 20 \) and \( x + 3y = -4 \) for \( x \).
   
   A 5  
   B \( -3 \)  
   C 10  
   D \( -40 \)  

9. Use elimination to solve the system \( 8x - 7y = 5 \) and \( 3x - 5y = 9 \) for \( y \).
   
   F \( -2 \)  
   G 8  
   H \( -3 \)  
   J \( -1 \)  

10. Use elimination to solve the system \( 4x + 6y = 10 \) and \( 2x + 5y = 1 \) for \( x \).
    
    A 11  
    B \( 5 \frac{1}{2} \)  
    C \( -2 \)  
    D \( \frac{1}{2} \)  

11. The substitution method should be used to solve which system of equations?
    
    F \( 4x + 3y = 6 \)  
    G \( 2x + 5y = 1 \)  
    H \( 6x + 2y = 1 \)  
    J \( y = 3x + 1 \)  
    5x - 3y = 2  
    2x - 3y = 4  
    3x + 4y = 7  
    2x + 4y = 5  

11. _____
12. Use substitution to solve the system \( x + 2y = 1 \) and \( 2x + 5y = 3 \).

\[
\begin{align*}
A \quad & (−1, 1) \quad B \quad (1, −1) \quad C \quad (−5, 3) \quad D \quad (−1, −1) \\
\end{align*}
\]

13. For Questions 13 and 14, solve the system and find values of \( y \).

\[
\begin{align*}
13. \quad 3x - 5y &= -35 \quad F \quad 4 \quad G \quad \frac{4}{5} \quad H \quad -4 \quad J \quad -\frac{4}{5} \\
& 2x - 5y = -30
\end{align*}
\]

14. \( 3x + 4y = -30 \quad A \quad -6 \quad B \quad 6 \quad C \quad -12 \quad D \quad 12 \)

\[
\begin{align*}
& 2x - 5y = 72
\end{align*}
\]

15. Coffee Cafe makes 90 pounds of coffee that costs $6 per pound. The types of coffee used to make this mixture cost $7 per pound and $4 per pound. How many pounds of the $7-per-pound coffee should be used in this mixture?

\[
\begin{align*}
F \quad 30 \text{ lb} \quad G \quad 40 \text{ lb} \quad H \quad 50 \text{ lb} \quad J \quad 60 \text{ lb}
\end{align*}
\]

16. Your teacher is giving a test that has 5 more four-point questions than six-point questions. The test is worth 120 points. Which system represents this information?

\[
\begin{align*}
A \quad x + 5 &= y \quad B \quad x + y &= 5 \quad C \quad x - y &= 5 \quad D \quad x - y &= 5 \\
& 4x + 6y = 120 \quad 6x + 4y = 120 \quad 6x + 4y = 120 \quad 4x + 6y = 120
\end{align*}
\]

17. What system of inequalities is represented in the graph?

\[
\begin{align*}
F \quad y &< -2x + 1 \\
& y \leq \frac{1}{5} x - 1
\end{align*}
\]

\[
\begin{align*}
G \quad y &> -2x + 1 \\
& y \leq \frac{1}{5} x - 1
\end{align*}
\]

\[
\begin{align*}
H \quad y &< -2x + 1 \\
& y \geq \frac{1}{5} x - 1
\end{align*}
\]

\[
\begin{align*}
J \quad y &> -2x + 1 \\
& y \geq \frac{1}{5} x - 1
\end{align*}
\]

18. Bonus Where on the graph of \( 2x - 6y = 7 \) is the \( x \)-coordinate twice the \( y \)-coordinate?

B: ________
Write the letter for the correct answer in the blank at the right of each question.

Use the graph for Questions 1–4.

For Questions 1 and 2, determine how many solutions exist for each system of equations.

- A  no solution
- B  one solution
- C  infinitely many solutions
- D  cannot be determined

1. \(3x - 3y = -6\)  \(y = x + 2\)
2. \(x + y = 0\)  \(3x - y = 2\)

3. The solution to which system of equations has an \(x\) value of \(-1\)?
   - F  \(x + y = 0\)  \(G 3x - y = 2\)  \(H  x + y = 0\)  \(J y = -x - 2\)
   - 3._____

4. The solution to which system of equations has a \(y\) value of \(-2\)?
   - A  \(x + y = 0\)  \(B 3x - y = 2\)  \(C  x + y = 0\)  \(D y = -x - 2\)
   - 4._____

5. When solving the system of equations,
   - \(3x + y = 14\)
   - \(x + 4y = 3\)
   - which expression could be substituted for \(y\) in the second equation?
   - F  \(3 - 4y\)  \(G \frac{3 - x}{4}\)  \(H \frac{14 - y}{3}\)  \(J 14 - 3x\)
   - 5._____

6. If \(x = 5y - 1\) and \(2x + 5y = -32\), what is the value of \(y\)?
   - A  \(-2\)  \(B 2\)  \(C 1\)  \(D -1\)
   - 6._____

7. Use elimination to solve the system \(3x + 5y = 16\) and \(8x - 5y = 28\) for \(x\).
   - F  \(-6\)  \(G 5\)  \(H 4\)  \(J \frac{4}{5}\)
   - 7._____

8. Use elimination to solve the system \(x - 4y = 1\) and \(x + 2y = 19\) for \(x\).
   - A  \(-11\)  \(B 3\)  \(C 25\)  \(D 13\)
   - 8._____

9. Use elimination to solve the system \(4x + 7y = -14\) and \(8x + 5y = 8\) for \(x\).
   - F  \(3\frac{1}{2}\)  \(G -1\frac{1}{2}\)  \(H 8\)  \(J -4\)
   - 9._____

10. Use elimination to solve the system \(5x + 4y = -10\) and \(3x + 6y = -6\) for \(y\).
    - A  \(-2\)  \(B -5\)  \(C 0\)  \(D 2\)
    - 10._____
12. Use substitution to solve the system \( x - 2y = 1 \) and \( 6x - 5y = 20 \).
   A (2, 5)  B (-5, -2)  C (5, 2)  D (-2, -5)

For Questions 13 and 14, solve the system and find the value of \( y \).

13. \( 2x + 3y = 1 \)
   \( 5x - 4y = -32 \)
   F \(-\frac{3}{7}\)  G \(\frac{3}{7}\)  H -3  J 3

14. \( 6x + 3y = 12 \)
   \( 5x + 3y = 0 \)
   A -20  B -\(\frac{9}{11}\)  C 20  D \(1\frac{9}{11}\)

15. Colortime Bakers wants to make 30 pounds of a berry mix that costs $3 per pound to use in their pancake mix. They are using blueberries that cost $2 per pound and blackberries that cost $3.50 per pound. How many pounds of blackberries should be used in this mixture?
   F 15 lb  G 20 lb  H 10 lb  J 30 lb

16. Your teacher is giving a test that has 12 more three-point questions than five-point questions. The test is worth 100 points. Which system represents this information?
   A \( x + y = 12 \)  B \( x + y = 12 \)  C \( x - y = 12 \)  D \( x - y = 12 \)
   \( 3x + 5y = 100 \)  \( 5x + 3y = 100 \)  \( 3x + 5y = 100 \)  \( 5x + 3y = 100 \)

17. What system of inequalities is represented in the graph?
   F \( y \geq -3x \)  G \( y \geq -3x \)  H \( y \leq -3x \)  J \( y \leq -3x \)
   \( y < -\frac{1}{2}x - 2 \)  \( y > -\frac{1}{2}x - 2 \)  \( y < -\frac{1}{2}x - 2 \)  \( y > -\frac{1}{2}x - 2 \)

**Bonus** Manuel is 8 years older than his sister. Three years ago he was 3 times older than his sister. How old is each now? B: ______________
Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

1. \[ y = -x \]
   \[ x + y = 3 \]

2. \[ x - 2y = -3 \]
   \[ 4x + y = 6 \]

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

3. \[ y = -x + 4 \]
   \[ y = x - 4 \]

4. \[ 2x - y = -3 \]
   \[ 6x - 3y = -9 \]

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

5. \[ y = 3x \]
   \[ x + y = 4 \]

6. \[ 5x - y = 10 \]
   \[ 7x - 2y = 11 \]

Use elimination to solve each system of equations.

7. \[ x + 4y = -8 \]
   \[ x - 4y = -8 \]

8. \[ 2x + 5y = 3 \]
   \[ -x + 3y = -7 \]

9. \[ 2x - 5y = -16 \]
   \[ -2x + 3y = 12 \]

10. \[ 2x + 5y = 9 \]
    \[ 2x + y = 13 \]

Determine the best method to solve each system of equations. Then solve the system.

11. \[ x = 2y - 1 \]
    \[ 3x + y = 11 \]

12. \[ 5x - y = 17 \]
    \[ 3x - y = 13 \]

13. The sum of two numbers is 17 and their difference is 29. What are the two numbers?

14. Adult tickets for the school musical sold for $3.50 and student tickets sold for $2.50. Three hundred twenty-one tickets were sold altogether for $937.50. How many of each kind of ticket were sold?

15. Ayana has $2.35 in nickels and dimes. If she has 33 coins in all, find the number of nickels and dimes.
For Questions 16 and 17, solve the system of inequalities by graphing.

16. \( y > x + 2 \)
   \[ y \leq -2x - 1 \]

17. \( y \leq \frac{1}{4}x + 2 \)
   \[ y \geq 2x \]

18. To qualify for a certain need-based scholarship, a student must get a score of at least 75 on a qualification test. In addition, the student’s family must make less than $40,000 a year. Define the variables, write a system of inequalities to represent this situation, and name one possible solution.

Bonus Mavis is 5 years older than her brother. Five years ago she was 2 times older than her brother. How old is each now?

B: ___________________
Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

1. \(x + y = 3\)
   \(y = -x + 3\)

2. \(3x - y = -3\)
   \(2x - 3y = -2\)

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

3. \(y = -x + 3\)
   \(y = x - 3\)

4. \(2x - y = 5\)
   \(4x - 2y = 10\)

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

5. \(y = 2x\)
   \(2x + y = 8\)

6. \(2x - y = 3\)
   \(5x + 7y = 17\)

Use elimination to solve each system of equations.

7. \(2x + 3y = 19\)
   \(2x - 3y = 1\)

8. \(6x + 4y = 20\)
   \(4x - 2y = 4\)

9. \(2x + 2y = 6\)
   \(3x - 2y = -11\)

10. \(7x + 3y = 1\)
   \(9x + 3y = -3\)

Determine the best method to solve each system of equations. Then solve the system.

11. \(y = 3x + 1\)
    \(x - 2y = 8\)

12. \(5x - 15y = -20\)
    \(5x - 4y = -9\)

13. The sum of two numbers is 16 and their difference is 20. What are the two numbers?

14. Kyle just started a new job that pays $7 per hour. He had been making $5 per hour at his old job. Kyle worked a total of 54 hours last month and made $338 before deductions. How many hours did he work at his new job?

15. Brent has $3.35 in quarters and dimes. If he has 23 coins in all, find the number of quarters and dimes.
For Questions 16 and 17, solve the system of inequalities by graphing.

16. \( y < \frac{1}{3}x + 1 \)
   \( y \leq 2x - 3 \)

17. \( y \leq x + 3 \)
   \( y > -\frac{1}{2}x - 2 \)

18. To qualify for a certain car loan, a customer must have a credit score of at least 600. In addition, the cost of the car must be at least $5000. Define the variables, write a system of inequalities to represent this situation, and name one possible solution.

Bonus Find the point on the graph of \( 3x - 4y = 9 \) where the \( y \)-coordinate is 3 times the \( x \)-coordinate.

B: ________________
Graph each system of equations. Determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

1. \( \frac{1}{3}y = x \)
   \( y + x + 4 = 0 \)

2. \( x + 3y = 3 \)
   \( 3y = -x + 9 \)

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

3. \( y = 2x - 7 \)
   \( 3x - 4y = 8 \)

4. \( 4y - 3x = 5 \)
   \( \frac{3}{4}x = y - 4 \)

5. \( x - 2y = -3 \)
   \( y = 3x - 1 \)

6. \( y = -x + 3 \)
   \( x + y = -1 \)

Use elimination to solve each system of equations.

7. \( 6x - 7y = 21 \)
   \( 3x + 7y = 6 \)

8. \( 0.2x + 0.5y = 0.7 \)
   \( -0.2x - 0.6y = -1.4 \)

9. \( 2x + \frac{2}{3}y = -8 \)
   \( \frac{1}{2}x - \frac{1}{3}y = 1 \)

10. \( \frac{1}{2}x + \frac{2}{5}y = -10 \)
    \( 3x + 6y = -6 \)

Determine the best method to solve each system of equations. Then solve the system.

11. \( x + y = 147 \)
    \( 25x + 10y = 2415 \)

12. \( 7y = 2\frac{1}{2} - 2x \)
    \( 5x = 3y - 4 \)

13. Three times one number added to five times a second number is 68. Three times the second number minus four times the first number is 6. What are the two numbers?

14. The difference of two numbers is 5. Five times the lesser number minus the greater number is 9. What are the two numbers?

15. A trail mix that costs $2.45 per pound is mixed with a trail mix that costs $2.30 per pound. How much of each type of trail mix must be used to have 30 pounds of a trail mix that costs $2.35 per pound?
16. A boat travels 60 miles downstream in the same time it takes to go 36 miles upstream. The speed of the boat in still water is 15 mph greater than the speed of the current. Find the speed of the current.

For Questions 17 and 18, solve the system of inequalities by graphing.

17. $y < -\frac{1}{4}x + 2$
   $y \leq x - 1$

18. $-2x - y > 1$
   $\frac{1}{2} x - y \leq 1$

19. Mr. Nordmann gets a commission of $2.50 on each pair of women’s shoes he sells, and a commission of $3 on each pair of men’s shoes he sells. To meet his sales targets, he must sell at least 10 pairs of women’s shoes and at least 5 pairs of men’s shoes. He also wants to make at least $60 a week in commissions. Define the variables, write a system of inequalities to represent this situation, and name one possible solution.

Bonus  Find the area of the polygon formed by the system of equations $y = 0$, $y = 2x + 4$ and $-3y - 4x = -12$.  B: _______________
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Ruth invests $10,000 in two accounts. One account has an annual interest rate of 7%, and the other account has an annual interest rate of 5%. Let $I$ represent the total interest earned in one year. Then Ruth’s investments can be modeled by the system of equations $x + y = 10,000$ and $0.07x + 0.05y = I$.
   a. Determine a solution for this system of equations that represents a possible investment, and find the value of $I$ that corresponds to your solution.
   b. Find the solution for the system if $I = 800$. Explain why this solution does not represent a possible investment.

2. A car rental company wants to charge a rate of $A$ per day plus $B$ per mile to rent a compact car. Their leading competitor charges $15 per day plus $0.25 per mile to rent a compact car.
   a. Explain how the system of equations $y = A + Bx$ and $y = 15 + 0.25x$ compares the total cost of renting a compact car for one day from this company and from their leading competitor.
   b. Describe how the value of $A$ and the value of $B$ affects the comparison of the total cost of any one-day rental from these two companies.

3. A bookstore makes a profit of $2.50 on each book they sell, and $0.75 on each magazine they sell. Each week the store sells $x$ books and $y$ magazines. Let $P$ be the weekly profit, and $S$ be the weekly sales for the bookstore.
   a. Write a system of equations that models the possible weekly sales and weekly profit for the book store. Then describe possible values for $P$ and $S$.
   b. Choose a value for $P$ and a value for $S$, and substitute the values into the system of equations from part a. Make a graph of this system of equations, and describe what the graph represents.

4. A bird observatory is keeping a tally on the number of adult and baby condors nesting on their grounds. There are 21 birds in total. The number of adult condors is 3 fewer than 5 times the number of babies.
   a. Write a system of linear equations to model the situation.
   b. How many adult and baby condors are there?
1. Evaluate \([1 + 4(5)] + [3(9) - 7]\). (Lesson 1-2)
   \[A \ 45 \quad B \ 27 \quad C \ 41 \quad D \ 31\]

2. Rewrite \(3(4a - 2b + c)\) using the Distributive Property. (Lesson 1-4)
   \[F \ 12a - 2b + c \quad G \ 12a - 2b + 3c \quad H \ 12a - 6b + c \quad J \ 12a - 6b + 3c\]

3. How many liters of a 10% saline solution must be added to 4 liters of a 40% saline solution to obtain a 15% saline solution? (Lesson 2-9)
   \[A \ 20 \text{ L} \quad B \ 4 \text{ L} \quad C \ 2 \text{ L} \quad D \ 48 \text{ L}\]

4. When Nick was traveling in Montreal, the currency exchange rate between the U.S. and Canada could be modeled by \(d = 1.02c\) where \(d\) represents the number of U.S. dollars and \(c\) represents the number of Canadian dollars. Solve the equation for Canadian dollar amounts of $1, $2, $5, and $20. (Lesson 3-1)
   \[F \{(1, 1.02), (2, 2.04), (5, 5.10), (20, 20.40)\} \quad G \{(1, 1), (2, 2), (5, 5), (20, 20)\} \quad H \{(1, 0.98), (2, 1.96), (5, 4.9), (20, 19.61)\} \quad J \{(1, 2.02), (2, 3.02), (5, 6.02), (20, 21.02)\}\]

5. If a line passes through \((0, -6)\) and has a slope of \(-3\), what is the equation of the line? (Lesson 4-2)
   \[A \ y = -6x - 3 \quad B \ x = -6y - 3 \quad C \ y = -3x - 6 \quad D \ x = -3y - 6\]

6. If \(r\) is the slope of a line, and \(m\) is the slope of a line perpendicular to that line, what is the relationship between \(r\) and \(m\)? (Lesson 4-4)
   \[F \text{ There is no relationship.} \quad H \ r = m \quad J \ r = -\frac{1}{m}\]

7. Which inequality does not have the solution \(\{t \mid t > 4\}\)? (Lesson 5-2)
   \[A \ -t < -4 \quad B \ 3t > 12 \quad C \ t \frac{1}{2} > 2 \quad D \ -\frac{t}{8} > -\frac{1}{2}\]

8. Solve \(4 - 2r \geq 3(5 - r) + 7(r + 1)\). (Lesson 5-3)
   \[F \ \{r \mid r \leq -\frac{3}{2}\} \quad G \ \{r \mid r \leq -3\} \quad H \ \{r \mid r \leq -2\} \quad J \ \{r \mid r \leq -\frac{9}{4}\}\]

9. Write a compound inequality for the graph. (Lesson 5-4)
   \[A \ x < -1 \text{ and } x \geq 2 \quad C \ x \leq -1 \text{ or } x > 2 \quad B \ x < -1 \text{ or } x \geq 2 \quad D \ x \leq -1 \text{ and } x > 2\]

10. Evaluate \(x^2 + 5(y - 3)\) when \(x = -3\) and \(y = 14\). (Lesson 1-2)
    \[F \ 64 \quad G \ 58 \quad H \ 61 \quad J \ 29\]
11. Solve $8x - 5 = 19$. (Lesson 2-3)
   A $\frac{7}{2}$  B -3  C 3  D 6  
   11. ⬜️ ⬜️ ⬜️ ⬜️

12. Solve $-\frac{4}{x} = \frac{6}{9}$. (Lesson 2-3)
   F -6  G 6  H 12  J -12  
   12. ⬜️ ⬜️ ⬜️ ⬜️

13. Use the graph to determine how many solutions exist for the system $-4x + 3y = 12$ and $x + y = 2$. (Lesson 6-1)
   A 0  C 2  B 1  D infinitely many  
   13. ⬜️ ⬜️ ⬜️ ⬜️

14. Use elimination to solve the system $2x + y = -8$ and $-2x + 3y = -8$ for $x$. (Lesson 6-3)
   F -2  G 2  H -4  J 4  
   14. ⬜️ ⬜️ ⬜️ ⬜️

15. The substitution method could be used to solve which system of equations? (Lesson 6-2)
   A $5x - 7y = 8$
   C $3x - 3y = 5$
   B $2x + 6y = 6$
   D $-2x + 2y = -6$
   $3x + 2y = 8$
   $4x + 3y = 5$

   15. ⬜️ ⬜️ ⬜️ ⬜️

16. Which graph represents the solution of $|n + 5| > 1$? (Lesson 5-5)
   F  H  J  
   16. ⬜️ ⬜️ ⬜️ ⬜️

**Part 2: Gridded Response**

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

For Questions 17 and 18, determine the value that is missing.

17. The solution set is $\{n \mid n \geq 15\}$ for the inequality $n - 7 \geq \_$. (Lesson 5-1)

18. If $|a - 8| = 17$, then $a = \_\_$ or $a = -9$. (Lesson 2-5)
19. State whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change.
   original: 76; new: 57  (Lesson 2-7)

20. Express the relation {(-2, 1), (3, -1), (2, -2), (-2, 0)} as a mapping.  (Lesson 1-6)

21. Determine whether -6, -3, 0, 3 ... is an arithmetic sequence. If it is, state the common difference. (Lesson 3-4)

22. Find the slope of the line that passes through (-2, 0) and (5, -8). (Lesson 3-3)

23. The Lopez family drove 165 miles in 3 hours. Write a direct variation equation for the distance driven in any time. How far can the Lopez family drive in 5 hours? (Lesson 3-4)

24. Write an equation of a line that passes through (-2, -1) with slope 3. (Lesson 4-3)

25. Solve the system of inequalities by graphing.  (Lesson 6-6)
   \[ 2x - y \geq 4 \]
   \[ x - 2y < 4 \]

26. Solve \( 12 + r < 15 \). Then graph the solution. (Lesson 5-1)

27. Define a variable, write a compound inequality, and solve the problem. (Lesson 5-4)
   Seven less than twice a number is greater than 13 or less than or equal to -5.

28. Three times a first number minus a second number equals negative forty. The first number plus twice the second number equals negative four. (Lesson 6-5)
   a. Define variables and formulate a system of linear equations from this situation.
   b. What are the numbers?
1. Graph $3x - y = 1$.

2. Solve $4x + 9 = 4x + 13$.

3. Find the value of $r$ so that the line through $(2, -3)$ and $(-4, r)$ has a slope of $-\frac{1}{2}$.

4. A giraffe can travel 800 feet in 20 seconds. Write a direct variation equation for the distance traveled in any time.

5. Find the 25th term of the arithmetic sequence with first term 7 and common difference $-2$.

6. Write an equation of the line whose slope is 2 and whose $y$-intercept is 9.

7. Write an equation of the line that passes through $(-1, -7)$ and $(1, 3)$.

8. Write $y - 4 = -\frac{3}{2}(x + 6)$ in standard form.

9. Write the slope-intercept form of an equation of the line that passes through $(-2, 0)$ and is parallel to the graph of $y = -3x - 2$.

10. The table below shows the distance driven during four different trips and the duration of each trip. Draw a scatter plot and determine what relationship exists, if any, in the data. Write an equation for a line of fit for the data.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>1</th>
<th>2</th>
<th>2.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles)</td>
<td>50</td>
<td>85</td>
<td>120</td>
<td>180</td>
</tr>
</tbody>
</table>
11. The table below shows the cost to ride the New York City subway in various years.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fare</td>
<td>$0.90</td>
<td>$1.00</td>
<td>$1.15</td>
<td>$1.25</td>
<td>$1.50</td>
<td>$2.00</td>
</tr>
</tbody>
</table>

Source: Metropolitan Transportation Authority (MTA)

Use a regression line to estimate the cost of a subway ride in 2014.

12. Solve each inequality.
   12. $4x - 5 < 7x + 10$
   13. $2(5a - 4) - 3(6 + 2a) \leq 6$

13. Solve each compound inequality.
   14. $5 < 2t + 7 < 11$
   15. $13 < 4 - 3v$ or $2v - 14 > 8$

For Questions 16 and 17, solve each open sentence. Then graph the solution set.

16. $|3b - 5| \leq 7$
17. $|w + 5| > 1$

18. Use a graph to determine whether the system $x - y = 4$ and $y = x$ has no solution, one solution, or infinitely many solutions.

For Questions 19–22, determine the best method to solve each system of equations. Then solve the system.

19. $x + y = 2$
   $y = 2x - 1$
20. $-x - 5y = 7$
   $x + y = 1$

21. $3x + y = 26$
   $3x + 3y = 18$
22. $4x - 8y = 52$
   $7x + 4y = 1$

23. Solve the system of inequalities by graphing.
   $2x - y > 3$
   $x + 2y \leq 4$
Anticipation Guide
Solving Systems of Linear Equations

Step 1
Before you begin Chapter 6
• Read each statement.
• Decide whether you Agree (A) or Disagree (D) with the statement.
• Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>Statement</th>
<th>STEP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, or NS</td>
<td></td>
<td>A or D</td>
</tr>
<tr>
<td>1.</td>
<td>A solution of a system of equations is any ordered pair that satisfies one of the equations</td>
<td>D</td>
</tr>
<tr>
<td>2.</td>
<td>A system of equations of parallel lines will have no solutions.</td>
<td>A</td>
</tr>
<tr>
<td>3.</td>
<td>A system of equations of two perpendicular lines will have infinitely many solutions.</td>
<td>D</td>
</tr>
<tr>
<td>4.</td>
<td>It is not possible to have exactly two solutions to a system of linear equations</td>
<td>A</td>
</tr>
<tr>
<td>5.</td>
<td>The most accurate way to solve a system of equations is to graph the equations to see where they intersect.</td>
<td>D</td>
</tr>
<tr>
<td>6.</td>
<td>To solve a system of equations, such as (2x - y = 21) and (3y = 2x - 6), by substitution, solve one of the equations for one variable and substitute the result into the other equation.</td>
<td>A</td>
</tr>
<tr>
<td>7.</td>
<td>When solving a system of equations, a result that is a true statement, such as (-5 = -5), means the equations do not share a common solution.</td>
<td>D</td>
</tr>
<tr>
<td>8.</td>
<td>Adding the equations (3x - 4y = 8) and (2x + 4y = 7) results in a 0 coefficient for (y).</td>
<td>A</td>
</tr>
<tr>
<td>9.</td>
<td>The equation (7x - 2y = 12) can be multiplied by 2 so that the coefficient of (y) is (-4).</td>
<td>A</td>
</tr>
<tr>
<td>10.</td>
<td>The result of multiplying (-7x - 3y = 11) by (-3) is (-1x + 9y = 11).</td>
<td>D</td>
</tr>
</tbody>
</table>

Step 2
After you complete Chapter 6
• Reread each statement and complete the last column by entering an A or a D.
• Did any of your opinions about the statements change from the first column?
• For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

1. \(y = -x + 2\) \(y = x + 1\)
   Since the graphs of \(y = -x + 2\) and \(y = x + 1\) intersect, there is one solution. Therefore, the system is consistent and independent.

2. \(3x + 3y = -3\)
   Since the graphs of \(3x + 3y = -3\) are parallel, there are no solutions. Therefore, the system is inconsistent.

3. \(y = -x - 1\)
   Since the graphs of \(3x + 3y = -3\) and \(y = -x - 1\) coincide, there are infinitely many solutions. Therefore, the system is consistent and dependent.

Exercises
Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.

1. \(y = -x - 3\)
   \(y = x - 1\)
   consistent; independent

2. \(2x + 2y = -6\)
   \(y = -x - 3\)
   consistent; dependent

3. \(y = -x - 3\)
   \(2x + 2y = 4\)
   inconsistent

4. \(2x + 2y = -6\)
   \(3x + y = 3\)
   consistent; independent
Example

Graph each system and determine the number of solutions that it has.

1. \( x - y = -4 \) consistent; independent
2. \( 2x - y = 4 \) consistent; independent
3. \( y = x + 4 \) consistent; independent
4. \( y = 2x - 3 \)
5. \( 3y = -2x + 1 \)
6. \( 2x + y = 4 \) consistent; independent
7. \( 3y + x = -3 \)
8. \( y = x + 2 \) consistent; independent
9. \( y = -x - 3 \)
10. \( y = -x + 1 \)
11. \( x + y = 3 \)
12. \( x = y = 2 \) consistent; independent

Exercises

Graph each system and determine the number of solutions that it has.

1. \( y = x - 2 \) one; (2, -1)
2. \( y = \frac{1}{2}x + 3 \) one; (6, 4)
3. \( 2x + y = 6 \) no solution
4. \( x = y = 6 \) two; (2, -1)
5. \( 3x + 2y = 6 \) infinitely; \( x = 2, y = 0 \)
6. \( 2x + y = 4 \) infinitely; \( x = 2, y = 0 \)
7. \( x = y = 3 \) infinitely; \( x = 2, y = 0 \)
**6-1 Practice**

**Graphing Systems of Equations**

Use the graph at the right to determine whether each system is consistent or inconsistent and if it is independent or dependent.

1. \(x + y = 3\) and \(x + y = -3\)
   - Inconsistent

2. \(2x - y = -3\) and \(4x - 2y = -6\)
   - Consistent and dependent

3. \(x + 3y = 3\) and \(4x + 3y = 3\)
   - Consistent and independent

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

5. \(3x - y = -2\) — No solution

6. \(y = 2x - 3\) — Infinitely many solutions

7. \(x + 2y = 3\) — One solution: \((-1, 2)\)

8. **BUSINESS** Nick plans to start a home-based business producing and selling gourmet dog treats. He figures it will cost $20 in operating costs per week plus $0.50 to produce each treat. He plans to sell each treat for $1.50.
   a. Graph the system of equations \(y = 0.5x + 20\) and \(y = 1.5x\) to represent the situation.
   b. How many treats does Nick need to sell per week to break even? 20

9. **SALES** A used book store also started selling used CDs and videos. In the first week, the store sold 40 used CDs and videos, at $4.00 per CD and $6.00 per video. The sales for both CDs and videos totaled $180.00.
   a. Write a system of equations to represent the situation: \(c + v = 40\), \(4c + 6v = 180\)
   b. Graph the system of equations.
   c. How many CDs and videos did the store sell in the first week? 30 CDs and 10 videos

**6-1 Word Problem Practice**

**Graphing Problem Practice**

1. **BUSINESS** The widget factory will sell a total of 600 widgets after \(x\) days according to the equation \(y = 200x + 300\). The gadget factory will sell \(y\) gadgets after \(x\) days according to the equation \(y = 200x + 100\). Look at the graph of the system of equations and determine if it has no solution, one solution, or infinitely many solutions.
   - No solution

2. **ARCHITECTURE** An office building has two elevators. One elevator starts out on the 4th floor, 35 feet above the ground, and is descending at a rate of 2.2 feet per second. The other elevator starts out at ground level and is rising at a rate of 1.7 feet per second. Write a system of equations to represent the situation.
   - Sample answer: \(y = 35 - 2.2x\), \(y = 1.7x\)

3. **FITNESS** Olivia and her brother William had a bicycle race. Olivia rode at a speed of 20 feet per second while William rode at a speed of 15 feet per second. To be fair, Olivia decided to give William a 150-foot head start. The race ended in a tie. How far away was the finish line from where Olivia started? 600 ft

4. **AVIATION** Two planes are in flight near a local airport. One plane is at an altitude of 1000 meters and is ascending at a rate of 400 meters per minute. The second plane is at an altitude of 5900 meters and is descending at a rate of 300 meters per minute.
   a. Write a system of equations that represents the progress of each plane: \(y = 400x + 1000\), \(y = 5900 - 300x\)
   b. Make a graph that represents the progress of each plane.
Chapter 6

6-1 Enrichment

Graphing a Trip
The distance formula, \( d = rt \), is used to solve many types of problems. If you graph an equation such as \( d = 50t \), the graph is a model for a car going at 50 mph. The time the car travels is \( t \); the distance in miles the car covers is \( d \). The slope of the line is the speed.

Suppose you drive to a nearby town and return. You average 50 mph on the trip out but only 25 mph on the trip home. The round trip takes 5 hours. How far away is the town?

The graph at the right represents your trip. Notice that the return trip is shown with a negative slope because you are driving in the opposite direction.

Solve each problem.

1. Estimate the answer to the problem in the above example. About how far away is the town?

Graph each trip and solve the problem.

2. An airplane has enough fuel for 3 hours of safe flying. On the trip out the pilot averages 200 mph flying against a headwind. On the trip back, the pilot averages 250 mph. How long a trip out can the pilot make?

about \( \frac{10}{3} \) hours and 330 miles

3. You drive to a town 100 miles away. On the trip out you average 25 mph. On the trip back you average 50 mph. How many hours do you spend driving?

6 hours

4. You drive at an average speed of 50 mph to a discount shopping plaza, spend 2 hours shopping, and then return at an average speed of 25 mph. The entire trip takes 8 hours. How far away is the shopping plaza?

100 miles

Graphing Calculator Activity

Solution to a System of Linear Equations
A graphing calculator can be used to solve a system of linear equations graphically. The solution of a system of linear equations can be found by using the TRACE feature or by using the intersect command under the CALC menu.

Example
Solve each system of linear equations.

a. \( x + y = 0 \)
\( x - y = -4 \)

Using TRACE: Solve each equation for \( y \) and enter each equation into \( Y_1 \). Then graph using Zoom 8: ZInteger. Use TRACE to find the solution.

Keystrokes: \( Y_1 = \) \( \frac{x}{2} \), \( Y_2 = 2 - x \)

The solution is \((2, 2)\).

b. \( 2x + y = 4 \)
\( 4x + 3y = 3 \)

Using CALC: Solve each equation for \( x \) and enter each into the calculator, and graph. Use CALC to determine the solution.

Keystrokes: \( Y_1 = - \frac{2}{3} x + \frac{4}{3} \), \( Y_2 = - \frac{8}{3} x + 4 \)

To change the \( x \)-value to a fraction, press \( \text{MATH} \), 1, \( \text{QUIT} \), \( \text{MATH} \), 2, \( \text{QUIT} \).

The solution is \((4.5, -0.5)\) or \((\frac{9}{2}, -\frac{1}{2})\).

Exercises
Solve each system of linear equations.

1. \( y = -x + 2 \)
\( 5x + 6y = 18 \)
\((2, 2)\)

2. \( y = -x + 3 \)
\( y = x + 1 \)
\((1, 2)\)

3. \( x + y = -1 \)
\( 2x - y = -8 \)
\((-3, 2)\)

4. \(-3x + y = 10 \)
\(-x + 2y = 0 \)
\((-4, -2)\)

5. \(-4x + 3y = 10 \)
\(7x + y = 20 \)
\((2, 6)\)

6. \(5x + 3y = 11 \)
\(x - 5y = 5 \)
\((2.5, -0.5)\)

7. \(3x - 2y = -4 \)
\(-4x + 3y = 5 \)
\((-2, -1)\)

infinite solutions

8. \(3x + 2y = 4 \)
\(-6x - 4y = -8 \)
\((5, 4)\)

9. \(4x - 5y = 0 \)
\(6x - 5y = 10 \)

Chapter 6
**Example 1** Use substitution to solve the system of equations.

\[
y = 2x
\]

4x – y = -4

Substitute 2x for y in the second equation.

4x − 2x = -4

2x = -4

x = -2

The solution is (-2, -4).

**Example 2** Solve for one variable, then substitute.

\[
x + 3y = 7
\]

2x − 4y = -6

Solve the first equation for x since the coefficient of x is 1.

\[
x = 7 - 3y
\]

Find the value of y by substituting 7 − 3y for x in the second equation.

\[
2(7 - 3y) - 4y = -6
\]

27 − 10y − 4y = -6

14 − 10y = -6

Combine like terms.

14 − 10y − 14 = -6 − 14

-10y = -20

Simplify

y = 2

Divide each side by -10 and simplify.

Use y = 2 to find the value of x.

\[
x = 7 - 3y
\]

x = 7 − 3(2)

x = 1

The solution is (1, 2).

**Exercises**

Use substitution to solve each system of equations.

1. y = 4x
   
   3x − y = 1
   
   (-1, -4)

2. x = 2y
   
   y = x − 2
   
   (4, 2)

3. x = 2y − 3
   
   x = 2y + 4
   
   no solution

4. x = -2y = 1
   
   (6, 3)

5. 5x = 4y = 1
   
   infinitely

6. x + 2y = 0

7. 2b = 6a − 14
   
   infinitely

8. 8x + y = 16

9. 3b − b = 7
   
   many

10. x = 2y (20, 20)
   
   (20, 10)

11. 5x + 0.5y = 10
   
   0.25x + 0.5y = 10

12. -0.2x + y = 0.5
   
   0.4x + y = 1.1
   
   (1, 0.7)

**Exercises (continued)**

**Substitution**

Solve Real-World Problems

Substitution can also be used to solve real-world problems involving systems of equations. It may be helpful to use tables, charts, diagrams, or graphs to help you organize data.

**Example** CHEMISTRY How much of a 10% saline solution should be mixed with a 20% saline solution to obtain 1000 milliliters of a 12% saline solution?

Let s = the number of milliliters of 10% saline solution.

Let t = the number of milliliters of 20% saline solution.

Use a table to organize the information.

<table>
<thead>
<tr>
<th>Total milliliters</th>
<th>10% saline</th>
<th>20% saline</th>
<th>12% saline</th>
</tr>
</thead>
<tbody>
<tr>
<td>s + t = 1000</td>
<td>0.10s</td>
<td>0.20t</td>
<td>0.12(1000)</td>
</tr>
<tr>
<td>Milliliters of saline</td>
<td>first equation</td>
<td>second equation</td>
<td>third equation</td>
</tr>
<tr>
<td>s</td>
<td>t</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

Write a system of equations.

\[
s + t = 1000
\]

\[
0.10s + 0.20t = 0.12(1000)
\]

Use substitution to solve this system.

\[
s + t = 1000\]

\[
s = 1000 - t\]

Solve for s.

\[
\begin{align*}
0.10s + 0.20t &= 0.12(1000) \\
0.10s + 0.20t &= 1200 \\
0.10s &= 1200 - 1000 \\
s &= 200
\end{align*}
\]

Divide each side by 0.10.

\[
0.10s = 200
\]

\[
t = 200
\]

Use each side by 0.10.

\[
s = 200
\]

800 milliliters of 10% solution and 20 milliliters of 20% solution should be used.

**Exercises**

1. SPORTS At the end of the 2007–2008 football season, 38 Super Bowl games had been played with the current two football leagues, the American Football Conference (AFC) and the National Football Conference (NFC). The AFC won 2 more games than the NFC. How many games did each conference win? AFC 18; NFC 20

2. CHEMISTRY A lab needs to make 100 gallons of an 18% acid solution by mixing a 12% acid solution with a 20% solution. How many gallons of each solution are needed?

25 gal of 12% solution and 75 gal of 20% solution

3. GEOMETRY The perimeter of a triangle is 24 inches. The longest side is 4 inches longer than the shortest side, and the shortest side is three-fourths the length of the middle side. Find the length of each side of the triangle. 6 in., 8 in., 10 in.
**6-2 Skills Practice**

### Substitution

Use substitution to solve each system of equations.

1. \(y = 4x\)
   \(x + y = 5\)
   \((1, 4)\)
   \(2. y = 2x\)
   \(x + 3y = -14\)
   \((-2, -4)\)

3. \(y = 3x\)
   \(2x + y = 15\)
   \((3, 9)\)

4. \(x = -3y\)
   \(3x + 2y = 20\)
   \((8, -2)\)

5. \(y = x - 1\)
   \(x + y = 3\)
   \((2, 1)\)

6. \(x = y - 7\)
   \(x + 8y = 2\)
   \((-6, 1)\)

7. \(y = 2x - 5\)
   \((-2, -9)\)

8. \(y = 3x + 8\)
   \(5x + 2y = 5\)
   \((-1, 5)\)

9. \(2x - 3y = 21\)
   \(y = 3 - x\)
   \((-6, -3)\)

10. \(y = 5x - 8\)
    \(4x + 3y = 33\)
    \((3, 7)\)

11. \(x + 2y = 13\)
    \(3x - 5y = 6\)
    \((7, 3)\)

12. \(x + 5y = 4\)
    \(3x + 15y = -1\)
    **no solution**

13. \(3x - y = 4\)
    \(2x - 3y = -9\)
    \((3, 5)\)

14. \(x + 4y = 8\)
    \(2x - 5y = 29\)
    \((-12, -1)\)

15. \(x - 5y = 10\)
    \(2x - 10y = 20\)
    **infinitely many**

16. \(5x - 2y = 14\)
    \(2x - y = 5\)
    \((4, 3)\)

17. \(2x + 5y = 38\)
    \(x - 3y = -3\)
    \((9, 4)\)

18. \(s - 4y = 27\)
    \(3x + y = -23\)
    \((-5, -8)\)

19. \(2x + 2y = 7\)
    \(x - 2y = -1\)
    \((2, 3)\)

20. \(2.5x + y = -2\)
    \(3x + 2y = 0\)
    \((-2, 3)\)

---

**6-2 Practice**

Use substitution to solve each system of equations.

1. \(y = 6x\)
   \(2x + 3y = -20\)
   \((-1, -6)\)

2. \(y = 3y\)
   \(3x - 5y = 12\)
   \((9, 3)\)

3. \(x = 2y + 7\)
   \(y = x + 4\)
   \((1, -3)\)

4. \(y = 2x - 2\)
   \(y = x + 2\)
   \((4, 6)\)

5. \(y = 2x + 6\)
   \(2x - y = 2\)
   **no solution**

6. \(x + y = 12\)
   \(y = -x - 2\)
   \((-1, 10)\)

7. \(2x - 3y = -18\)
   \(x - 2y = 12\)
   **many**

8. \(x - 2y = 3\)
   \(4x - 8y = 12\)
   **infinitely**

9. \(x - 5y = 36\)
   \(-2x - 3y = -18\)
   **no solution**

10. \(2x - 3y = -24\)
    \(x + 6y = 18\)
    \((-6, 4)\)

11. \(x + 14y = 84\)
    \(2x - 7y = -7\)
    \((14, 5)\)

12. \(0.3x - 0.2y = 0.5\)
    \(x - 2y = -5\)
    \((5, 5)\)

13. \(0.5x + 4y = -1\)
    \(3x - 2y = 11\)
    \((6, -1)\)

14. \(x + 2.5y = 3.5\)
    \(x - \frac{1}{2}y = 4\)
    \((5, 2)\)

15. \(x - 2y = 6\)
    \(x - 2y = 6\)
    \((12, 3)\)

16. \(\frac{1}{2}x - y = 3\)
    \(4x - 5y = -7\)
    \((12, 3)\)

17. \(2x + y = 25\)
    \(y = 5x\)
    **no solution**

18. \(x + 3y = -4\)

19. **EMPLOYMENT** Kenisha sells athletic shoes part-time at a department store. She can earn either $500 per month plus a 4% commission on her total sales, or $400 per month plus a 3% commission on total sales.

   a. Write a system of equations to represent the situation.
      
      \[
      \begin{align*}
      y &= 0.04x + 500 \\
      y &= 0.03x + 400
      \end{align*}
      \]

   b. What is the total price of the athletic shoes Kenisha needs to sell to earn the same income from each pay scale? $10,000.

   c. Which is the better offer? the first offer if she expects to sell less than $10,000 in shoes, and the second offer if she expects to sell more than $10,000 in shoes.

20. **MOVIE TICKETS** Tickets to a movie cost $7.25 for adults and $5.50 for students. A group of friends purchased 8 tickets for $52.75.

   a. Write a system of equations to represent the situation.
      
      \[
      \begin{align*}
      x + y &= 8 \\
      7.25x + 5.5y &= 52.75
      \end{align*}
      \]

   b. How many adult tickets and student tickets were purchased? 5 adult and 3 student.
6-2 Word Problem Practice

**Substitution**

1. **BUSINESS** Mr. Randolph finds that the supply and demand for gasoline at his station are generally given by the following equations.
   \[ x - y = -2 \]
   \[ x + y = 10 \]
   Use substitution to find the equilibrium point where the supply and demand lines intersect. \((4, 6)\)

2. **GEOMETRY** The measures of complementary angles have a sum of 90 degrees. Angle A and angle B are complementary, and their measures have a difference of 20°. What are the measures of the angles? \(35° \) and \(55°\)

3. **MONEY** Harvey has some $1 bills and some $5 bills. In all, he has 6 bills worth $22. Let \(x\) be the number of $1 bills and let \(y\) be the number of $5 bills. Write a system of equations to represent the information and use substitution to determine how many bills of each denomination Harvey has.
   \[ x + y = 6 \]
   \[ x + 5y = 22 \]
   He has four $5 bills and two $1 bills.

4. **POPULATION** Sanjay is researching population trends in South America. He found that the population of Ecuador increased by 1,000,000 and the population of Chile increased by 600,000 from 2004 to 2009. The table displays the information he found.

<table>
<thead>
<tr>
<th>Country</th>
<th>2004 Population</th>
<th>5-Year Population Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ecuador</td>
<td>13,000,000</td>
<td>+1,000,000</td>
</tr>
<tr>
<td>Chile</td>
<td>16,000,000</td>
<td>+600,000</td>
</tr>
</tbody>
</table>

*Source: World Almanac*

If the population growth for each country continues at the same rate, in what year are the populations of Ecuador and Chile predicted to be equal? 2041

5. **CHEMISTRY** Shelby and Calvin are doing a chemistry experiment. They need 5 ounces of a solution that is 65% acid and 35% distilled water. There is no undiluted acid in the chemistry lab, but they do have two flasks of diluted acid:

- Flask A contains 70% acid and 30% distilled water.
- Flask B contains 20% acid and 80% distilled water.

**a.** Write a system of equations that Shelby and Calvin could use to determine how many ounces they need to pour from each flask to make their solution.

Sample answer: \(a + b = 5; 0.7a + 0.2b = 0.65(5)\)

**b.** Solve your system of equations. How many ounces from each flask do Shelby and Calvin need? 4.5 oz from Flask A and 0.5 oz from Flask B.

6-2 Enrichment

**Intersection of Two Parabolas**

Substitution can be used to find the intersection of two parabolas. Replace the \(y\)-value in one of the equations with the \(y\)-value in terms of \(x\) from the other equation.

**Example** Find the intersection of the two parabolas.

\[ y = x^2 + 5x + 6 \]
\[ y = x^2 + 4x + 3 \]

Graph the equations.

From the graph, notice that the two graphs intersect in one point.

Use substitution to solve for the point of intersection.

\[ x^2 + 5x + 6 = x^2 + 4x + 3 \]

Subtract \(x^2\) from each side.

\[ 5x + 6 = 4x + 3 \]

Subtract 4\(x\) from each side.

\[ x = -3 \]

The graphs intersect at \(x = -3\).

Replace \(x\) with \(-3\) in either equation to find the \(y\)-value.

\[ y = (-3)^2 + 5(-3) + 6 \]

Original equation

\[ y = 9 - 15 + 6 \]

Replace \(x\) with \(-3\).

\[ y = 0 \]

Simplify

The point of intersection is \((-3, 0)\).

**Exercises**

Use substitution to find the point of intersection of each pair of equations.

1. \(y = x^2 + 8x + 7\)
   \(y = x^2 + 2x + 1\)
   \((-1, 0)\)

2. \(y = x^2 + 6x + 8\)
   \(y = x^2 + 4x + 4\)
   \((-2, 0)\)

3. \(y = x^2 + 5x + 6\)
   \(y = x^2 + 7x + 6\)
   \((0, 6)\)
Elimination Using Addition

In systems of equations where the coefficients of the x or y terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called elimination.

Example 1

Use elimination to solve the system of equations.

\[ \begin{align*}
2x - 3y &= 7 \\
3x + 3y &= 9
\end{align*} \]

Write the equations in column form and add to eliminate y.

\[ \begin{align*}
(+x) 2x + 0y &= 24 \\
2x + 3y &= 94
\end{align*} \]

\[ x = 47 \]

Solve for x.

\[ 4x = 16 \]

\[ x = 4 \]

Substitute 4 for x in either equation and solve for y.

\[ \begin{align*}
4 - 3y &= 7 \\
-3y &= 3 \\
y &= -1
\end{align*} \]

The solution is (4, -1).

Example 2

The sum of two numbers is 70 and their difference is 24. Find the numbers.

Let \( x \) represent one number and \( y \) represent the other number.

\[ \begin{align*}
x + y &= 70 \\
(-x - y) &= 24
\end{align*} \]

\[ \begin{align*}
2x &= 94 \\
x &= 47
\end{align*} \]

Substitute 47 for \( x \) in either equation.

\[ \begin{align*}
y &= 70 - 47 \\
y &= 23
\end{align*} \]

The numbers are 47 and 23.

Exercises

Use elimination to solve each system of equations.

1. \[ \begin{align*}
x + y &= 4 \\
x - y &= 0
\end{align*} \]

2. \[ \begin{align*}
x + y &= 14 \\
x - y &= 2
\end{align*} \]

3. \[ \begin{align*}
x - y &= 9 \\
x - y &= 2
\end{align*} \]

4. \[ \begin{align*}
x + y &= 1 \\
x - y &= 3
\end{align*} \]

5. \[ \begin{align*}
x + y &= 11 \\
x - y &= 2
\end{align*} \]

6. \[ \begin{align*}
x + y &= 16 \\
x - y &= 10
\end{align*} \]

7. \[ \begin{align*}
x + y &= 13 \\
x - y &= 7
\end{align*} \]

8. \[ \begin{align*}
x + y &= 6 \\
x - y &= 5
\end{align*} \]

9. \[ \begin{align*}
x + y &= 10 \\
x - y &= 8
\end{align*} \]

10. \[ \begin{align*}
x + y &= 12 \\
x - y &= 1
\end{align*} \]

11. \[ \begin{align*}
x + y &= 25 \\
x - y &= 9
\end{align*} \]

12. \[ \begin{align*}
x + y &= 24 \\
x - y &= 12
\end{align*} \]

13. Rema is older than Ken. The difference of their ages is 12 and the sum of their ages is 50. Find the age of each. Rema is 31 and Ken is 19.

14. The sum of the digits of a two-digit number is 12. The difference of the digits is 2. Find the number if the units digit is larger than the tens digit. 57
6-3 Skills Practice

Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

1. \(x - y = 1\)
   \(x + y = 3\)  \((2, 1)\)

2. \(-x + y = 1\)
   \(x + y = 11\)  \((5, 6)\)

3. \(x + 4y = 11\)
   \(x - 6y = 11\)  \((11, 0)\)

4. \(-x + 3y = 6\)
   \(x + 3y = 8\)  \((6, 4)\)

5. \(3x + 4y = 19\)
   \(3x + 6y = 33\)  \((-3, 7)\)

6. \(x + 4y = -8\)
   \(-x - 4y = -8\)  \((-8, 0)\)

7. \(3x + 4y = 2\)
   \(-4x - 4y = 12\)  \((-2, -1)\)

8. \(3x - y = -1\)
   \(-3x - y = 5\)  \((-1, -2)\)

9. \(2x - 3y = 9\)
   \(-5x - 3y = 30\)  \((-3, -5)\)

10. \(x - y = 4\)
    \(2x + y = -4\)  \((0, -4)\)

11. \(3x - y = 26\)
    \(-2x - y = -24\)  \((10, 4)\)

12. \(5x - y = -6\)
    \(-x + y = 2\)  \((-1, 1)\)

13. \(6x - 2y = 32\)
    \(-4x - 2y = 18\)  \((7, 5)\)

14. \(3x + 2y = -19\)
    \(-3x - 5y = 25\)  \((-5, -2)\)

15. \(7x + 4y = 2\)
    \(7x + 2y = 8\)  \((2, -3)\)

16. \(2x - 5y = -28\)
    \(4x + 3y = 4\)  \((-4, 4)\)

17. The sum of two numbers is 28 and their difference is 4. What are the numbers? 12, 16

18. Find the two numbers whose sum is 29 and whose difference is 15. 7, 22

19. The sum of two numbers is 24 and their difference is 2. What are the numbers? 13, 11

20. Find the two numbers whose sum is 54 and whose difference is 4. 25, 29

21. Two times a number added to another number is 25. Three times the first number minus the other number is 20. Find the numbers. 9, 7
**6-3 Word Problem Practice**

**Elimination Using Addition and Subtraction**

1. **NUMBER FUN** Ms. Simms, the sixth grade math teacher, gave her students this challenge problem.
   
   Twice a number added to another number is 15. The sum of the two numbers is 11.
   
   Lorenzo, an algebra student who was Ms. Simms aide, realized he could solve the problem by writing the following system of equations.
   
   \[
   2x + y = 15 \\
   x + y = 11
   \]
   
   Use the elimination method to solve the system and find the two numbers. (4, 7)

2. **GOVERNMENT** The Texas State Legislature is comprised of state senators and state representatives. The sum of the number of senators and representatives is 181. There are 119 more representatives than senators. How many senators and how many representatives make up the Texas State Legislature? 31 state senators and 150 state representatives

3. **RESEARCH** Melissa wondered how much it would cost to send a letter by mail in 1990, so she asked her father. Rather than answer directly, Melissa’s father gave her the following information. It would have cost $3.70 to send 13 postcards and 7 letters, and it would have cost $2.65 to send 6 postcards and 7 letters. Use a system of equations and elimination to find how much it cost to send a letter in 1990. $0.25

4. **SPORTS** As of 2010, the New York Yankees had won more World Series Championships than any other team. In fact, the Yankees had won 3 fewer than 3 times the number of World Series championships won by the second most-winning team, the St. Louis Cardinals. The sum of the two teams’ World Series championships is 37. How many times has each team won the World Series?
   
   The Cardinals have 10 wins and the Yankees have 27 wins.

**6-3 Enrichment**

**Solving Systems of Equations in Three Variables**

Systems of equations can involve more than 2 equations and 2 variables. It is possible to solve a system of 3 equations and 3 variables using elimination.

**Example** Solve the following system.

\[
\begin{align*}
3x - y + z &= 8 \\
x - z &= 2
\end{align*}
\]

**Step 1:** Use elimination to get rid of the \( y \) in the first two equations.

\[
\begin{align*}
3x - y + z &= 8 \\
3x - 2x &= 14
\end{align*}
\]

Multiply the second equation by 2 so that the \( z \)'s will eliminate.

\[
\begin{align*}
3x - y + z &= 8 \\
6x &= 18
\end{align*}
\]

So, \( x = 3 \).

**Step 3:** Replace \( x \) with 3 in the third original equation to determine \( z \).

\[
3 - z = 2, \quad z = 1
\]

**Step 4:** Replace \( x \) with 3 and \( z \) with 1 in either of the first two original equations to determine the value of \( y \).

\[
3 + y + 1 = 6 \quad \text{or} \quad 4 + y = 6, \quad y = 2, \quad y = 2
\]

So, the solution to the system of equations is \((3, 2, 1)\).

**Exercises**

Solve each system of equations.

\[
\begin{align*}
\text{1. } & 3x + 2y + z = 42 \\
& 2y + z = 12 = 3x \\
& x - 3y = 0
\end{align*}
\]

\[
\begin{align*}
\text{2. } & x + y + z = -3 \\
& 2x + 3y + 5z = -4 \\
& 2y - z = 4
\end{align*}
\]

\[
\begin{align*}
\text{3. } & x + y + z = 7 \\
& 2x + y + z = 10 \\
& 2y + z = 5
\end{align*}
\]

\[
\begin{align*}
& x = 9 \\
& y = 3 \\
& z = 9
\end{align*}
\]

\[
\begin{align*}
& x = -5 \\
& y = 2 \\
& z = 0
\end{align*}
\]

\[
\begin{align*}
& x = 5 \\
& y = 3 \\
& z = -1
\end{align*}
\]
During a canoeing trip, it takes Raymond 4 hours to paddle 12 miles upstream. It takes him 3 hours to make the return trip paddling downstream. Find the speed of the canoe in still water.

**Example 1** Use elimination to solve the system of equations.

\[
\begin{align*}
x + 10y &= 3 \\
4x + 5y &= 5
\end{align*}
\]

If you multiply the second equation by -2, you can eliminate the \( y \) terms.

\[
\begin{align*}
7x - 20y &= -10 \\
-7x &= 14
\end{align*}
\]

Substitute 1 for \( x \) in either equation.

\[
\begin{align*}
1 + 10y &= 3 \\
10y &= 2
\end{align*}
\]

The solution is \( (1, \frac{1}{5}) \).

**Example 2** Use elimination to solve the system of equations.

\[
\begin{align*}
x - 2y &= -7 \\
2x - 5y &= 10
\end{align*}
\]

If you multiply the first equation by 2 and the second equation by -3, you can eliminate the \( x \) terms.

\[
\begin{align*}
6x - 4y &= -14 \\
11y &= 44
\end{align*}
\]

Substitute \(-4\) for \( y \) in either equation.

\[
\begin{align*}
x - 2(-4) &= -7 \\
x &= -5
\end{align*}
\]

The solution is \( (-5, -4) \).

**Exercises**

Use elimination to solve each system of equations.

1. \( 2x + 3y = 6 \) \( (3, 4) \) \( 2x + 5 = 10 \) \( (1, 2) \) \( x + 2y = 5 \) \( (3, 2) \) \( 3a - b = 2 \) \( (1, 1) \)
2. \( 2m + 3n = 4 \) \( -m + 2n = 5 \) \( (1, 2) \) \( a + 2b = 3 \) \( (1, 3) \)
3. \( 4x + 5y = 6 \) \( 5x - 3y = 22 \) \( 6x - 7y = -20 \) \( (4, -2) \) \( 6x - 2y = 8 \) \( (2, -1) \) \( 10x + 2y = 8 \) \( (1, 2) \) \( 12x + 2y = -5 \) \( (3, 1) \)
4. \( 4x - y = 9 \) \( 8x - 3y = -8 \) \( 5x + 2y = 8 \) \( (2, -1) \) \( 2a + 2b = 3 \) \( (\frac{1}{2}, 2) \) \( 5x + 2y = -5 \) \( (\frac{1}{2}, 2) \) \( 2a + 2b = 3 \) \( (\frac{1}{2}, 2) \) \( 11x + 2y = -5 \) \( (\frac{3}{2}, 2) \) \( 12x + 2y = -5 \) \( (\frac{3}{2}, 2) \)
5. \( 5x - y = 9 \) \( 8x - 3y = -8 \) \( 4x - y = 9 \) \( (\frac{1}{2}, 2) \) \( 4x - y = 1 \) \( (\frac{3}{2}, 2) \)

**Exercises (continued)**

1. **FLIGHT** An airplane traveling with the wind flies 450 miles in 2 hours. On the return trip, the plane takes 3 hours to travel the same distance. Find the speed of the airplane if the wind is still. \( 187.5 \text{ miles per hour} \)

2. **FUNDRAISING** Benji and Joel are raising money for their class trip by selling gift wrapping paper. Benji raises $38 by selling 5 rolls of red wrapping paper and 2 rolls of foil wrapping paper. Joel raises $57 by selling 3 rolls of red wrapping paper and 6 rolls of foil wrapping paper. For how much do Benji and Joel sell each roll of red and foil wrapping paper? red: $5; foil: $7
6-4 Practice

Use elimination to solve each system of equations.

1. \(x + y = -9\)
   \(5x - 2y = 32\) \((2, -11)\)
2. \(3x + 2y = -9\)
   \(x - y = -13\) \((-7, 6)\)

3. \(2x + 5y = 3\)
   \(-x + 3y = -7\) \(4, -1)\)
4. \(2x + y = 3\)
   \(-4x - 4y = -8\) \((1, 1)\)

5. \(4x - 2y = -14\)
   \(3x - y = -8\) \((-1, 5)\)
6. \(2x + y = 0\)
   \(5x + 3y = 2\) \((-2, 4)\)

7. \(5x + 3y = -10\)
   \(3x + 5y = -6\) \((-2, 0)\)
8. \(2x + 3y = 14\)
   \(3x - 4y = 4\) \((4, 2)\)

9. \(2x - 3y = 21\)
   \(5x - 2y = 25\) \((3, -5)\)
10. \(3x + 2y = -26\)
   \(4x - 5y = -4\) \((-6, -4)\)

11. \(3x - 6y = -3\)
    \(2x + 4y = 30\) \((7, 4)\)
12. \(5x + 2y = -3\)
    \(3x + 3y = 9\) \((-3, 6)\)

13. Two times a number plus three times another number equals 13. The sum of the two numbers is 7. What are the numbers? \(8, -1\)
14. Four times a number minus twice another number is -16. The sum of the two numbers is -1. Find the numbers. \(-3, 2\)

15. **Fundraising** Trisha and Byron are washing and vacuuming cars to raise money for a canoe trip. Trisha raised $38 washing 5 cars and vacuuming 4 cars. Byron raised $28 by washing 4 cars and vacuuming 2 cars. Find the amount they charged to wash a car and vacuum a car.

   **Wash:** $6, **Vacuum:** $2

16. **Canoeing** Laura and Brent paddled a canoe 6 miles upstream in 4 hours. The return trip took three hours. Find the rate at which Laura and Brent paddled the canoe in still water. \(1.75 \text{ mi/h}\)

17. **Number Theory** The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 more than the original number. Find the number. \(38\)
1. **SOCCER** Suppose a youth soccer field has a perimeter of 320 yards and its length measures 40 yards more than its width. Ms. Hughey asks her players to determine the length and width of their field. She gives them the following system of equations to represent the situation. Use elimination to solve the system to find the length and width of the field.

\[
2L + 2W = 320 \\
L - W = 40
\]

width = 60 yd; length = 100 yd

2. **SPORTS** The Fan Cost Index (FCI) tracks the average costs for attending sporting events, including tickets, drinks, food, parking, programs, and souvenirs. According to the FCI, a family of four would spend a total of $592.30 to attend two Major League Baseball (MLB) games and one National Basketball Association (NBA) game. The family would spend $691.31 to attend one MLB and two NBA games. Write and solve a system of equations to find the family’s costs for each kind of game according to the FCI.

3. **ART** Mr. Santee, the curator of the children’s museum, recently made two purchases of clay and wood for a visiting artist to sculpt. Use the table to find the cost of each product per kilogram.

<table>
<thead>
<tr>
<th>Clay (kg)</th>
<th>Wood (kg)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>$36.50</td>
</tr>
<tr>
<td>3.5</td>
<td>6</td>
<td>$60.45</td>
</tr>
</tbody>
</table>

Clay was $0.70 per kilogram and wood was $8.00 per kilogram.

4. **TRAVEL** Antonio flies from Houston to Philadelphia, a distance of about 1340 miles. His plane travels with the wind and takes 2 hours and 20 minutes. At the same time, Paul is on a plane from Philadelphia to Houston. Since his plane is heading against the wind, Paul’s flight takes 2 hours and 50 minutes. What was the speed of the wind in miles per hour? About 50.67 mph

5. **BUSINESS** Suppose you start a business assembling and selling motorized scooters. It costs you $1500 for tools and equipment to get started, and the materials cost $200 for each scooter. Your scooters sell for $300 each. Your scooters sell for $300 each.

a. Write and solve a system of equations representing the total costs and revenue of your business.

\[
x + y = 2800 \\
x - y = 10
\]

b. Describe what the solution means in terms of the situation. The solution represents the point at which cost equals revenue. This is the break-even point for the business owner.

c. Give an example of a reasonable number of scooters you could assemble and sell in order to make a profit, and find the profit you would make for that number of scooters. Sample answer: 20 scooters: It costs $5500 to assemble these scooters, which sell for $6000, leaving a $500 profit.

**George Washington Carver and Percy Julian**

In 1990, George Washington Carver and Percy Julian became the first African Americans elected to the National Inventors Hall of Fame. Carver (1864–1943) was an agricultural scientist known worldwide for developing hundreds of uses for the peanut and the sweet potato. His work revitalized the economy of the southern United States because it was no longer dependent solely upon cotton. Julian (1898–1975) was a research chemist who became famous for inventing a method of making a synthetic cortisone from soybeans. His discovery has had many medical applications, particularly in the treatment of arthritis.

There are dozens of other African American inventors whose accomplishments are not as well known. Their inventions range from common household items like the ironing board to complex devices that have revolutionized manufacturing. The exercises that follow will help you identify just a few of these inventors and their inventions.

Match the inventors with their inventions by matching each system with its solution. (Not all the solutions will be used.)

<table>
<thead>
<tr>
<th>Inventor</th>
<th>System</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sara Boone</td>
<td></td>
<td>A. (1, 4) automatic traffic signal</td>
</tr>
<tr>
<td>Sarah Goode</td>
<td></td>
<td>B. (4, -2) eggbeater</td>
</tr>
<tr>
<td>Frederick M. Jones</td>
<td></td>
<td>C. (-2, 3) fire extinguisher</td>
</tr>
<tr>
<td>J. L. Love</td>
<td></td>
<td>D. (-5, 7) folding cabinet bed</td>
</tr>
<tr>
<td>T. J. Marshall</td>
<td></td>
<td>E. (6, -4) ironing board</td>
</tr>
<tr>
<td>Jan Matzeliger</td>
<td></td>
<td>F. (-2, 4) pencil sharpener</td>
</tr>
<tr>
<td>Garrett A. Morgan</td>
<td></td>
<td>G. (-3, 0) portable x-ray machine</td>
</tr>
<tr>
<td>Norbert Rillieux</td>
<td></td>
<td>H. (2, -3) player piano</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>L. no solution evaporating pan for refining sugar</td>
</tr>
<tr>
<td>J</td>
<td></td>
<td>M. infinitely many solutions manufacturing shoes</td>
</tr>
</tbody>
</table>
A T-shirt printing company sells $1500 worth of T-shirts. The company incurs a fixed cost for the printing machine used to print the T-shirts and an additional cost per T-shirt sold. To determine how many T-shirts need to be sold for the income to equal the expenses, you can use the method of solving systems of linear equations.

You have learned five methods for solving systems of linear equations: graphing, substitution, elimination using addition, elimination using subtraction, and elimination using multiplication. For an exact solution, an algebraic method is best.

Example: BUSINESS A T-shirt printing company sells $3000 worth of T-shirts for $15 each. The company incurs a fixed cost for the printing machine used to print the T-shirts and an additional cost per T-shirt sold. To determine how many T-shirts need to be sold for the income to equal the expenses, you can use the method of solving systems of linear equations.

You know the initial income and the initial expense and the rates of change of each quantity with each T-shirt sold. Therefore, you can write an equation to represent the income and the expenses. Then solve to find how many T-shirts need to be sold for both values to be equal.

Check: Does the solution make sense in the context of the problem? After selling 300 T-shirts, the income would be about 300 × $15 or $4500. The costs would be about $3000 + $5 or $3050. This means that if 300 T-shirts are sold, the income and expenses of the T-shirt company are equal.

Exercises
1. Refer to the example above. If the costs of the T-shirt company change to the given values and the selling price remains the same, determine the number of T-shirts the company must sell in order for income to equal expenses.

   a. Printing machine: $5000.00; T-shirt: $10.00 each

   b. Printing machine: $2100.00; T-shirt: $8.00 each

   c. Printing machine: $1000.00; T-shirt: $12.00 each

   d. Printing machine: $800.00; T-shirt: $4.00 each
### 6-5 Applying Systems of Linear Equations

#### Determining the best method to solve each system of equations.

Then solve the system.

1. \[ \begin{align*}
5x + 3y &= 16 \\
x - 5y &= -4
\end{align*} \]
   - Elimination \((\times); (2, 2)\)

2. \[ \begin{align*}
3x - 5y &= 7 \\
x + 2y &= 13
\end{align*} \]
   - Elimination \((\div); (4, 1)\)

3. \[ \begin{align*}
y &= 3x - 24 \\
x - y &= 8
\end{align*} \]
   - Substitution \((-8, -48)\)

4. \[ \begin{align*}
y &= -11x - 10y = 17 \\
x + 2y &= 50
\end{align*} \]
   - Elimination \((\times); (3, -5)\)

5. \[ \begin{align*}
x + y &= 24 \\
x - y &= 12
\end{align*} \]
   - Elimination \((\div); (4, 8)\)

6. \[ \begin{align*}
x &= 4 - 2y \\
x - y &= 145
\end{align*} \]
   - Substitution \((-22, 13)\)

#### 7. VEGETABLE STAND

A roadside vegetable stand sells pumpkins for $5 each and squashes for $3 each. One day they sold 6 more squashes than pumpkins, and their sales totaled $88. Write and solve a system of equations to find how many pumpkins and squashes they sold.

\[ \begin{align*}
y &= 6 + x \\
5x + 3y &= 98
\end{align*} \]

10 pumpkins, 16 squashes

#### 8. INCOME

Ramiro earns $20 per hour during the week and $30 per hour for overtime on the weekends. One week Ramiro earned a total of $650. He worked 5 times as many hours during the week as he did on the weekend. Write and solve a system of equations to determine how many hours of overtime Ramiro worked on the weekend.

\[ \begin{align*}
20x + 30y &= 650 \\
x &= 5y
\end{align*} \]

5 hours

#### 9. BASKETBALL

Anya makes 14 baskets during her game. Some of these baskets were worth 2-points and others were worth 3-points. In total, she scored 30 points. Write and solve a system of equations to find how many 2-points baskets she made.

\[ \begin{align*}
x + y &= 14 \\
2x + 3y &= 30
\end{align*} \]

12 points

---

### 6-5 Practice

#### Determining the best method to solve each system of equations. Then solve the system.

1. \[ \begin{align*}
5x - 3y &= 29 \\
x - 0.9y &= 4.5
\end{align*} \]
   - Substitution \((63, 65)\)

2. \[ \begin{align*}
1.2x - 0.8y &= -6 \\
4.8x + 2.4y &= 60
\end{align*} \]
   - Elimination \((\times); (5, 15)\)

3. \[ \begin{align*}
18x - 16y &= -312 \\
78x - 16y &= 408
\end{align*} \]
   - Elimination \((\div); (12, 33)\)

4. \[ \begin{align*}
14x + 7y &= 217 \\
14x + 3y &= 189
\end{align*} \]
   - Elimination \((\times); (12, 7)\)

5. \[ \begin{align*}
x &= 3.6y + 0.7 \\
2x + 0.2y &= 38.4
\end{align*} \]
   - Substitution \((18.7, 5)\)

6. \[ \begin{align*}
5.3x - 4y &= 43.5 \\
x + 7y &= 78
\end{align*} \]
   - Substitution \((15, 9)\)

7. BOOKS

A library contains 2000 books. There are 3 times as many non-fiction books as fiction books. Write and solve a system of equations to determine the number of non-fiction and fiction books. \[ x + y = 2000 \text{ and } x = 3y; 1500 \text{ non-fiction, } 500 \text{ fiction}\]

8. SCHOOL CLUBS

The chess club has 16 members and gains a new member every month. The film club has 4 members and gains 4 new members every month. Write and solve a system of equations to find when the number of members in both clubs will be equal. \[ y = 16 + x \text{ and } y = 4 + 4x; 4 \text{ months}\]

9. Tia and Ken each sold snack bars and magazine subscriptions for a school fund-raise, as shown in the table. Tia earned $132 and Ken earned $190.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>snack bars</td>
<td>16</td>
</tr>
<tr>
<td>magazine</td>
<td>4</td>
</tr>
<tr>
<td>subscriptions</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Define variable and formulate a system of linear equations from this situation. Let \( x \) = the cost per snack bar and let \( y \) = the cost per magazine subscription; \( 16x + 4y = 132 \text{ and } 20x + 6y = 190 \).

b. What was the price per snack bar? Determine the reasonableness of your solution.

\$2; Sample answer: $2 is a reasonable price for a snack bar, and the solution checks if the magazine subscriptions are $25.
### 6-5 Word Problem Practice

**Applying Systems of Linear Equations**

1. **MONEY** Veronica has been saving dimes and quarters. She has 84 coins in all, and the total value is $19.30. How many dimes and how many quarters does she have?

   28 dimes; 66 quarters

2. **CHEMISTRY** How many liters of 15% and 33% acid should be mixed to make 40 liters of 21% acid solution?

   **| Concentration of Solution | Amount of Solution (L) | Amount of Add |
   ├───┬────────┬───────────┬───────────┐
   │ 15% | x       |          |           |
   │ 33% | y       |          |           |
   │ 21% | 40      |          |           |

   26 \( \frac{2}{3} \) L of 15%; 13 \( \frac{1}{3} \) L of 33%

3. **BUILDINGS** The Sears Tower in Chicago is the tallest building in North America. The total height of the tower \( t \) and the antenna that stands on top of it is 1729 feet. The difference in heights between the building and the antenna is 279 feet. How tall is the Sears Tower?

   1450 ft

4. **PRODUCE** Roger and Trevor went shopping for produce on the same day. They each bought some apples and some potatoes. The amount they bought and the total price they paid are listed in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Apples (lb)</th>
<th>Potatoes (lb)</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roger</td>
<td>8</td>
<td>7</td>
<td>18.85</td>
</tr>
<tr>
<td>Trevor</td>
<td>2</td>
<td>10</td>
<td>12.88</td>
</tr>
</tbody>
</table>

   What was the price of apples and potatoes per pound? **Apples:** $1.49 per lb; **Potatoes:** $0.99 per lb

5. **SHOPPING** Two stores are having a sale on T-shirts that normally sell for $20. Store S is advertising a 10% discount, and Store T is advertising a $10 dollar discount. Rose spends $63 for three T-shirts from Store S and one from Store T. Manny spends $140 on five T-shirts from Store S and four from Store T. Find the discount at each store.

   **Store S:** 20%; **Store T:** 5%

6. **TRANSPORTATION** A Speedy River barge bound for New Orleans leaves Baton Rouge, Louisiana, at 9:00 a.m. and travels at a speed of 10 miles per hour. A Rail Transport freight train also bound for New Orleans leaves Baton Rouge at 1:30 p.m. The train travels at 25 miles per hour, and the river barge travels at 10 miles per hour. Both the barge and the train will travel 100 miles to reach New Orleans.

   a. How far will the train travel before catching up to the barge? **75 mi**

   b. Which shipment will reach New Orleans first? At what time? **The train will arrive first. It will arrive in New Orleans at 5:30 p.m. the same day.**

   c. If both shipments take an hour to unload before heading back to Baton Rouge, what is the earliest time that either one of the companies can begin to load grain to ship in Baton Rouge? **10:30 p.m. the same day**

### 6-5 Enrichment

**Cramer’s Rule**

Cramer’s Rule is a method for solving a system of equations. To use Cramer’s Rule, set up a matrix to represent the equations. A matrix is a way of organizing data.

**Example**

Solve the following system of equations using Cramer's Rule.

\[
2x + 3y = 13 \\
x + y = 5
\]

**Step 1:** Set up a matrix representing the coefficients of \( x \) and \( y \).

\[
A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}
\]

**Step 2:** Find the determinant of matrix \( A \).

If \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), then the determinant, \( \det(A) = ad - bc \).

\( \det(A) = (2)(1) - (1)(3) = -1 \)

**Step 3:** Replace the first column in \( A \) with 13 and 5 and find the determinant of the new matrix.

\( A_1 = \begin{bmatrix} 13 & 3 \\ 1 & 1 \end{bmatrix} \)

\( \det(A_1) = 13(1) - 5(3) = -2 \)

**Step 4:** To find the value of \( x \) in the solution to the system of equations, determine the value of \( \frac{\det(A_1)}{\det(A)} \).

\( \frac{\det(A_1)}{\det(A)} = \frac{-2}{-1} = 2 \)

**Step 5:** Repeat the process to find the value of \( y \). This time, replace the second column with 13 and 5 and find the determinant.

\( A_2 = \begin{bmatrix} 10 & 5 \\ 1 & 1 \end{bmatrix} \)

\( \det(A_2) = (10)(1) - (5)(1) = 5 \)

\( \det(A) = (2)(1) - (1)(3) = -1 \)

\( \frac{\det(A_2)}{\det(A)} = \frac{5}{-1} = -5 \)

So, the solution to the system of equations is \((2, 3)\).

**Exercises**

Use Cramer’s Rule to solve each system of equations.

1. \( 2x + y = 1 \) \( x + y = 4 \) \( 3x - y = 8 \)
   \( (0, 1) \) \( (2, 2) \) \( (6, 2) \)

2. \( 4x - y = 3 \) \( 5x + 2y = 7 \) \( 6x - 5y = 1 \)
   \( (2, 5) \) \( (5, 4) \) \( (1, 1) \)
### Systems of Inequalities

The solution of a system of inequalities is the set of all ordered pairs that satisfy both inequalities. If you graph the inequalities in the same coordinate plane, the solution is the region where the graphs overlap.

#### Example 1

Solve the system of inequalities by graphing.

\[
y > x + 2 \\
y \leq -2x - 1
\]

The solution includes the ordered pairs in the intersection of the graphs. This region is shaded at the right. The graphs of \(y = x + 2\) and \(y = -2x - 1\) are boundaries of this region. The graph of \(y = x + 2\) is dashed and is not included in the graph of \(y > x + 2\).

#### Example 2

Solve the system of inequalities by graphing.

\[
x + y > 4 \\
x + y < -1
\]

The graphs of \(x + y = 4\) and \(x + y = -1\) are parallel. Because the two regions have no points in common, the system of inequalities has no solution.

### Exercises

Solve each system of inequalities by graphing.

1. \(y > -1\)
   \(x < 0\)

2. \(y > -2x + 2\)
   \(y \leq x + 1\)

3. \(y < x + 1\)
   \(3x + 4y \geq 12\)

4. \(2x + y \geq 1\)
   \(x - y \geq -2\)

5. \(y \leq 2x + 3\)
   \(y \geq -1 + 2x\)

6. \(5x - 2y < 6\)
   \(y > -x + 1\)

---

### Systems of Inequalities (continued)

#### Example

**BUSINESS** AAA Gem Company produces necklaces and bracelets. In a 40-hour week, the company has 400 gems to use. A necklace requires 40 gems and a bracelet requires 10 gems. It takes 2 hours to produce a necklace and a bracelet requires one hour. How many of each type can be produced in a week?

Let \(n\) = the number of necklaces that will be produced and \(b\) = the number of bracelets that will be produced. Neither \(n\) or \(b\) can be a negative number, so the following system of inequalities represents the conditions of the problems.

\[
\begin{align*}
n & \geq 0 \\
b & \geq 0 \\
b + 2n & \leq 40 \\
10b + 40n & \leq 400
\end{align*}
\]

The solution is the set ordered pairs in the intersection of the graphs. This region is shaded at the right. Only whole-number solutions, such as \((5, 20)\) make sense in this problem.

#### Exercises

For each exercise, graph the solution set. List three possible solutions to the problem.

**1. HEALTH** Mr. Flowers is on a restricted diet that allows him to have between 1600 and 2000 Calories per day. His daily fat intake is restricted to between 45 and 55 grams. What daily Calorie and fat intakes are acceptable?

Sample answers: 1600 Calories, 45 fat grams; 1800 Calories, 50 fat grams; 2000 Calories, 55 fat grams

**2. RECREATION** Maria had $150 in gift certificates to use at a record store. She bought fewer than 20 recordings. Each tape cost $5.95 and each CD cost $8.95. How many of each type of recording might she have bought?

Sample answers: 10 tapes, 9 CDs; 0 tapes, 16 CDs; 14 tapes, 5 CDs
Chapter 6 Skills Practice

Systems of Inequalities

Solve each system of inequalities by graphing.

1. \(x > -1\)  \(\ y \leq -3\)
2. \(y > 2\)  \(\ x < -2\)
3. \(y > x + 3\)  \(\ y \leq -1\)
4. \(x < 2\)  \(\ y - x \leq 2\)
5. \(x + y \leq -1\)  \(\ x + y \geq 3\)
6. \(y - x > 4\)  \(\ x + y > 2\)
7. \(y > x + 1\)  \(\ y \geq -x + 1\)
8. \(y \geq -x + 2\)  \(\ y < 2x - 2\)
9. \(y < 2x + 4\)  \(\ y \geq x + 1\)

10. \(y \leq x + 2\)  \(\ y \geq x - 3\)
11. \(y > -x, y > x\)
12. \(y \geq x + 1, y < 1\)

7. FITNESS Diego started an exercise program in which each week he works out at the gym between 4.5 and 6 hours and walks between 9 and 12 miles.
   a. Make a graph to show the number of hours Diego works out at the gym and the number of miles he walks per week.
   b. List three possible combinations of working out and walking that meet Diego’s goals. Sample answers:
      - Gym 5 h, walk 9 mi; gym 6 h, walk 10 mi, gym 5.5 h, walk 11 mi

8. SOUVENIRS Emily wants to buy turquoise stones on her trip to New Mexico to give to at least 4 of her friends. The gift shop sells stones for either $4 or $6 per stone. Emily has no more than $30 to spend.
   a. Make a graph showing the numbers of each price of stone Emily can purchase.
   b. List three possible solutions. Sample answer:
      - one $4 stone and four $6 stones; three $4 stones and three $6 stones; five $4 stones and one $6 stone
### Word Problem Practice

**Systems of Inequalities**

1. **PETS** Renée’s Pet Store never has more than a combined total of 20 cats and dogs and never more than 8 cats. This is represented by the inequalities $x \leq 8$ and $x + y \leq 20$. Solve the system of inequalities by graphing.

2. **WAGES** The minimum wage for one group of workers in Texas is $7.25 per hour effective Sept. 1, 2008. The graph below shows the possible weekly wages for a person who makes at least minimum wages and works at most 40 hours. Write the system of inequalities for the graph.

   \[ x \leq 40, \quad y \geq 7.25x \]

3. **FUND RAISING** The Camp Courage Club plans to sell tins of popcorn and peanuts as a fundraiser. The Club members have $900 to spend on products to sell and want to order up to 200 tins in all. They also want to order at least as many tins of popcorn as tins of peanuts. Each tin of popcorn costs $3 and each tin of peanuts costs $4. Write a system of equations to represent the conditions of this problem. $3x + 4y \leq 900$; $x \geq y$; $x + y \leq 200$; $x \geq y$; $x \geq 0$ and $y \geq 0$.

4. **BUSINESS** For maximum efficiency, a factory must have at least 100 workers, but no more than 200 workers on a shift. The factory also must manufacture at least 30 units per worker.
   a. Let $x$ be the number of workers and $y$ be the number of units. Write four inequalities expressing the conditions in the problem given above. $x \leq 200$; $x \geq 100$; $y \geq 3000$; $y \geq 30x$.
   b. Graph the systems of inequalities.
   c. List at least three possible solutions. Sample answer: $(110, 3410), (150, 5100), (180, 6300)$

### Enrichment

**Describing Regions**

The shaded region inside the triangle can be described with a system of three inequalities.

\[ y < -x + 1 \]
\[ y > \frac{3}{3}x - 3 \]
\[ y > -9x - 31 \]

Write systems of inequalities to describe each region. You may first need to divide a region into triangles or quadrilaterals.
6-6 Graphing Calculator Activity

Shading Absolute Value Inequalities

Absolute value inequalities of the form \( y \leq a|x - h| + k \) or \( y \geq a|x - h| + k \) can be graphed using the SHADE command or by using the shading feature in the Y= screen.

Example 1
Graph \( y \leq 2|x - 3| + 2 \).

Method 1: SHADE command
Enter the boundary equation \( y = 2|x - 3| + 2 \) into Y1. From the home screen, use SHADE to shade the region under the boundary equation because the inequality uses \( \leq \). Use Ymin as the lower boundary and the boundary equation, Y1, as the upper boundary.

Keystrokes: \( \text{DRAW} \rightarrow \text{TRACE} \rightarrow \text{SHIFT} \rightarrow \text{GRAPH} \rightarrow \text{YES} \rightarrow \text{YES} \)

Method 2: Y=
On the Y= screen, move the cursor to the far left and repeatedly press \( \text{DRAW} \) to toggle through the graph choices. Be sure to select the option to shade below. Then enter the boundary equation and graph.

The same technique can be used to solve a system of absolute value inequalities.

Example 2
Solve the system \( y \geq |x + 5| - 4 \) and \( y \leq 3|x - 6| + 1 \).

Enter \( y = |x + 5| - 4 \) in Y1 and set the graph to shade above. Then enter \( y = 3|x - 6| + 1 \) in Y2 and set the graph to shade below. Graph the inequalities. Notice that a different pattern is used for the second inequality. The region where the two patterns overlap is the solution to the system.

Exercises
Solve each system of inequalities by graphing.

1. \( y \leq 2|x - 4| + 1 \)
2. \( y < \frac{3}{2}|x - 3| + 2 \)
3. \( y \leq |x - 4| + 3 \)
4. \( y < \frac{3}{4}|x + 1| + 1 \)
5. \( y \leq |x + 3| + 5 \)

6-6 Spreadsheet Activity

Systems of Inequalities

Example
TopSport Shoe Company has a total of 9600 minutes of machine time each week to cut the materials for the two types of athletic shoes they make. There are a total of 28,000 minutes of worker time per week for assembly. It takes 3 minutes to cut and 12 minutes to assemble a pair of Runners and 2 minutes to cut and 10 minutes to assemble a pair of Flyers. Is it possible for the company to make 1200 pairs of Runners and 1400 pairs of Flyers in a week?

Step 1
Represent the situation using a system of inequalities. Let \( r \) represent the number of Runners and \( f \) represent the number of Flyers.

\[ 3r + 2f \leq 9600 \]
\[ 12r + 10f \leq 28,000 \]

Step 2
Columns A and B contain the values of \( r \) and \( f \). Columns C and D contain the formulas for the inequalities. The formulas will return TRUE or FALSE. If both inequalities are true for an ordered pair, then the ordered pair is a solution to the system of inequalities.

Since one of the inequalities is false for (1200, 1400), the ordered pair is not part of the solution set. The company cannot make 1200 pairs of Runners and 1400 pairs of Flyers in a week.

Exercises
Use the spreadsheet to determine whether TopSport can make the following combinations of shoes.

1. 1000 Runners, 1100 Flyers \( \text{yes} \)
2. 1200 Runners, 1000 Flyers \( \text{yes} \)
3. 2000 Runners, 500 Flyers \( \text{no} \)
4. 300 Runners, 1500 Flyers \( \text{yes} \)
5. 450 Runners, 1400 Flyers \( \text{yes} \)
6. 1800 Runners, 1300 Flyers \( \text{no} \)

If TopSport can either buy another cutting machine or hire more assemblers, which would make more combinations of shoe production possible? Explain. Hire more assemblers. Each ordered pair that was not a solution of the system was true for the first inequality. Changing the second inequality may make some of those ordered pairs solutions of the system.
### Chapter 6 Assessment Answer Key

**Quiz 1** (Lessons 6-1 and 6-2)  
**Page 47**

1. \( y = -x + 5 \)  
   - one solution; (2, 3)

2. \( x - 2y = -2 \)  
   - no solution

3. \( (1, 5) \)

4. infinitely many solutions

5. \( A \)

**Quiz 2** (Lessons 6-3 and 6-4)  
**Page 47**

1. \( \left( \frac{1}{2}, -\frac{1}{2} \right) \)

2. \( \left( -\frac{1}{2}, 2 \right) \)

3. \( (2, -3) \)

4. \( (2, -1) \)

5. \( D \)

**Quiz 3** (Lessons 6-5)  
**Page 48**

1. **substitution;** \( (5, 2) \)

2. elimination \( (x); \) \( (2, 3) \)

3. elimination \( (-); \) \( (-2, 2) \)

4. elimination \( (+); \) \( \left( \frac{1}{4}, 1 \right) \)

5. \( D \)

**Quiz 4** (Lessons 6-6)  
**Page 48**

1. **substitution;** \( (3, 2) \)

2. elimination \( (x); \) \( (3, \frac{3}{5}, \frac{3}{5}) \)

3. \( (1, -4) \)

4. \( (1, -4) \)

5. \( G \)

**Mid-Chapter Test**  
**Page 49**

1. \( B \)

2. \( A \)

3. \( F \)

4. \( C \)

5. \( A \)

6. \( D \)

7. one solution; \( (3, 2) \)

8. \( (1, -4) \)

9. \( (-9, -5) \)

10. \( \left( \frac{3}{5}, \frac{3}{5} \right) \)

11. \( (-1, -4) \)

12. \( 34 \) games
Vocabulary Test

1. __________ system of equations
2. __________ consistent
3. __________ dependent
4. __________ elimination
   Sample answer: Substitution is a method of solving a system of equations where one variable is solved in terms of the other variable.
5. __________
   Sample answer: A system of equations that has exactly one solution is called independent.
6. __________
### Chapter 6 Assessment Answer Key

**Form 2A**

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<th>Form 2B</th>
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**B:**

Manuel is 15 years old; his sister is 7 years old.

**Form 2B**

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<td>2. <strong>B</strong></td>
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<td>11.</td>
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</table>

**B:**

Manuel is 15 years old; his sister is 7 years old.

\([-7, -3 \frac{1}{2}]\)

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1. no solution
2. one solution
3. one solution; (4, 0)
4. infinitely many solutions
5. (1, 3)
6. (3, 5)
7. (−8, 0)
8. (4, −1)
9. (−3, 2)
10. (7, −1)
11. substitution; (3, 2)
12. elimination (−); (2, −7)
13. 23 and −6
14. 186 student tickets; 135 adult tickets
15. 14 dimes; 19 nickels

16. x = test score, y = family income; x ≥ 75, y ≤ 40,000; sample answer: (80, 38,000)
17. Mavis is 15 years old; her brother is 10 years old
18. B: ______
Chapter 6 Assessment Answer Key

Form 2D
Page 59

1. infinitely many solutions

2. one solution

3. one solution; (3, 0)

4. infinitely many solutions

5. (2, 4)

6. (2, 1)

7. (5, 3)

8. (2, 2)

9. (−1, 4)

10. (−2, 5)

11. substitution; (−2, −5)

12. elimination (−);

13. 18 and −2

14. 34 hours

15. 16 dimes;

16. 7 quarters

17. x = credit score, y = cost of car; x ≥ 600, y ≥ 5000; sample answer: (650, 8000)

18. B: (−1, −3)

Page 60
Chapter 6 Assessment Answer Key

Form 3
Page 61

1. one solution; \((-1, -3)\)

2. no solution

3. \((4, 1)\)

4. no solution

5. \((1, 2)\)

6. no solution

7. \((3, -\frac{3}{7})\)

8. \((-14, 7)\)

9. \((-2, -6)\)

10. \((-32, 15)\)

11. substitution; \((63, 84)\)

12. elimination \((x); (-\frac{1}{2}, \frac{1}{2})\)

13. 6, 10

14. \(3\frac{1}{2}, 8\frac{1}{2}\)

15. 10 lb of $2.45 mix;

16. 5 mph

17. \(x = \) women’s shoes sold; \(y = \) men’s shoes sold; \(x \geq 10, y \geq 5, 2.5x + 3y \geq 60; \) sample answer: \((15, 10)\)

18. B: 10 square units

19. 

20. 20 lb of $2.30 mix
## Chapter 6 Assessment Answer Key

### Page 63, Extended-Response Test

#### Scoring Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
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<tbody>
<tr>
<td>4</td>
<td><strong>Superior</strong>&lt;br&gt;A correct solution that is supported by well-developed, accurate explanations</td>
<td>• Shows thorough understanding of the concepts of <em>solving systems of equations</em>.&lt;br&gt;• Uses appropriate strategies to solve problems.&lt;br&gt;• Computations are correct.&lt;br&gt;• Written explanations are exemplary.&lt;br&gt;• Graphs are accurate and appropriate.&lt;br&gt;• Goes beyond requirements of some or all problems.</td>
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<td>3</td>
<td><strong>Satisfactory</strong>&lt;br&gt;A generally correct solution, but may contain minor flaws in reasoning or computation</td>
<td>• Shows an understanding of the concepts of <em>solving systems of equations</em>.&lt;br&gt;• Uses appropriate strategies to solve problems.&lt;br&gt;• Computations are mostly correct.&lt;br&gt;• Written explanations are effective.&lt;br&gt;• Graphs are mostly accurate and appropriate.&lt;br&gt;• Satisfies all requirements of problems.</td>
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<td><strong>Nearly Satisfactory</strong>&lt;br&gt;A partially correct interpretation and/or solution to the problem</td>
<td>• Shows an understanding of most of the concepts of <em>solving systems of equations</em>.&lt;br&gt;• May not use appropriate strategies to solve problems.&lt;br&gt;• Computations are mostly correct.&lt;br&gt;• Written explanations are satisfactory.&lt;br&gt;• Graphs are mostly accurate.&lt;br&gt;• Satisfies the requirements of most of the problems.</td>
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<td>1</td>
<td><strong>Nearly Unsatisfactory</strong>&lt;br&gt;A correct solution with no supporting evidence or explanation</td>
<td>• Final computation is correct.&lt;br&gt;• No written explanations or work is shown to substantiate the final computation.&lt;br&gt;• Graphs may be accurate but lack detail or explanation.&lt;br&gt;• Satisfies minimal requirements of some of the problems.</td>
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<tr>
<td>0</td>
<td><strong>Unsatisfactory</strong>&lt;br&gt;An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given</td>
<td>• Shows little or no understanding of most of the concepts of <em>solving systems of equations</em>.&lt;br&gt;• Does not use appropriate strategies to solve problems.&lt;br&gt;• Computations are incorrect.&lt;br&gt;• Written explanations are unsatisfactory.&lt;br&gt;• Graphs are inaccurate or inappropriate.&lt;br&gt;• Does not satisfy requirements of problems.&lt;br&gt;• No answer may be given.</td>
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Chapter 6 Assessment Answer Key

Page 63, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating extended-response assessment items.

1a. Sample answer: (5000, 5000); $600

1b. (15,000, −5000); To represent a possible investment both x and y must be positive. Thus, there is no interpretation for a negative value for y.

2a. The student should recognize that the total cost of a one-day rental of a compact car from the car rental company is modeled by the equation \( y = A + Bx \), where y is the total cost and x is the number of miles driven during the day. Likewise, the total cost of a one-day rental of a compact car from the leading competitor is modeled by the equation \( y = 15 + 0.25x \). The solution to the system is the number of miles driven during the day that makes the cost of renting a compact car from the two companies for one day equal, and the corresponding total cost.

2b. The value of A determines if the one-day fee for the rental of the compact car from the car rental company will be less than, greater than, or equal to the one-day fee from their leading competitor. The value of B determines whether the number of miles driven will keep the total cost less than, greater than, or equal to the total cost of renting from the leading competitor, or change which company has the higher total cost for the rental.

3a. \( x + y = S \)
\( 2.5x + 0.75y = P \)
Both S and P must be positive. The student may recognize that the maximum profit is 2.5 times the weekly sales, and the minimum profit is 0.75 times the weekly sales. i.e., \( 0.75S \leq P \leq 2.5S \).

3b. Sample answer:

The graph of this system of equations shows the ordered pair that corresponds to weekly sales of 250 and a weekly profit of $450. In order to have a profit of $450 from magazine and book sales, the store would have to sell 150 books and 100 magazines.

4a. \( a + b = 21 \)
\( a - 5b = 3 \)

4b. 17 adults; 4 babies
## Chapter 6 Assessment Answer Key

### Standardized Test Practice

**Page 64**

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### Diagrams

**Diagram 17**: [Blank diagram]

**Diagram 18**: [Blank diagram]
Chapter 6 Assessment Answer Key

Standardized Test Practice
Page 66

19. decrease; 25%

20. yes; common difference is 3

22. \(-\frac{8}{7}\)

23. \(d = 55t; 275\) miles

24. \(y = 3x + 5\)

25.

26. \(\{r \mid r < 3\}\)

27. Sample answer: Let \(x = \) first number and \(y = \) second number;
\[3x - y = -40\]
\[x + 2y = -4\]

28a. \(-12, 4\)
Chapter 6 Assessment Answer Key

Unit 2 Test
Page 67

1. no solution

2. 0

3. \( d = 40t \)

4. -41

5. \( y = 2x + 9 \)

6. \( y = 5x - 2 \)

7. \( 3x + 2y = -10 \)

8. \( y = -3x - 6 \)

9. 

10. \( y = \frac{130}{3}x + \frac{20}{3} \)

11. about $2.25

12. \( \{x \mid x > -5\} \)

13. \( \{a \mid a \leq 8\} \)

14. \( \{t \mid -1 < t < 2\} \)

15. \( \{v \mid v < -3 \text{ or } v > 11\} \)

16. \( \{b \mid \frac{2}{3} \leq b \leq 4\} \)

17. \( \{w \mid w < -6 \text{ or } w > -4\} \)

18. no solution

19. substitution; (1, 1)

20. elimination (+); (3, -2)

21. elimination (-); (10, -4)

22. elimination (x); (3, -5)

23. positive; sample answer: \( y = \frac{130}{3}x + \frac{20}{3} \)