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Chapter 9 Resource Masters

Algebra 1

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CONSUMABLE WORKBOOKS Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

	MHID	ISBN
Study Guide and Intervention Workbook	0-07-660292-3	978-0-07-660292-6
Homework Practice Workbook	0-07-660291-5	978-0-07-660291-9
Spanish Version		
Homework Practice Workbook	0-07-660294-X	978-0-07-660294-0

Answers For Workbooks The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

Spanish Assessment Masters (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 9 Test Form 2A and Form 2C.

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8787 Orion Place
Columbus, OH 43240

ISBN: 978-0-07-660283-4
MHID: 0-07-660283-4

Printed in the United States of America.

1 2 3 4 5 6 7 8 9 DOH 16 15 14 13 12 11

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Teacher's Guide to Using the *Chapter 9 Resource Masters*

The *Chapter 9 Resource Masters* includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 9-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire, or Spreadsheet Activities
These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the ***Chapter 9 Resource Masters*** offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- ***Form 1*** contains multiple-choice questions and is intended for use with below grade level students.
- ***Forms 2A and 2B*** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Forms 2C and 2D*** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Form 3*** is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.

NAME ______________________ DATE ____________ PERIOD ____________

9 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
absolute value function		
axis of symmetry (SIH·muh·tree)		
common ratio		
completing the square		
compound interest		
discriminant		
double root		
geometric sequence		
greatest integer function		
maximum		

(continued on the next page)

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9 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
minimum		
nonlinear function		
parabola (puh·RA·buh·luh)		
piecewise-defined function		
piecewise-linear function		
Quadratic Formula (kwah·DRA·tihk)		
quadratic function		
step function		
transformation		
vertex		

NAME ______________________________ DATE ____________ PERIOD ____________

9 Anticipation Guide

Quadratic Functions and Equations

Step 1 *Before you begin Chapter 9*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, o NS	Statement	STEP 2 A o D
	1. The graph of a quadratic function is a parabola.	
	2. The graph of $y = 4x^2 - 2x + 7$ will be a parabola opening downward since the coefficient of x^2 is positive.	
	3. A quadratic function's axis of symmetry is either the x-axis or the y-axis.	
	4. The graph of a quadratic function opening upward has no maximum value.	
	5. The x-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.	
	6. All quadratic equations have two real solutions.	
	7. Any quadratic expression can be written as a perfect square by a method called *completing the square.*	
	8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.	
	9. The graph of a step function is a series of disjointed line segments.	
	10. It is not possible to identify data as linear based on patterns of behavior of their y-values.	

Step 2 *After you complete Chapter 9*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE ______________________________ FECHA ____________ PERÍODO ____________

9 Ejercicios preparatorios

Funciones y Ecuaciones Cuadráticas

Paso 1 *Antes de comenzar el Capítulo 9*

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

PASO 1 A, D, or NS	Enunciado	PASO 2 A or D
	1. La gráfica de una función cuadrática es una parábola.	
	2. La gráfica de $y = 4x^2 - 2x + 7$ será una parábola que se abre hacia abajo, puesto que el coeficiente de x^2 es positivo.	
	3. El eje de simetría de una función cuadrática es el eje x o el eje y.	
	4. La gráfica de una función cuadrática que se abre hacia arriba no tiene un valor máximo.	
	5. Las intersecciones x de la gráfica de una función cuadrática son las soluciones de la ecuación cuadrática relacionada.	
	6. Todas las ecuaciones cuadráticas tienen dos soluciones reales.	
	7. Cualquier expresión cuadrática puede escribirse como un cuadrado perfecto mediante el método denominado *completar el cuadrado*.	
	8. La fórmula cuadrática sólo puede usarse para resolver ecuaciones cuadráticas que no pueden resolverse mediante factorización o gráficas.	
	9. La gráfica de una función de paso es una serie de segmentos de líneas inconexas.	
	10. No es posible identificar los datos como lineal basado en patrones de comportamiento de sus valores de y.	

Paso 2 *Después de completar el Capítulo 9*

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

9-1 Study Guide and Intervention

Graphing Quadratic Functions

Characteristics of Quadratic Functions

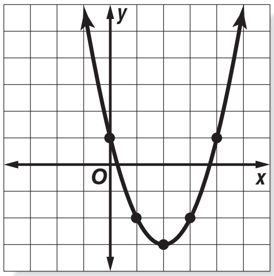
Quadratic Function	a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$	**Example:** $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic functions have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a is positive, and a parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1

a. Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
−1	6
0	1
1	−2
2	−3
3	−2
4	1



Graph the ordered pairs in the table and connect them with a smooth curve.

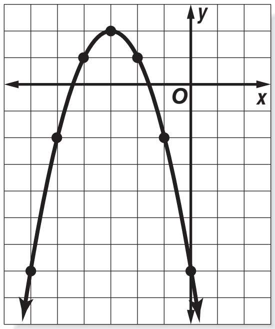
b. What are the domain and range of this function?

The domain is all real numbers. The range is all real numbers greater than or equal to −3, which is the minimum.

Example 2

a. Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
−6	−7
−5	−2
−4	1
−3	2
−2	1
−1	−2
0	−7



Graph the ordered pairs in the table and connect them with a smooth curve.

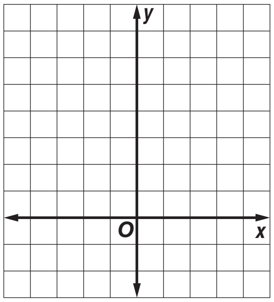
b. What are the domain and range of this function?

The domain is all real numbers. The range is all real numbers less than or equal to 2, which is the maximum.

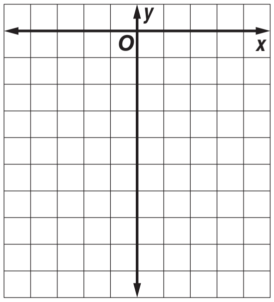
Exercises

Use a table of values to graph each function. Determine the domain and range.

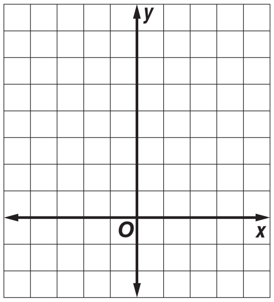
1. $y = x^2 + 2$



2. $y = -x^2 - 4$



3. $y = x^2 - 3x + 2$



9-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Symmetry and Vertices

Parabolas have a geometric property called **symmetry**. That is, if the figure is folded in half, each half will match the other half exactly. The vertical line containing the fold line is called the **axis of symmetry**. The axis of symmetry contains the minimum or maximum point of the parabola, the **vertex**.

Axis of Symmetry	For the parabola $y = ax^2 + bx + c$, where $a \neq 0$, the line $x = -\frac{b}{2a}$ is the axis of symmetry.	**Example:** The axis of symmetry of $y = x^2 + 2x + 5$ is the line $x = -1$.

Example **Consider the graph of $y = 2x^2 + 4x + 1$.**

a. Write the equation of the axis of symmetry.

In $y = 2x^2 + 4x + 1$, $a = 2$ and $b = 4$. Substitute these values into the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$

$$x = -\frac{4}{2(2)} = -1$$

The axis of symmetry is $x = -1$.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate of the vertex is -1.

$y = 2x^2 + 4x + 1$ Original equation

$y = 2(-1)^2 + 4(-1) + 1$ Substitute.

$y = 2(1) - 4 + 1$ Simplify.

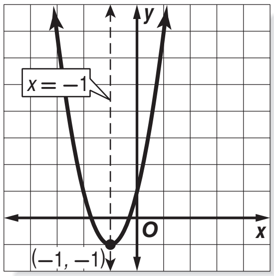
$y = -1$

The vertex is at $(-1, -1)$.

c. Identify the vertex as a maximum or a minimum.

Since the coefficient of the x^2-term is positive, the parabola opens upward, and the vertex is a minimum point.

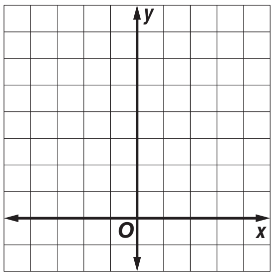
d. Graph the function.



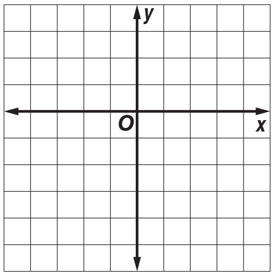
Exercises

Consider each equation. Determine whether the function has *maximum* or *minimum* value. State the maximum or minimum value and the domain and range of the function. Find the equation of the axis of symmetry. Graph the function.

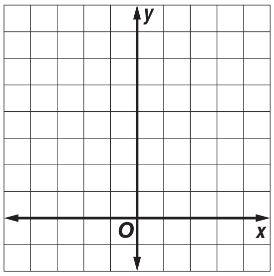
1. $y = x^2 + 3$



2. $y = -x^2 - 4x - 4$



3. $y = x^2 + 2x + 3$



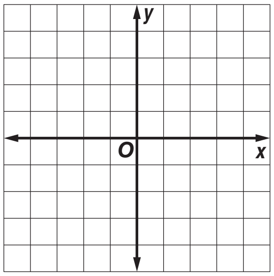
NAME ______________________ DATE ____________ PERIOD ____________

9-1 Skills Practice

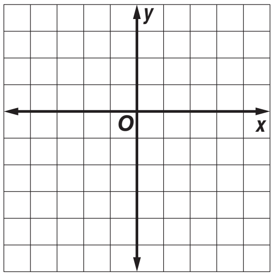
Graphing Quadratic Functions

Use a table of values to graph each function. State the domain and the range.

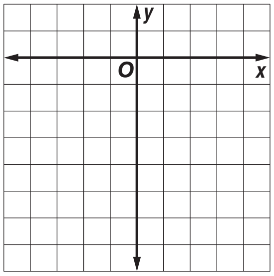
1. $y = x^2 - 4$



2. $y = -x^2 + 3$



3. $y = x^2 - 2x - 6$



Find the vertex, the equation of the axis of symmetry, and the *y*-intercept of the graph of each function.

4. $y = 2x^2 - 8x + 6$

5. $y = x^2 + 4x + 6$

6. $y = -3x^2 - 12x + 3$

Consider each equation.

a. Determine whether the function has a *maximum* or a *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

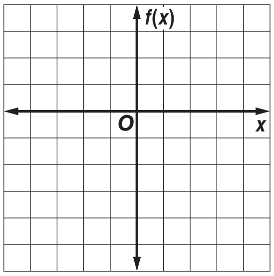
7. $y = 2x^2$

8. $y = x^2 - 2x - 5$

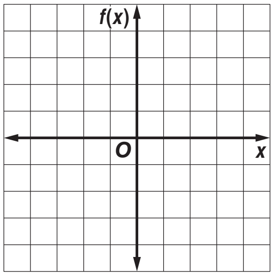
9. $y = -x^2 + 4x - 1$

Graph each function.

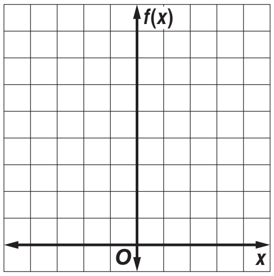
10. $f(x) = -x^2 - 2x + 2$



11. $f(x) = 2x^2 + 4x - 2$



12. $f(x) = -2x^2 - 4x + 6$



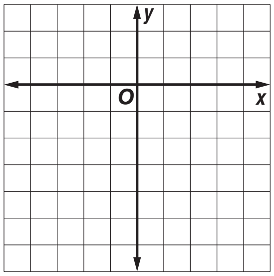
NAME ______________________________ DATE ____________ PERIOD ____________

9-1 Practice

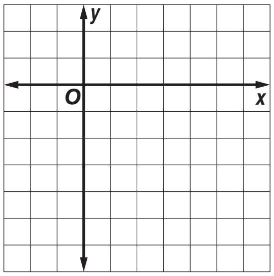
Graphing Quadratic Functions

Use a table of values to graph each function. Determine the domain and range.

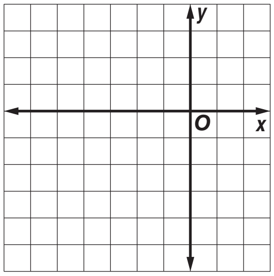
1. $y = -x^2 + 2$



2. $y = x^2 - 6x + 3$



3. $y = -2x^2 - 8x - 5$



Find the vertex, the equation of the axis of symmetry, and the *y*-intercept of the graph of each function.

4. $y = x^2 - 9$

5. $y = -2x^2 + 8x - 5$

6. $y = 4x^2 - 4x + 1$

Consider each equation. Determine whether the function has a *maximum* or a *minimum* value. State the maximum or minimum value. What are the domain and range of the function?

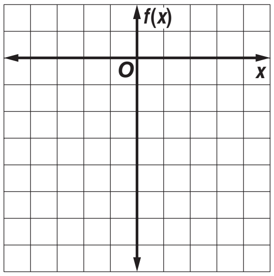
7. $y = 5x^2 - 2x + 2$

8. $y = -x^2 + 5x - 10$

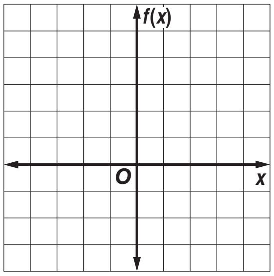
9. $y = \frac{3}{2}x^2 + 4x - 9$

Graph each function.

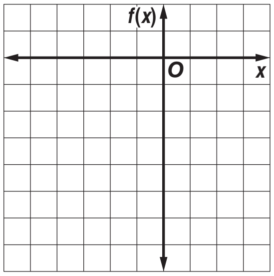
10. $f(x) = -x^2 + 1$



11. $f(x) = -2x^2 + 8x - 3$



12. $f(x) = 2x^2 + 8x + 1$



13. BASEBALL The equation $h = -0.005x^2 + x + 3$ describes the path of a baseball hit into the outfield, where h is the height and x is the horizontal distance the ball travels.

a. What is the equation of the axis of symmetry?

b. What is the maximum height reached by the baseball?

c. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it?

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9-1 Word Problem Practice

Graphing Quadratic Functions

1. **OLYMPICS** Olympics were held in 1896 and have been held every four years except 1916, 1940, and 1944. The winning height y in men's pole vault at any number Olympiad x can be approximated by the equation $y = 0.37x^2 + 4.3x + 126$. Complete the table to estimate the pole vault heights in each of the Olympic Games. Round your answers to the nearest tenth.

Year	Olympiad (x)	Height (y inches)
1896	1	
1900	2	
1924	7	
1936	10	
1964	15	
2008	26	

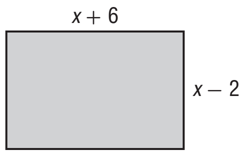
Source: National Security Agency

2. **PHYSICS** Mrs. Capwell's physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that $y = -x^2 + 6x$ accurately predicts the ball's position y after rolling x seconds. On the graph of the equation, what would be the y value when $x = 4$?

3. **ARCHITECTURE** A hotel's main entrance is in the shape of a parabolic arch. The equation $y = -x^2 + 10x$ models the arch height y for any distance x from one side of the arch. Use a graph to determine its maximum height.

4. **SOFTBALL** Olympic softball gold medalist Michele Smith pitches a curveball with a speed of 64 feet per second. If she throws the ball straight upward at this speed, the ball's height h in feet after t seconds is given by $h = -16t^2 + 64t$. Find the coordinates of the vertex of the graph of the ball's height and interpret its meaning.

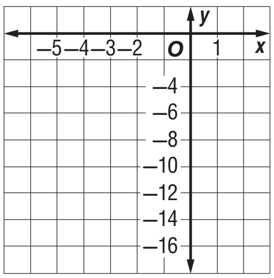
5. **GEOMETRY** Teddy is building the rectangular deck shown below.



a. Write an equation representing the area of the deck y.

b. What is the equation of the axis of symmetry?

c. Graph the equation and label its vertex.



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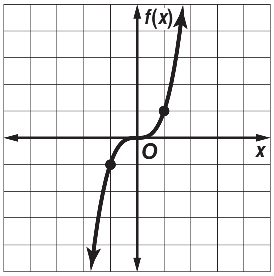
9-1 Enrichment

Graphing Cubic Functions

A **cubic function** is a polynomial written in the form of $f(x) = ax^3 + bx^2 + cx + n$, where $a \neq 0$. Cubic functions do not have absolute minimum and maximum values like quadratic functions do, but they can have a **local minimum** and a **local maximum** point.

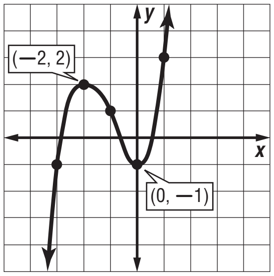
Parent Function: $f(x) = x^3$

Domain: {all real numbers}
Range: {all real numbers}



Example **Use a table of values to graph $y = x^3 + 3x^2 - 1$. Then use the graph to estimate the locations of the local minimum and local maximum points.**

x	–3	–2	–1	0	1
y	–1	2	1	–1	2



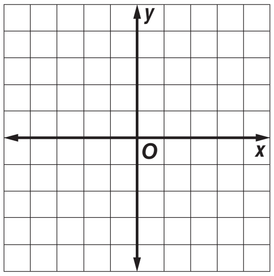
Graph the ordered pairs, and connect them to create a smooth curve. The end behavior of the "S" shaped curve shows that as x increases, y increases, and as x decreases, y decreases.

The local minimum is located at (0, –1). The local maximum is located at (–2, 2).

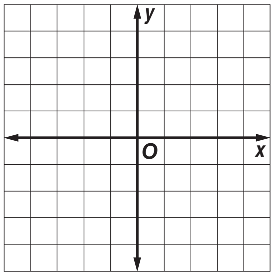
Exercises

Use a table of values to graph each equation. Then use the graph to estimate the locations of the local minimum and local maximum points.

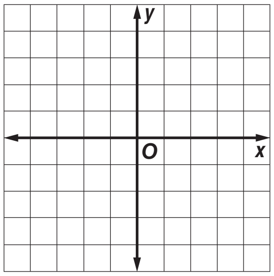
1. $y = 0.5x^3 + x^2 - 1$



2. $y = -2x^3 - 3x^2 - 1$



3. $y = x^3 + 3x^2 + x - 4$



9-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

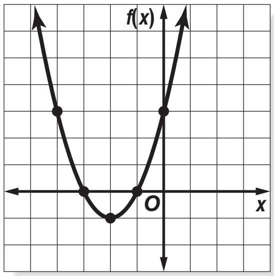
Solve by Graphing

Quadratic Equation	an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by graphing the related quadratic function $f(x) = ax^2 + bx + c$ and finding the x-intercepts or **zeros** of the function.

Example 1 **Solve $x^2 + 4x + 3 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + 4x + 3$. The equation of the axis of symmetry is $x = -\frac{4}{2(1)}$ or -2. The vertex is at $(-2, -1)$. Graph the vertex and several other points on either side of the axis of symmetry.

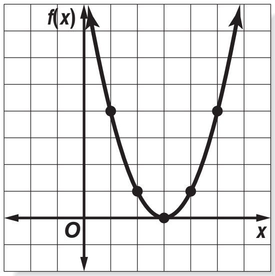


To solve $x^2 + 4x + 3 = 0$, you need to know where $f(x) = 0$. This occurs at the x-intercepts, -3 and -1.

The solutions are -3 and -1.

Example 2 **Solve $x^2 - 6x + 9 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - 6x + 9$. The equation of the axis of symmetry is $x = \frac{6}{2(1)}$ or 3. The vertex is at $(3, 0)$. Graph the vertex and several other points on either side of the axis of symmetry.

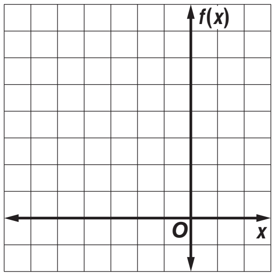


To solve $x^2 - 6x + 9 = 0$, you need to know where $f(x) = 0$. The vertex of the parabola is the x-intercept. Thus, the only solution is 3.

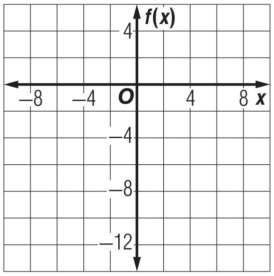
Exercises

Solve each equation by graphing.

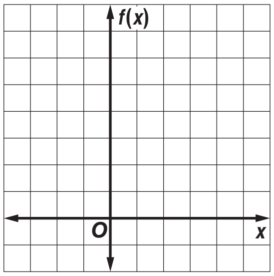
1. $x^2 + 7x + 12 = 0$



2. $x^2 - x - 12 = 0$



3. $x^2 - 4x + 5 = 0$



9-2 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Graphing

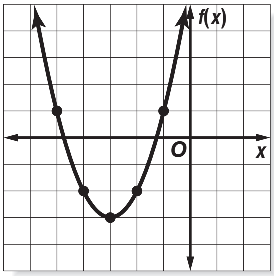
Estimate Solutions The roots of a quadratic equation may not be integers. If exact roots cannot be found, they can be estimated by finding the consecutive integers between which the roots lie.

Example **Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(x) = x^2 + 6x + 6$.

x	$f(x)$
−5	1
−4	−2
−3	−3
−2	−2
−1	1

Notice that the value of the function changes from negative to positive between the x-values of −5 and −4 and between −2 and −1.

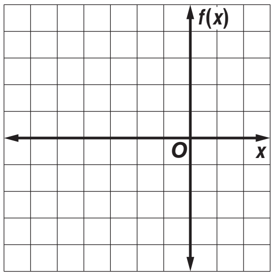


The x-intercepts of the graph are between −5 and −4 and between −2 and −1. So one root is between −5 and −4, and the other root is between −2 and −1.

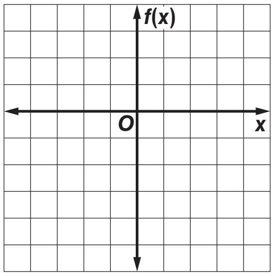
Exercises

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

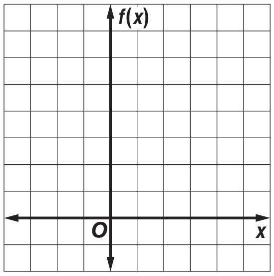
1. $x^2 + 7x + 9 = 0$



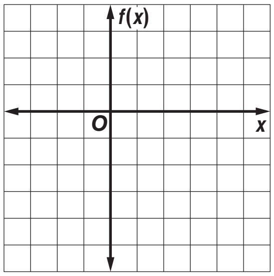
2. $x^2 - x - 4 = 0$



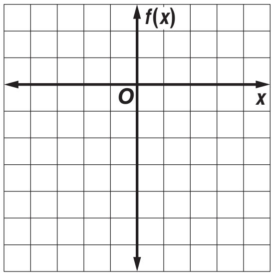
3. $x^2 - 4x + 6 = 0$



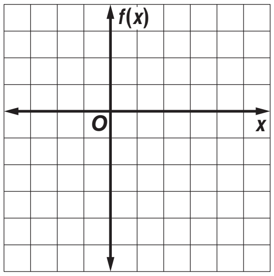
4. $x^2 - 4x - 1 = 0$



5. $4x^2 - 12x + 3 = 0$



6. $x^2 - 2x - 4 = 0$

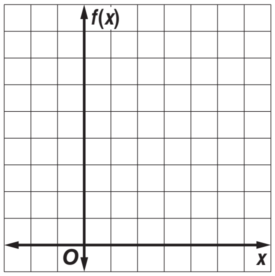


9-2 Skills Practice

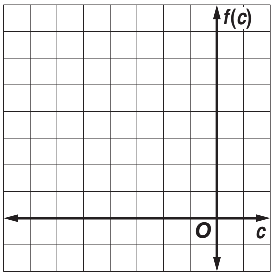
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

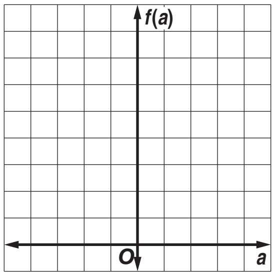
1. $x^2 - 2x + 3 = 0$



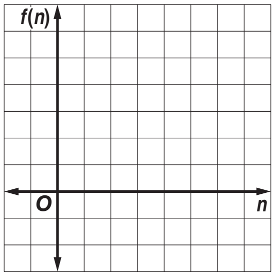
2. $c^2 + 6c + 8 = 0$



3. $a^2 - 2a = -1$

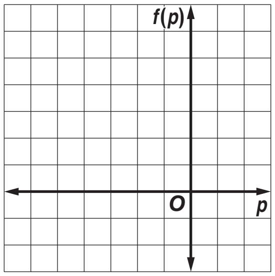


4. $n^2 - 7n = -10$

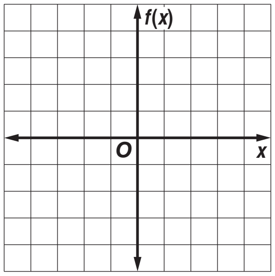


Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

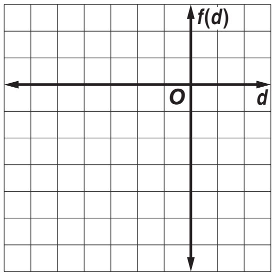
5. $p^2 + 4p + 2 = 0$



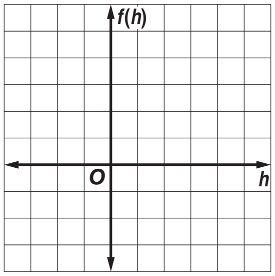
6. $x^2 + x - 3 = 0$



7. $d^2 + 6d = -3$



8. $h^2 + 1 = 4h$



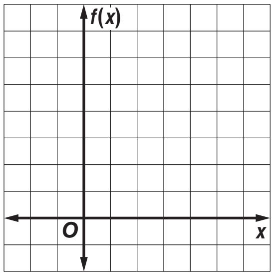
NAME ____________________ DATE ____________ PERIOD ____________

9-2 Practice

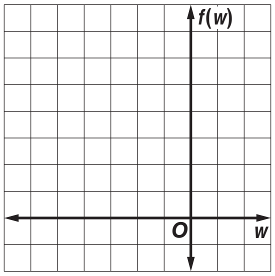
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

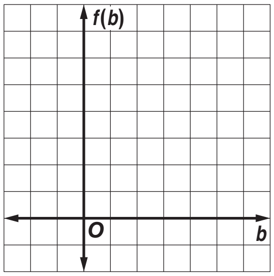
1. $x^2 - 5x + 6 = 0$



2. $w^2 + 6w + 9 = 0$

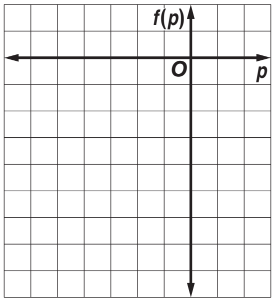


3. $b^2 - 3b + 4 = 0$

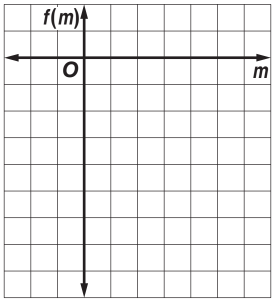


Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

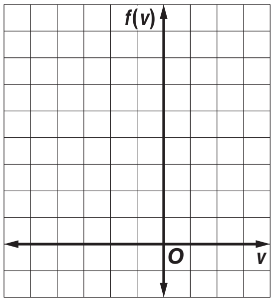
4. $p^2 + 4p = 3$



5. $2m^2 + 5 = 10m$



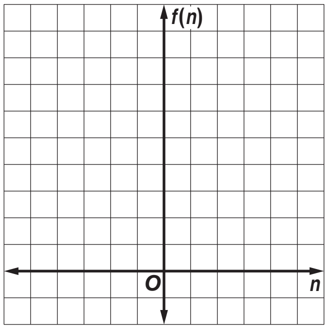
6. $2v^2 + 8v = -7$



7. NUMBER THEORY Two numbers have a sum of 2 and a product of −8. The quadratic equation $-n^2 + 2n + 8 = 0$ can be used to determine the two numbers.

a. Graph the related function $f(n) = -n^2 + 2n + 8$ and determine its x-intercepts.

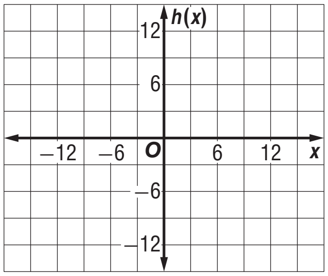
b. What are the two numbers?



8. DESIGN A footbridge is suspended from a parabolic support. The function $h(x) = -\frac{1}{25}x^2 + 9$ represents the height in feet of the support above the walkway, where $x = 0$ represents the midpoint of the bridge.

a. Graph the function and determine its x-intercepts.

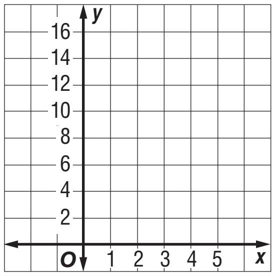
b. What is the length of the walkway between the two supports?



9-2 Word Problem Practice

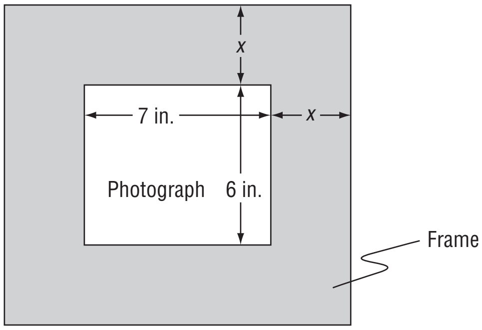
Solving Quadratic Equations by Graphing

1. FARMING In order for Mr. Moore to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. His crop yield y depends on the amount of fertilizer he applies to his fields x according to the equation $y = -x^2 + 4x + 12$. Graph the function, and find the point at which Mr. Moore gets the highest yield possible.



2. LIGHT Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the parabola and find that the equation $y = x^2 - 3x - 10$ matches the shape of the light beam. Determine the zeros of the function.

3. FRAMING A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using a material that is x inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material?

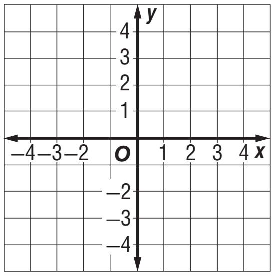


4. WRAPPING PAPER Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (*Hint:* Let length = $30 - w$.) Explain.

5. ENGINEERING The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation.

$$0.5x^2 = 2 + y$$

a. Approximate the zeros of this function by graphing.



b. On the coordinate plane above, translate the parabola so that there is only one zero. Label this curve A.

c. Translate the parabola so that there are no zeros. Label this curve B.

Lesson 9-2

9-2 Enrichment

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points (0, –2), (3, 0), and (5, 2).

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y, you get three equations.

(0, –2): $-2 = c$

(3, 0): $0 = 9a + 3b + c$

(5, 2): $2 = 25a + 5b + c$

First, substitute –2 for c in the second and third equations.
Then solve those two equations as you would any system of two equations.
Multiply the second equation by 5 and the third equation by –3.

$$\begin{array}{lcrcl}
0 = 9a + 3b - 2 & \xrightarrow{\text{Multiply by 5.}} & 0 = & 45a + 15b & - 10 \\
2 = 25a + 5b - 2 & \xrightarrow{\text{Multiply by } -3.} & -6 = & -75a - 15b & + 6 \\
\hline
 & & -6 = & -30a & - 4 \\
 & & a = & \frac{1}{15} &
\end{array}$$

To find b, substitute $\frac{1}{15}$ for a in either the second or third equation.

$$0 = 9\left(\frac{1}{15}\right) + 3b - 2$$

$$b = \frac{7}{15}$$

The equation of a parabola through the three points is

$$y = \frac{1}{15}x^2 + \frac{7}{15}x - 2.$$

Find the equation of a parabola through each set of three points.

1. (1, 5), (0, 6), (2, 3)

2. (–5, 0), (0, 0), (8, 100)

3. (4, –4), (0, 1), (3, –2)

4. (1, 3), (6, 0), (0, 0)

5. (2, 2), (5, –3), (0, –1)

6. (0, 4), (4, 0), (–4, 4)

9-3 Study Guide and Intervention

Transformations of Quadratic Functions

Translations A **translation** is a change in the position of a figure either up, down, left, right, or diagonal. Adding or subtracting constants in the equations of functions translates the graphs of the functions.

The graph of $g(x) = x^2 + k$ translates the graph of $f(x) = x^2$ vertically.

If $k > 0$, the graph of $f(x) = x^2$ is translated k units up.

If $k < 0$, the graph of $f(x) = x^2$ is translated $|k|$ units down.

The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

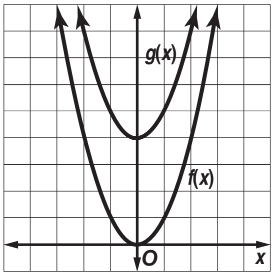
If $h > 0$, the graph of $f(x) = x^2$ is translated h units to the right.

If $h < 0$, the graph of $f(x) = x^2$ is translated $|h|$ units to the left.

Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

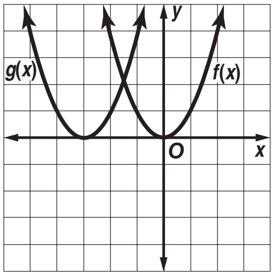
a. $g(x) = x^2 + 4$

The value of k is 4, and $4 > 0$. Therefore, the graph of $g(x) = x^2 + 4$ is a translation of the graph of $f(x) = x^2$ up 4 units.



b. $g(x) = (x + 3)^2$

The value of h is –3, and $-3 < 0$. Thus, the graph of $g(x) = (x + 3)^2$ is a translation of the graph of $f(x) = x^2$ to the left 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 1$

2. $g(x) = (x - 6)^2$

3. $g(x) = (x + 1)^2$

4. $g(x) = 20 + x^2$

5. $g(x) = (-2 + x)^2$

6. $g(x) = -\frac{1}{2} + x^2$

7. $g(x) = x^2 + \frac{8}{9}$

8. $g(x) = x^2 - 0.3$

9. $g(x) = (x + 4)^2$

Lesson 9-3

NAME ______________________________ DATE ____________ PERIOD ____________

9-3 Study Guide and Intervention *(continued)*

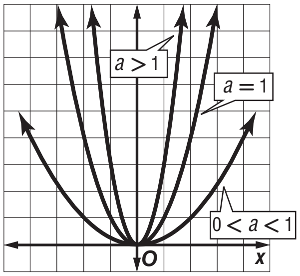
Transformations of Quadratic Functions

Dilations and Reflections A **dilation** is a transformation that makes the graph narrower or wider than the parent graph. A **reflection** flips a figure over the x- or y-axis.

The graph of $f(x) = ax^2$ stretches or compresses the graph of $f(x) = x^2$.

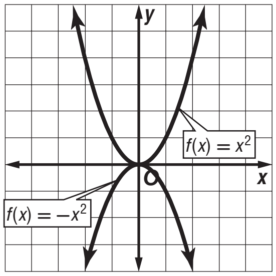
If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



The graph of the function $-f(x)$ flips the graph of $f(x) = x^2$ across the x-axis.

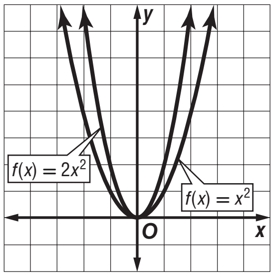
The graph of the function $f(-x)$ flips the graph of $f(x) = x^2$ across the y-axis.



Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

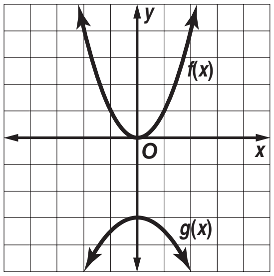
a. $g(x) = 2x^2$

The function can be written as $f(x) = ax^2$ where $a = 2$. Because $|a| > 1$, the graph of $y = 2x^2$ is the graph of $y = x^2$ that is stretched vertically.



b. $g(x) = -\frac{1}{2}x^2 - 3$

The negative sign causes a reflection across the x-axis. Then a dilation occurs in which $a = \frac{1}{2}$ and a translation in which $k = -3$. So the graph of $g(x) = -\frac{1}{2}x^2 - 3$ is reflected across the x-axis, dilated wider than the graph of $f(x) = x^2$, and translated down 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = -5x^2$

2. $g(x) = -(x + 1)^2$

3. $g(x) = -\frac{1}{4}x^2 - 1$

NAME ______________________________ DATE ____________ PERIOD ____________

9-3 Skills Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 2$

2. $g(x) = (x - 1)^2$

3. $g(x) = x^2 - 8$

4. $g(x) = 7x^2$

5. $g(x) = \frac{1}{5}x^2$

6. $g(x) = -6x^2$

7. $g(x) = -x^2 + 3$

8. $g(x) = 5 - \frac{1}{2}x^2$

9. $g(x) = 4(x - 1)^2$

Match each equation to its graph.

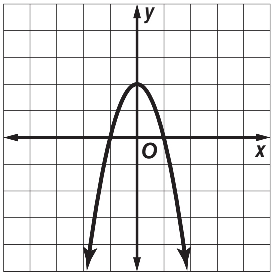
10. $y = 2x^2 - 2$

11. $y = \frac{1}{2}x^2 - 2$

12. $y = -\frac{1}{2}x^2 + 2$

13. $y = -2x^2 + 2$

A.



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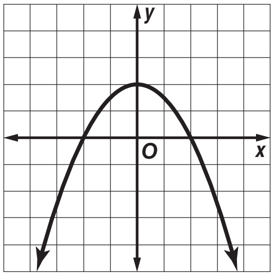

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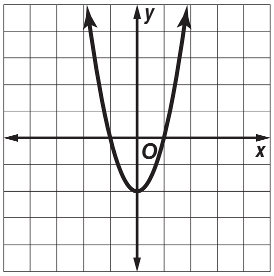

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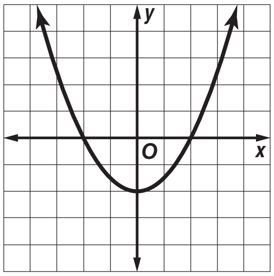

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9-3 Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = (10 + x)^2$

2. $g(x) = -\frac{2}{5} + x^2$

3. $g(x) = 9 - x^2$

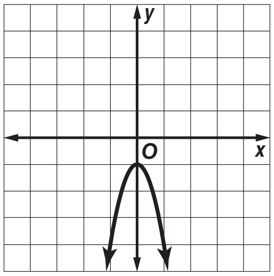
4. $g(x) = 2x^2 + 2$

5. $g(x) = -\frac{3}{4}x^2 - \frac{1}{2}$

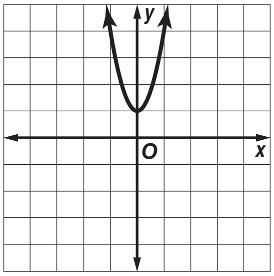
6. $g(x) = -3(x + 4)^2$

Match each equation to its graph.

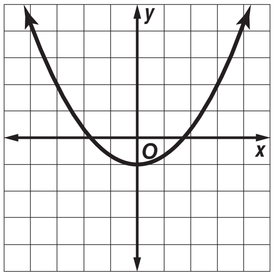
A.



B.



C.



7. $y = -3x^2 - 1$

8. $y = \frac{1}{3}x^2 - 1$

9. $y = 3x^2 + 1$

List the functions in order from the most vertically stretched to the least vertically stretched graph.

10. $f(x) = 3x^2, g(x) = \frac{1}{2}x^2, h(x) = -2x^2$

11. $f(x) = \frac{1}{2}x^2, g(x) = -\frac{1}{6}x^2, h(x) = 4x^2$

12. PARACHUTING Two parachutists jump at the same time from two different planes as part of an aerial show. The height h_1 of the first parachutist in feet after t seconds is modeled by the function $h_1 = -16t^2 + 5000$. The height h_2 of the second parachutist in feet after t seconds is modeled by the function $h_2 = -16t^2 + 4000$.

a. What is the parent function of the two functions given?

b. Describe the transformations needed to obtain the graph of h_1 from the parent function.

c. Which parachutist will reach the ground first?

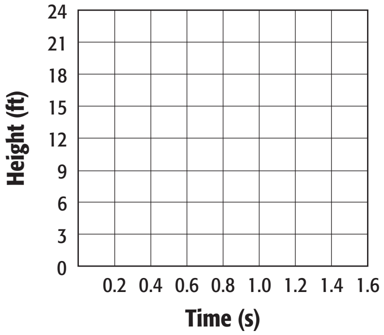
9-3 Word Problem Practice

Transformations of Quadratic Functions

1. **SPRINGS** The potential energy stored in a spring is given by $U_s = \frac{1}{2}kx^2$ where k is a constant known as the spring constant, and x is the distance the spring is stretched or compressed from its initial position. How is the graph of the function for a spring where $k = 2$ newtons/meter related to the graph of the function for a spring where $k = 10$ newtons/meter?

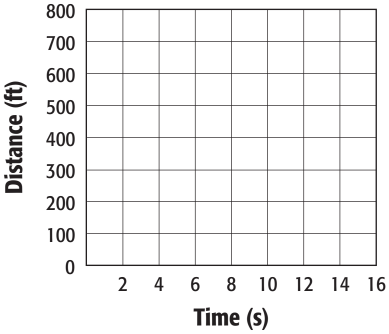
2. **BLUEPRINTS** The bottom left corner of a rectangular park is located at (–3, 5) on a construction blueprint. The park is 8 units long and has an area of 48 units^2 on the blueprint. Suppose the park is translated on the blueprint so that the top right corner is now located at the origin. Describe the translation of the graph.

3. **PHYSICS** A ball is dropped from a height of 20 feet. The function $h = -16t^2 + 20$ models the height of the ball in feet after t seconds. Graph the function and compare this graph to the graph of its parent function.



4. **ACCELERATION** The distance d in feet a car accelerating at 6 ft/s^2 travels after t seconds is modeled by the function $d = 3t^2$. Suppose that at the same time the first car begins accelerating, a second car begins accelerating at 4 ft/s^2 exactly 100 feet down the road from the first car. The distance traveled by second car is modeled by the function $d = 2t^2 + 100$.

   a. Graph and label each function on the same coordinate plane.



   b. Explain how each graph is related to the graph of $d = t^2$.

   c. After how many seconds will the first car pass the second car?

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9-3 Enrichment

Graphing Polynomial Functions

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable.

Polynomial Function
If n is a nonnegative integer, $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ are real numbers, and $a_n \neq 0$, then $$f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$$ is a polynomial function of degree n.

Notice that a quadratic function is a polynomial function of degree 2.

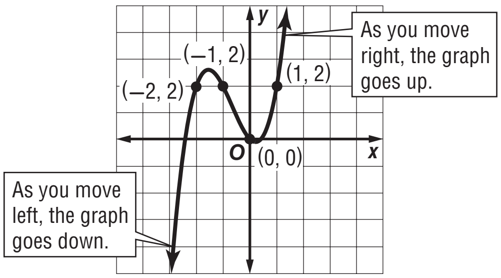
Example **Create a table of values and a graph for $y = x^3 + 2x^2 - x$. Then describe its end behavior.**

Create a table of values, and graph the ordered pairs. Connect the points with a smooth curve. Find and plot additional points to better approximate the curve's shape.

x	$x^3 + 2x^2 - x$	y
−4	$(-4)^3 + 2(-4)^2 - (-4)$	−28
−3	$(-3)^3 + 2(-3)^2 - (-3)$	−6
−2	$(-2)^3 + 2(-2)^2 - (-2)$	2
−1	$(-1)^3 + 2(-1)^2 - (-1)$	2
0	$(0)^3 + 2(0)^2 - 0$	0
1	$1^3 + 2(1)^2 - 1$	2
2	$2^3 + 2(2)^2 - 2$	14
3	$3^3 + 2(3)^2 - 3$	42

As x decreases, y decreases

As x increases, y increases.



As you move right, the graph goes up.

As you move left, the graph goes down.

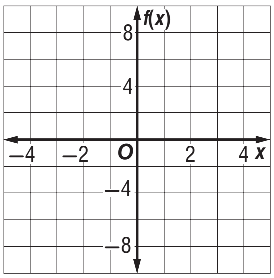
From the table and the graph we see that as x decreases, y decreases and as x increases, y increases.

Exercises

Create a table of values and a graph for each function. Then describe its end behavior.

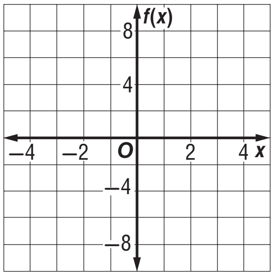
1. $f(x) = x^3 + 3x^2 - 1$

x	f(x)
−4	
−3	
−2	
−1	
0	
1	



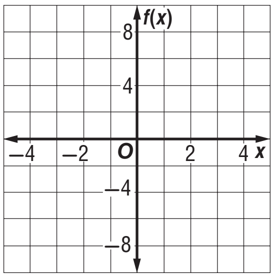
2. $f(x) = -2x^5 + 4x^3$

x	f(x)
−2	
−1	
−0.5	
0	
0.5	
1	
2	



3. $f(x) = 2x^4 - 4x^3 + 3x$

x	f(x)
−2	
−1	
−0.5	
0	
0.5	
1	
2	



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9-4 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Complete the Square Perfect square trinomials can be solved quickly by taking the square root of both sides of the equation. A quadratic equation that is not in perfect square form can be made into a perfect square by a method called **completing the square**.

Completing the Square

To complete the square for any quadratic equation of the form $x^2 + bx$:

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example **Find the value of c that makes $x^2 + 2x + c$ a perfect square trinomial.**

Step 1 Find $\frac{1}{2}$ of 2. $\frac{2}{2} = 1$

Step 2 Square the result of Step 1. $1^2 = 1$

Step 3 Add the result of Step 2 to $x^2 + 2x$. $x^2 + 2x + 1$

Thus, $c = 1$. Notice that $x^2 + 2x + 1$ equals $(x + 1)^2$.

Exercises

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 10x + c$

2. $x^2 + 14x + c$

3. $x^2 - 4x + c$

4. $x^2 - 8x + c$

5. $x^2 + 5x + c$

6. $x^2 + 9x + c$

7. $x^2 - 3x + c$

8. $x^2 - 15x + c$

9. $x^2 + 28x + c$

10. $x^2 + 22x + c$

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9-4 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Solve by Completing the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$ Original equation

$x^2 + 6x + 3 - 3 = 10 - 3$ Subtract 3 from each side.

$x^2 + 6x = 7$ Simplify.

$x^2 + 6x + 9 = 7 + 9$ Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.

$(x + 3)^2 = 16$ Factor $x^2 + 6x + 9$.

$x + 3 = \pm 4$ Take the square root of each side.

$x = -3 \pm 4$ Simplify.

$x = -3 + 4$ or $x = -3 - 4$

$= 1$ $= -7$

The solution set is {−7, 1}.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $x^2 - 4x + 3 = 0$

2. $x^2 + 10x = -9$

3. $x^2 - 8x - 9 = 0$

4. $x^2 - 6x = 16$

5. $x^2 - 4x - 5 = 0$

6. $x^2 - 12x = 9$

7. $x^2 + 8x = 20$

8. $x^2 = 2x + 1$

9. $x^2 + 20x + 11 = -8$

10. $x^2 - 1 = 5x$

11. $x^2 = 22x + 23$

12. $x^2 - 8x = -7$

13. $x^2 + 10x = 24$

14. $x^2 - 18x = 19$

15. $x^2 + 16x = -16$

16. $4x^2 = 24 + 4x$

17. $2x^2 + 4x + 2 = 8$

18. $4x^2 = 40x + 44$

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9-4 Skills Practice

Solving Quadratic Equations by Completing the Square

Find the value of *c* that makes each trinomial a perfect square.

1. $x^2 + 6x + c$

2. $x^2 + 4x + c$

3. $x^2 - 14x + c$

4. $x^2 - 2x + c$

5. $x^2 - 18x + c$

6. $x^2 + 20x + c$

7. $x^2 + 5x + c$

8. $x^2 - 70x + c$

9. $x^2 - 11x + c$

10. $x^2 + 9x + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

11. $x^2 + 4x - 12 = 0$

12. $x^2 - 8x + 15 = 0$

13. $x^2 + 6x = 7$

14. $x^2 - 2x = 15$

15. $x^2 - 14x + 30 = 6$

16. $x^2 + 12x + 21 = 10$

17. $x^2 - 4x + 1 = 0$

18. $x^2 - 6x + 4 = 0$

19. $x^2 - 8x + 10 = 0$

20. $x^2 - 2x = 5$

21. $2x^2 + 20x = -2$

22. $0.5x^2 + 8x = -7$

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9-4 Practice

Solving Quadratic Equations by Completing the Square

Find the value of *c* that makes each trinomial a perfect square.

1. $x^2 - 24x + c$

2. $x^2 + 28x + c$

3. $x^2 + 40x + c$

4. $x^2 + 3x + c$

5. $x^2 - 9x + c$

6. $x^2 - x + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. $x^2 - 14x + 24 = 0$

8. $x^2 + 12x = 13$

9. $x^2 - 30x + 56 = -25$

10. $x^2 + 8x + 9 = 0$

11. $x^2 - 10x + 6 = -7$

12. $x^2 + 18x + 50 = 9$

13. $3x^2 + 15x - 3 = 0$

14. $4x^2 - 72 = 24x$

15. $0.9x^2 + 5.4x - 4 = 0$

16. $0.4x^2 + 0.8x = 0.2$

17. $\frac{1}{2}x^2 - x - 10 = 0$

18. $\frac{1}{4}x^2 + x - 2 = 0$

19. NUMBER THEORY The product of two consecutive even integers is 728. Find the integers.

20. BUSINESS Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Jaime has made in month x for the first two years of his business.

a. Write an equation representing the month in which Jaime's profit is \$2400.

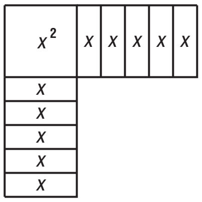
b. Use completing the square to find out in which month Jaime's profit is \$2400.

21. PHYSICS From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height H of the rock t seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace H with 0.)

9-4 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. INTERIOR DESIGN Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 foot by 1 foot squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below.



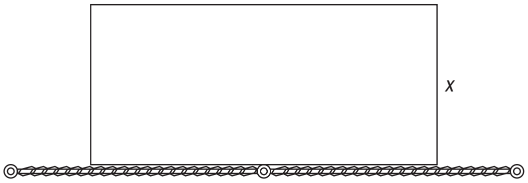
$x^2 + 10x + \_\_\_\_\_ = (x + \_\_\_\_\_)^2$

2. FALLING OBJECTS Keisha throws a rock down an old well. The distance d in feet the rock falls after t seconds can be represented by $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water?

3. MARS On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will hit the ground in about 5.4 seconds. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. How much longer will it take for the golf ball hit on Mars to reach the ground? Round your answer to the nearest tenth.

4. FROGS A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h in feet is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground?

5. GARDENING Peg is planning a rectangular vegetable garden using 200 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



a. Let x = the width of the rectangle. Write an equation to represent the area A of the garden if Peg uses all the fencing material.

b. For what widths would the area of Peg's garden equal 4800 square feet if she uses all the fencing material?

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9-4 Enrichment

Factoring Quartic Polynomials

Completing the square is a useful tool for factoring quadratic expressions. You can utilize a similar technique to factor simple **quartic polynomials** of the form $x^4 + c$.

Example **Factor the quartic polynomial $x^4 + 64$.**

Step 1 Find the value of the middle term needed to complete the square.
This value is $\left(2\sqrt{64}\right)(x^2)$, or $16x^2$.

Step 2 Rewrite the original polynomial in factorable form.
$\left(x^4 + 16x^2 + \left(\frac{16}{2}\right)^2\right) - 16x^2$

Step 3 Factor the polynomials. $(x^2 + 8)^2 - (4x)^2$

Step 4 Rewrite using the difference of two squares.
$(x^2 + 8 + 4x)(x^2 + 8 - 4x)$

The factored form of $x^2 + 64$ is $(x^2 + 4x + 8)(x^2 - 4x + 8)$. This could then be factored further, if needed, to find the solutions to a quartic equation.

Exercises

Factor each quartic polynomial.

1. $x^4 + 4$

2. $x^4 + 324$

3. $x^4 + 2500$

4. $x^4 + 9604$

5. $x^4 + 1024$

6. $x^4 + 5184$

7. $x^4 + 484$

8. $x^4 + 9$

9. $x^4 + 144$

10. $x^8 + 16{,}384$

11. Factor $x^4 + c$ to come up with a general rule for factoring quartic polynomials.

9-5 Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{16}}{2}$$

$$x = \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2}$$

$$= 1 \qquad\qquad = -3$$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{6 + \sqrt{44}}{2}$$

$$x = \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2}$$

$$\approx 6.3 \qquad\qquad \approx -0.3$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$

2. $x^2 - 8x = -16$

3. $16x^2 - 8x = -1$

4. $x^2 + 5x = 6$

5. $3x^2 + 2x = 8$

6. $8x^2 - 8x - 5 = 0$

7. $-4x^2 + 19x = 21$

8. $2x^2 + 6x = 5$

9. $48x^2 + 22x - 15 = 0$

10. $8x^2 - 4x = 24$

11. $2x^2 + 5x = 8$

12. $8x^2 + 9x - 4 = 0$

13. $2x^2 + 9x + 4 = 0$

14. $8x^2 + 17x + 2 = 0$

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9-5 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real solutions for a quadratic equation.

***Case 1*:** $b^2 - 4ac < 0$	no real solutions
***Case 2*:** $b^2 - 4ac = 0$	one real solution
***Case 3*:** $b^2 - 4ac > 0$	two real solutions

Example **State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.**

a. $\mathbf{12x^2 + 5x = 4}$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real solutions.

b. $\mathbf{2x^2 + 3x = -4}$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

Find the discriminant.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real solutions.

Exercises

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

1. $3x^2 + 2x - 3 = 0$

2. $3x^2 - 7x - 8 = 0$

3. $2x^2 - 10x - 9 = 0$

4. $4x^2 = x + 4$

5. $3x^2 - 13x = 10$

6. $6x^2 - 10x + 10 = 0$

7. $2x^2 - 20 = -x$

8. $6x^2 = -11x - 40$

9. $9 - 18x + 9x^2 = 0$

10. $12x^2 + 9 = -6x$

11. $9x^2 = 81$

12. $16x^2 + 16x + 4 = 0$

13. $8x^2 + 9x = 2$

14. $4x^2 - 4x + 4 = 3$

15. $3x^2 - 18x = - 14$

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9-5 Skills Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 49 = 0$

2. $x^2 - x - 20 = 0$

3. $x^2 - 5x - 36 = 0$

4. $x^2 + 11x + 30 = 0$

5. $x^2 - 7x = -3$

6. $x^2 + 4x = -1$

7. $x^2 - 9x + 22 = 0$

8. $x^2 + 6x + 3 = 0$

9. $2x^2 + 5x - 7 = 0$

10. $2x^2 - 3x = -1$

11. $2x^2 + 5x + 4 = 0$

12. $2x^2 + 7x = 9$

13. $3x^2 + 2x - 3 = 0$

14. $3x^2 - 7x - 6 = 0$

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

15. $x^2 + 4x + 3 = 0$

16. $x^2 + 2x + 1 = 0$

17. $x^2 - 4x + 10 = 0$

18. $x^2 - 6x + 7 = 0$

19. $x^2 - 2x - 7 = 0$

20. $x^2 - 10x + 25 = 0$

21. $2x^2 + 5x - 8 = 0$

22. $2x^2 + 6x + 12 = 0$

23. $2x^2 - 4x + 10 = 0$

24. $3x^2 + 7x + 3 = 0$

9-5 Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 + 2x - 3 = 0$

2. $x^2 + 8x + 7 = 0$

3. $x^2 - 4x + 6 = 0$

4. $x^2 - 6x + 7 = 0$

5. $2x^2 + 9x - 5 = 0$

6. $2x^2 + 12x + 10 = 0$

7. $2x^2 - 9x = -12$

8. $2x^2 - 5x = 12$

9. $3x^2 + x = 4$

10. $3x^2 - 1 = -8x$

11. $4x^2 + 7x = 15$

12. $1.6x^2 + 2x + 2.5 = 0$

13. $4.5x^2 + 4x - 1.5 = 0$

14. $\frac{1}{2}x^2 + 2x + \frac{3}{2} = 0$

15. $3x^2 - \frac{3}{4}x = \frac{1}{2}$

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

16. $x^2 + 8x + 16 = 0$

17. $x^2 + 3x + 12 = 0$

18. $2x^2 + 12x = -7$

19. $2x^2 + 15x = -30$

20. $4x^2 + 9 = 12x$

21. $3x^2 - 2x = 3.5$

22. $2.5x^2 + 3x - 0.5 = 0$

23. $\frac{3}{4}x^2 - 3x = -4$

24. $\frac{1}{4}x^2 = -x - 1$

25. CONSTRUCTION A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

a. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint*: Since the object is thrown down, the initial velocity is negative.)

b. How long does it take the tile to hit the ground?

26. PHYSICS Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint:* Substitute 25 for h and use the discriminant.)

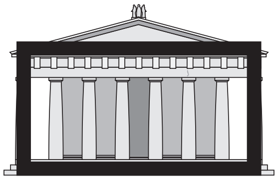
9-5 Word Problem Practice

Solving Quadratic Equations by Using the Quadratic Formula

1. **BUSINESS** Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit?

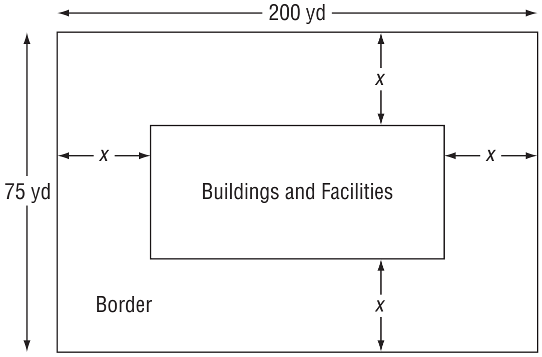
2. **AERONAUTICS** At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth.

3. **ARCHITECTURE** The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth.



4. **CRAFTS** Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x be the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.

5. **SITE DESIGN** The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x represent the width of the border.



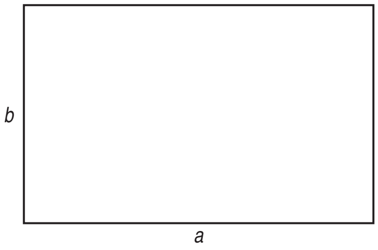
   a. Use an equation similar to $A = \ell \times w$ to represent the situation.

   b. Write the equation in standard quadratic form.

   c. What should be the width of the border? Round your answer to the nearest tenth.

9-5 Enrichment

Golden Rectangles



A **golden rectangle** has the property that its sides satisfy the following proportion.

$$\frac{a + b}{a} = \frac{a}{b}$$

Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.

1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation.

2. Solve the equation in Exercise 1 for b.

3. In the proportion, let $b = 1$. Write a quadratic equation in a.

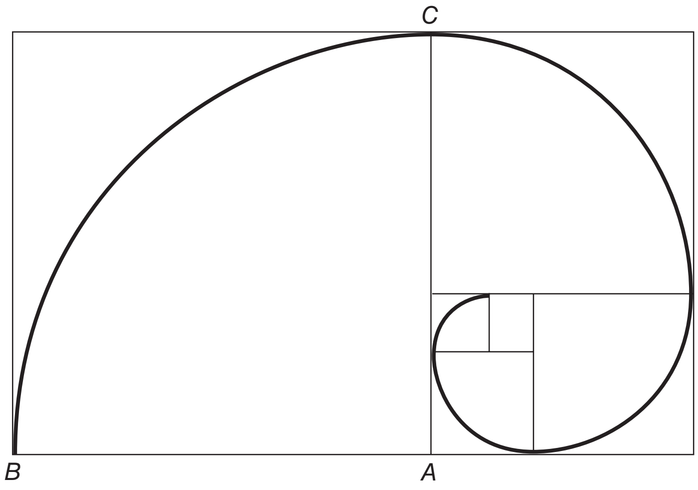
4. Solve the equation in Exercise 3 for a.

5. Explain why $\frac{1}{2}(\sqrt{5} + 1)$ and $\frac{1}{2}(\sqrt{5} - 1)$ are called golden ratios.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarters of circles.

For example, the arc from point B to point C is created by putting the point of a compass at point A. The radius of the arc is the length BA.



6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

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9-6 Study Guide and Intervention

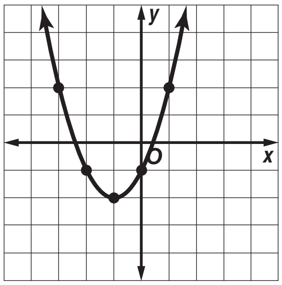
Analyzing Functions with Successive Differences and Ratios

Identify Functions Linear functions, quadratic functions, and exponential functions can all be used to model data. The general forms of the equations are listed at the right.

Linear Function	$y = mx + b$
Quadratic Function	$y = ax^2 + bx + c$
Exponential Function	$y = ab^x$

You can also identify data as linear, quadratic, or exponential based on patterns of behavior of their *y*-values.

Example 1 **Graph the set of ordered pairs {(–3, 2), (–2, –1), (–1, –2), (0, –1), (1, 2)}. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.**



The ordered pairs appear to represent a quadratic function.

Example 2 **Look for a pattern in the table to determine which model best describes the data.**

x	–2	–1	0	1	2
y	4	2	1	0.5	0.25

Start by comparing the first differences.

$$4 \xrightarrow[-2]{} 2 \xrightarrow[-1]{} 1 \xrightarrow[-0.5]{} 0.5 \xrightarrow[-0.25]{} 0.25$$

The first differences are not all equal. The table does not represent a linear function. Find the second differences and compare.

$$-2 \xrightarrow[+1]{} -1 \xrightarrow[+0.5]{} -0.5 \xrightarrow[+0.25]{} -0.25$$

The table does not represent a quadratic function. Find the ratios of the *y*-values.

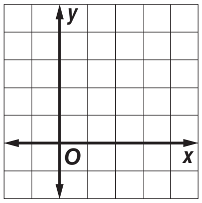
$$4 \xrightarrow[\times 0.5]{} 2 \xrightarrow[\times 0.5]{} 1 \xrightarrow[\times 0.5]{} 0.5 \xrightarrow[\times 0.5]{} 0.25$$

The ratios are equal. Therefore, the table can be modeled by an exponential function.

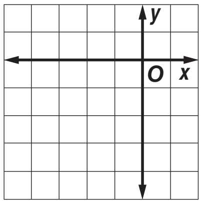
Exercises

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (0, –1), (1, 1), (2, 3), (3, 5)



2. (–3, –1), (–2, –4), (–1, –5), (0, –4), (1, –1)



Look for a pattern in each table to determine which model best describes the data.

3.

x	–2	–1	0	1	2
y	6	5	4	3	2

4.

x	–2	–1	0	1	2
y	6.25	2.5	1	0.4	0.16

9-6 Study Guide and Intervention *(continued)*

Analyzing Functions with Successive Differences and Ratios

Write Equations Once you find the model that best describes the data, you can write an equation for the function.

Basic Forms	Linear Function	$y = mx + b$
	Quadratic Function	$y = ax^2$
	Exponential Function	$y = ab^x$

Example **Determine which model best describes the data. Then write an equation for the function that models the data.**

x	0	1	2	3	4
y	3	6	12	24	48

Step 1 Determine whether the data is modeled by a linear, quadratic, or exponential function.

First differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24 \xrightarrow[+24]{} 48$

Second differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24$

y-value ratios: $3 \xrightarrow[\times 2]{} 6 \xrightarrow[\times 2]{} 12 \xrightarrow[\times 2]{} 24 \xrightarrow[\times 2]{} 48$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

Step 2 Write an equation for the function that models the data. The equation has the form $y = ab^x$. The y-value ratio is 2, so this is the value of the base.

$y = ab^x$ Equation for exponential function

$3 = a(2)^0$ $x = 0$, $y = 3$, and $b = 2$

$3 = a$ Simplify.

An equation that models the data is $y = 3 \cdot 2^x$. To check the results, you can verify that the other ordered pairs satisfy the function.

Exercises

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

1.

x	–2	–1	0	1	2
y	12	3	0	3	12

2.

x	–1	0	1	2	3
y	–2	1	4	7	10

3.

x	–1	0	1	2	3
y	0.75	3	12	48	192

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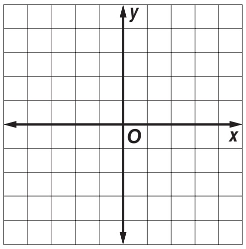
9-6 Skills Practice

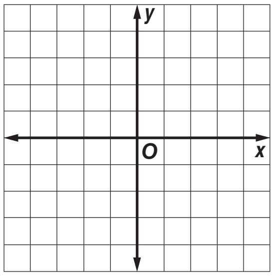
Analyzing Functions with Successive Differences and Ratios

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

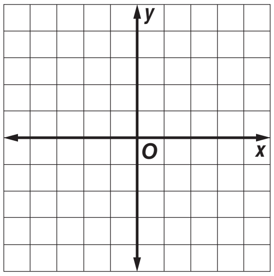
1. (2, 3), (1, 1), (0, –1), (–1, –3), (–3, –5)



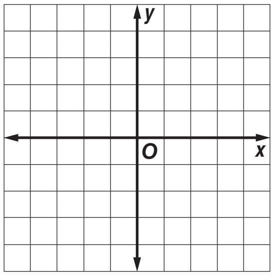
2. (–1, 0.5), (0, 1), (1, 2), (2, 4)



3. (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4)



4. (–3, 5), (–2, 2), (–1, 1), (0, 2), (1, 5)



Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5.

x	–3	–2	–1	0	1	2
y	32	16	8	4	2	1

6.

x	–1	0	1	2	3
y	7	3	–1	–5	–9

7.

x	–3	–2	–1	0	1
y	–27	–12	–3	0	–3

8.

x	0	1	2	3	4
y	0.5	1.5	4.5	13.5	40.5

9.

x	–2	–1	0	1	2
y	–8	–4	0	4	8

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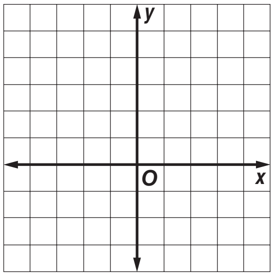
9-6 Practice

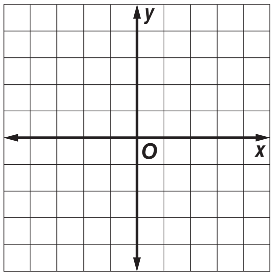
Analyzing Functions with Successive Differences and Ratios

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

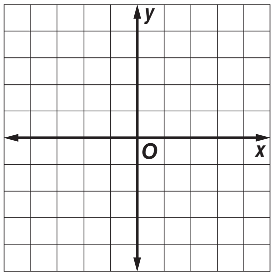
1. (4, 0.5), (3, 1.5), (2, 2.5), (1, 3.5), (0, 4.5)



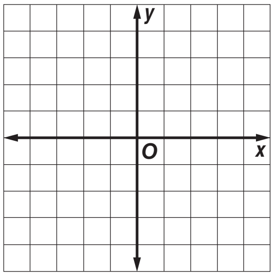
2. $\left(-1, \frac{1}{9}\right)$, $\left(0, \frac{1}{3}\right)$, (1, 1), (2, 3)



3. (–4, 4), (–2, 1), (0, 0), (2, 1), (4, 4)



4. (–4, 2), (–2, 1), (0, 0), (2, –1), (4, –2)



Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5.

x	–3	–1	1	3	5
y	–5	–2	1	4	7

6.

x	–2	–1	0	1	2
y	0.02	0.2	2	20	200

7.

x	–1	0	1	2	3
y	6	0	6	24	54

8.

x	–2	–1	0	1	2
y	18	9	0	–9	–18

9. INSECTS The local zoo keeps track of the number of dragonflies breeding in their insect exhibit each day.

Day	1	2	3	4	5
Dragonflies	9	18	36	72	144

a. Determine which function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine the number of dragonflies that will be breeding after 9 days.

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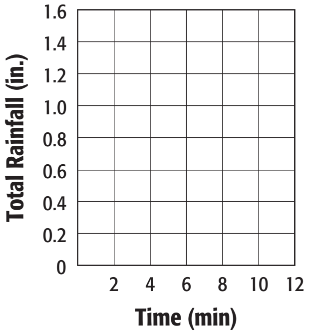
9-6 Word Problem Practice

Analyzing Functions with Successive Differences and Ratios

1. **WEATHER** The San Mateo weather station records the amount of rainfall since the beginning of a thunderstorm. Data for a storm is recorded as a series of ordered pairs shown below, where the *x* value is the time in minutes since the start of the storm, and the *y* value is the amount of rain in inches that has fallen since the start of the storm.

   (2, 0.3), (4, 0.6), (6, 0.9), (8, 1.2), (10, 1.5)

   Graph the ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.



2. **INVESTING** The value of a certain parcel of land has been increasing in value ever since it was purchased. The table shows the value of the land parcel over time.

Year Since Purchasing	0	1	2	3	4
Land Value (thousands $)	$1.05	$2.10	$4.20	$8.40	$16.80

   Look for a pattern in the table of values to determine which model best describes the data. Then write an equation for the function that models the data.

3. **BOATS** The value of a boat typically depreciates over time. The table shows the value of a boat over a period of time.

Years	0	1	2	3	4
Boat Value ($)	8250	6930	5821.20	4889.81	4107.44

   Write an equation for the function that models the data. Then use the equation to determine how much the boat is worth after 9 years.

4. **NUCLEAR WASTE** Radioactive material slowly decays over time. The amount of time needed for an amount of radioactive material to decay to half its initial quantity is known as its half-life. Consider a 20-gram sample of a radioactive isotope.

Half-Lives Elapsed	0	1	2	3	4
Amount of Isotope Remaining (grams)	20	10	5	2.5	1.25

   a. Is radioactive decay a *linear* decay, *quadratic* decay, or an *exponential* decay?

   b. Write an equation to determine how many grams *y* of a radioactive isotope will be remaining after *x* half-lives.

   c. How many grams of the isotope will remain after 11 half-lives?

   d. Plutonium-238 is one of the most dangerous waste products of nuclear power plants. If the half-life of plutonium-238 is 87.7 years, how long would it take for a 20-gram sample of plutonium-238 to decay to 0.078 gram?

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9-6 Enrichment

Sierpinski Triangle

Sierpinski Triangle is an example of a fractal that changes exponentially. Start with an equilateral triangle and find the midpoints of each side. Then connect the midpoints to form a smaller triangle. Remove this smaller triangle from the larger one.

Repeat the process to create the next triangle in the sequence. Find the midpoints of the sides of the three remaining triangles and connect them to form smaller triangles to be removed.

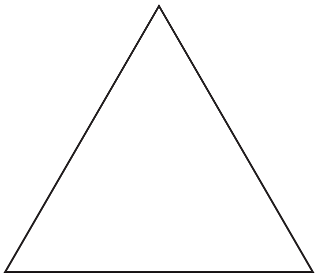


figure 1

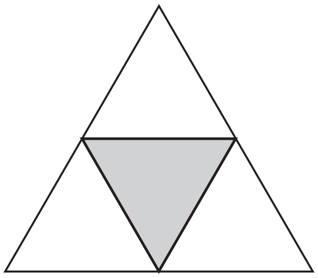


figure 2

Cut out: $\frac{1}{4}$

Area $= 1 - \frac{1}{4}$ or $\frac{3}{4}$

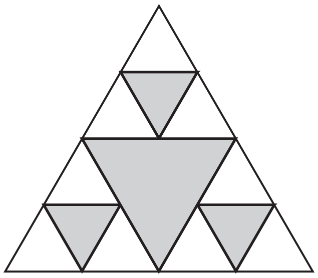


figure 3

Cut out: $\frac{1}{4} + \frac{3}{16}$ or $\frac{7}{16}$

Area $= 1 - \frac{7}{16}$ or $\frac{9}{16}$

1. Find the next triangle in the sequence. How much has been cut out? What is the area of the fourth figure in the sequence?

2. Make a conjecture as to what you need to multiply the previous amount cut by to find the new amount cut.

3. Fill in the chart to represent the amount cut and the area remaining for each triangle in the sequence.

Figure	1	2	3	4	5	6
Amount Cut	0	$\frac{1}{4}$	$\frac{7}{16}$			
Area Remaining	1	$\frac{3}{4}$	$\frac{9}{16}$			

4. Write an equation to represent the area that is left in the nth triangle in the sequence.

5. If this process is continued, make a conjecture as to the remaining area.

9-7 Study Guide and Intervention

Special Functions

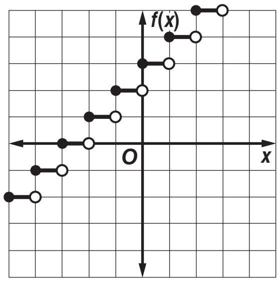
Step Functions The graph of a **step function** is a series of disjointed line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function.**

One example of a step function is the greatest integer function, written as $f(x) = \llbracket x \rrbracket$, where $f(x)$ is the greatest integer not greater than x.

Example **Graph $f(x) = \llbracket x + 3 \rrbracket$.**

Make a table of values using integer and noninteger values. On the graph, dots represent included points, and circles represent points that are excluded.

x	$x + 3$	$\llbracket x + 3 \rrbracket$
–5	–2	–2
–3.5	–0.5	–1
–2	1	1
–0.5	2.5	2
1	4	4
2.5	5.5	5



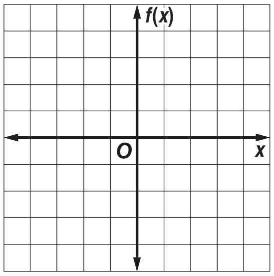
Because the dots and circles overlap, the domain is all real numbers. The range is all integers.

Lesson 9-7

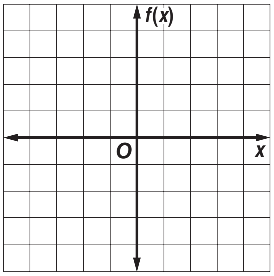
Exercises

Graph each function. State the domain and range.

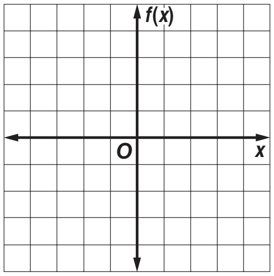
1. $f(x) = \llbracket x + 1 \rrbracket$



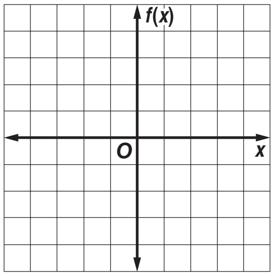
2. $f(x) = -\llbracket x \rrbracket$



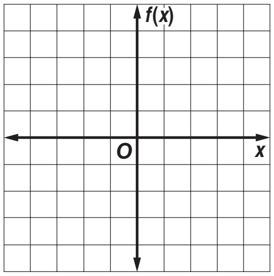
3. $f(x) = \llbracket x - 1 \rrbracket$



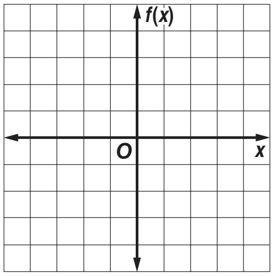
4. $f(x) = \llbracket x \rrbracket + 4$



5. $f(x) = \llbracket x \rrbracket - 3$



6. $f(x) = \llbracket 2x \rrbracket$



9-7 Study Guide and Intervention *(continued)*

Special Functions

Absolute Value Functions Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

The absolute value function is called a **piecewise-defined function** because it can be written using two or more expressions.

Example 1 **Graph $f(x) = |x + 2|$. State the domain and range.**

$f(x)$ cannot be negative, so the minimum point is $f(x) = 0$.

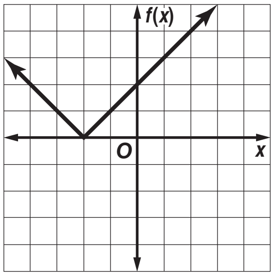
$f(x) = |x + 2|$ Original function

$0 = x + 2$ Replace $f(x)$ with 0.

$-2 = x$ Subtract 2 from each side.

Make a table. Include values for $x > -2$ and $x < -2$.

x	$f(x)$
–5	3
–4	2
–3	1
–2	0
–1	1
0	2
1	3
2	4

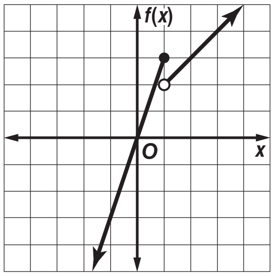


The domain is all real numbers. The range is all real numbers greater than or equal to 0.

Example 2 **Graph $f(x) = \begin{cases} x + 1 \text{ if } x > 1 \\ 3x \text{ if } x \leq 1 \end{cases}$. State the domain and range.**

Graph the first expression. When $x > 1$, $f(x) = x + 1$. Since $x \neq 1$, place an open circle at (1, 2).

Next, graph the second expression. When $x \leq 1$, $f(x) = 3x$. Since $x = 1$, place a closed circle at (1, 3).

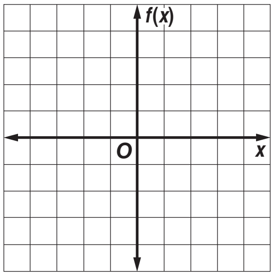


The domain and range are both all real numbers.

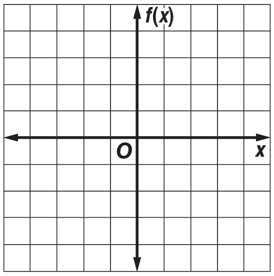
Exercises

Graph each function. State the domain and range.

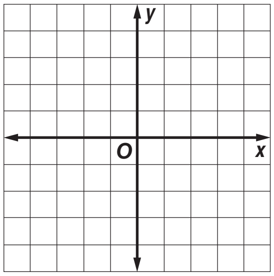
1. $f(x) = |x - 1|$



2. $f(x) = |-x + 2|$



3. $f(x) = \begin{cases} -x + 4 \text{ if } x \leq 1 \\ x - 2 \text{ if } x > 1 \end{cases}$



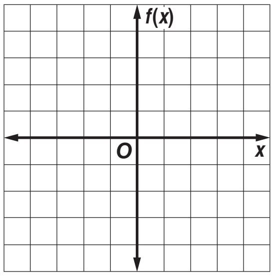
NAME ______________________ DATE __________ PERIOD __________

9-7 Skills Practice

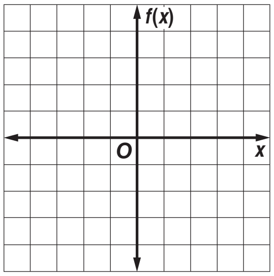
Special Functions

Graph each function. State the domain and range.

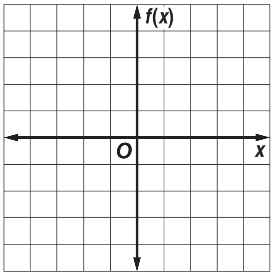
1. $f(x) = [\![x - 2]\!]$



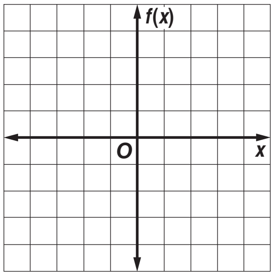
2. $f(x) = 3[\![x]\!]$



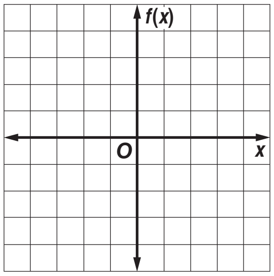
3. $f(x) = [\![2x]\!]$



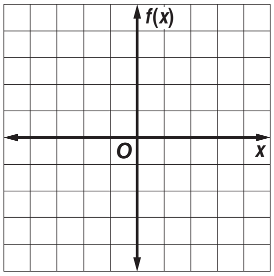
4. $f(x) = |x| - 3$



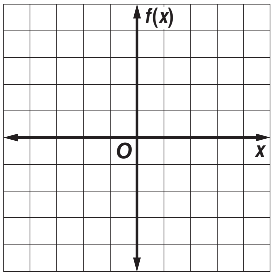
5. $f(x) = |2x|$



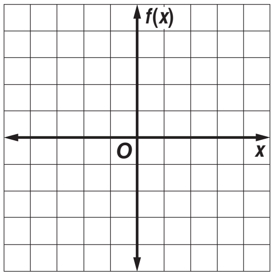
6. $f(x) = |2x + 5|$



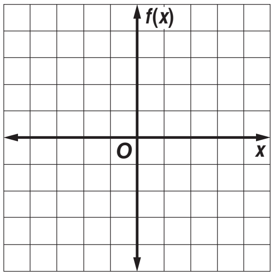
7. $f(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ -x + 3 & \text{if } x > 2 \end{cases}$



8. $f(x) = \begin{cases} x + 4 & \text{if } x \leq 1 \\ 0.25x + 1 & \text{if } x > 1 \end{cases}$



9. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ -0.5x + 1 & \text{if } x \geq 0 \end{cases}$

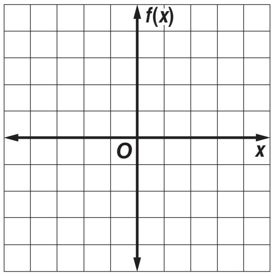


9-7 Practice

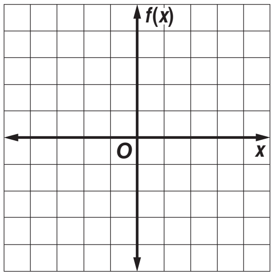
Special Functions

Graph each function. State the domain and range.

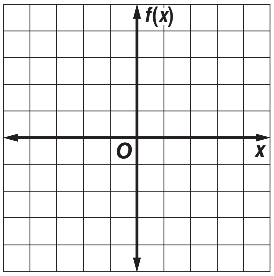
1. $f(x) = -2[\![x + 1]\!]$



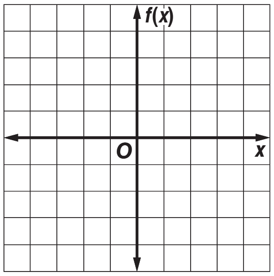
2. $f(x) = [\![x + 3]\!] - 2$



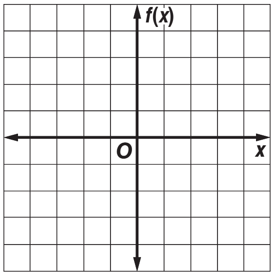
3. $f(x) = -\left|\frac{1}{2}x\right| + 1$



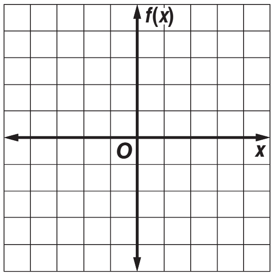
4. $f(x) = |2x + 4| - 3$



5. $f(x) = \begin{cases} 2 & \text{if } x > -1 \\ x + 4 & \text{if } x \leq -1 \end{cases}$

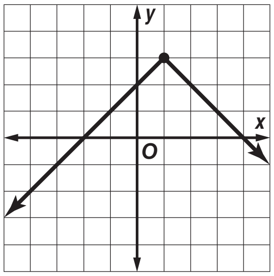


6. $f(x) = \begin{cases} -2x + 3 & \text{if } x > 0 \\ \frac{1}{2}x - 1 & \text{if } x \leq 0 \end{cases}$

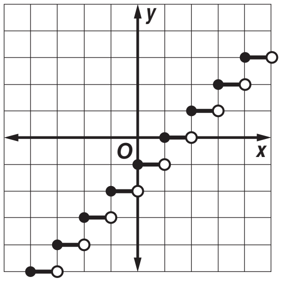


Determine the domain and range of each function.

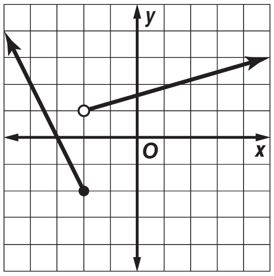
7.



8.



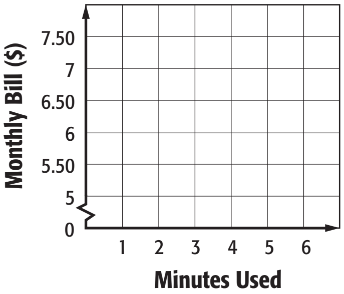
9.



10. CELL PHONES Jacob's cell phone service costs $5 each month plus $0.35 for each minute he uses. Every fraction of a minute is rounded up to the next minute.

a. Draw a graph to represent the cost of using the cell phone.

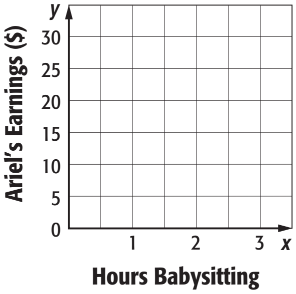
b. What is Jacob's monthly bill if he uses 124.8 minutes?



9-7 Word Problem Practice

Special Functions

1. BABYSITTING Ariel charges $4 per hour as a babysitter. She rounds every fraction of an hour up to the next half-hour. Draw a graph to represent Ariel's total earnings y after x hours.

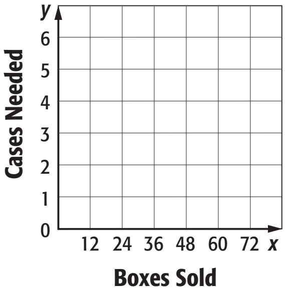


2. SHIPPING A package delivery service determines rates for express shipping by the weight of a package, with every fraction of a pound rounded up to the next pound. The table shows the cost of express shipping for packages between 1 and 9 pounds. Write a piecewise-linear function representing the cost to ship a package between 1 and 9 pounds. State the domain and range.

Weight (pounds)	Rate (dollars)
1	17.40
2	19.30
3	22.40
4	25.50
5	28.60
6	31.70
7	34.80
8	37.90
9	41.00

3. DISEASE PREVENTION Body Mass Index (BMI) is used by doctors to determine weight categories that may lead to health problems. According to the Centers for Disease Control and Prevention, 21.7 is a healthy BMI for adults. If the BMI differs from the desired 21.7 by more than x, there may be health risk involved. Write an equation that can be used to find the highest and lowest BMI for a healthy adult. Then solve if $x = 3.2$.

4. FUNDRAISING Students are selling boxes of cookies at a fund-raiser. The boxes of cookies can only be ordered by the case, with 12 boxes per case. Draw a graph to represent the number of cases needed y when x boxes of cookies are sold.

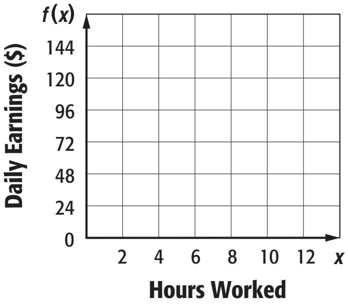


5. WAGES Kelly earns $8 per hour the first 8 hours she works in a day and $11.50 per hour each hour thereafter.

a. Organize the information into a table. Include a column for hours worked x, and a column for daily earnings $f(x)$.

b. Write the piecewise equation describing Kelly's daily earnings $f(x)$ for x hours.

c. Draw a graph to represent Kelly's daily earnings.



Lesson 9-7

NAME ______________________________ DATE ____________ PERIOD ____________

9-7 Enrichment

Parametric Equations

A **parametric equation** is a pair of functions $x = f(t)$ and $y = g(t)$ that describe both the x- and y-coordinates for the graph as a whole. Parametric functions allow the drawing of many complex curves and figures.

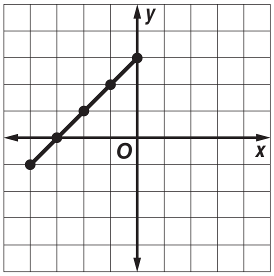
Example **Graph the parametric function given by $x = t - 2$ and $y = t + 1$ for $-2 \le t \le 2$.**

Step 1 Create a table of values for $-2 \le t \le 2$ by evaluating x and y for each value of t.

t	–2	–1	0	1	2
x	–4	–3	–2	–1	0
y	–1	0	1	2	3

Step 2 Plot the points.

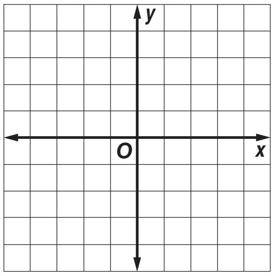
Step 3 Draw a line to connect the points between $-2 \le t \le 2$.



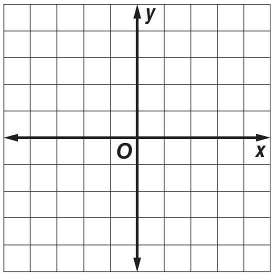
Exercises

Graph each pair of parametric equations over the given range of t values.

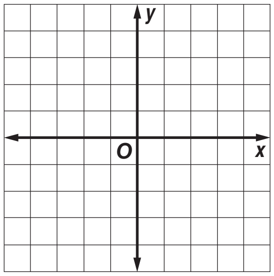
1. $x = t + 2$
$y = t + 2$
$-2 \le t \le 2$



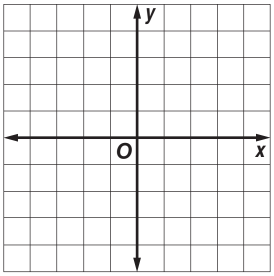
2. $x = 2t$
$y = \frac{1}{2}t - 1$
$-2 \le t \le 2$



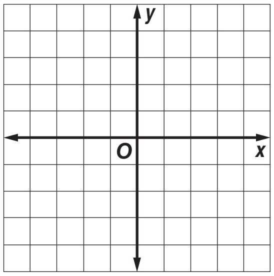
3. $x = t - 3$
$y = 2t - 1$
$-2 \le t \le 2$



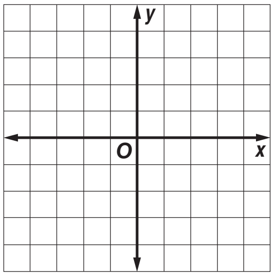
4. $x = 4t$
$y = 2t - 2$
$-1 \le t \le 1$



5. $x = t^2$
$y = t + 1$
$-2 \le t \le 2$



6. $x = t^2$
$y = t^2 - t - 1$
$-2 \le t \le 2$



NAME ______________________ DATE __________ PERIOD __________

9 Student Recording Sheet

Use this recording sheet with pages 614–615 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓕ Ⓖ Ⓗ Ⓙ

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

7. ______________ *(grid in)*

8a. ______________

8b. ______________

8c. ______________

8d. ______________

9a. ______________

9b. ______________

10a. ______________

10b. ______________

7.

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Extended Response

Record your answers for Question 11 on the back of this paper.

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9 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 11 Rubric

Score	Specific Criteria
4	A correct solution that is supported by well-developed, accurate explanations. In part **a**, the student correctly factors the polynomial as $(x - 2)(x - 5)$. In part **b**, x is found to be 2 and 5. Each step in solving the equations is shown. In part **c**, students correctly note that the graph of the equation crosses the x-axis at $x = 2$ and $x = 5$, the solutions to the equation (where $y = 0$).
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME ______________________ DATE ____________ PERIOD ____________

SCORE ________

9 Chapter 9 Quiz 1

(Lessons 9-1 through 9-3)

1. State the domain and range of $y = 2x^2 - 8x + 4$.

1. ______________________

2. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = -x^2 - 4x + 5$. Identify the vertex as a maximum or a minimum.

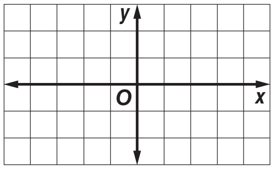
2. ______________________

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

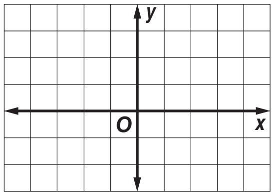
3. $x^2 + x - 2 = 0$

4. $3x^2 - 5x + 1 = 0$

3. ______________________



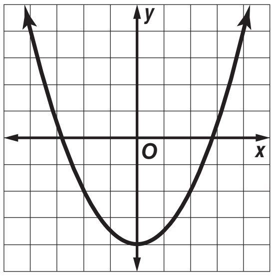
4. ______________________



5. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph?

A $y = -\frac{1}{2}x^2 - 4$ **C** $y = \frac{1}{2}x^2 - 4$

B $y = 2x^2 - 4$ **D** $y = 2x^2 - 2$



5. ______________________

NAME ______________________ DATE ____________ PERIOD ____________

SCORE ________

9 Chapter 9 Quiz 2

(Lessons 9-4 and 9-5)

1. Find the value of c that makes $p^2 + 9p + c$ a perfect square.

1. ______________________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

2. $w^2 + 8w - 10 = 10$

2. ______________________

3. $3x^2 - 2x = 27$

3. ______________________

4. **MULTIPLE CHOICE** Solve $x^2 + x - 7 = 0$ by using the Quadratic Formula.

A 6, 7 **C** $\frac{1 \pm \sqrt{29}}{2}$

B $\frac{-1 \pm \sqrt{-27}}{2}$ **D** $\frac{-1 \pm \sqrt{29}}{2}$

4. ______________________

5. State the value of the discriminant for $2x^2 - 3x + 8 = 0$. Then determine the number of real solutions of the equation.

5. ______________________

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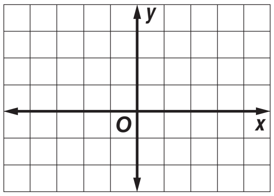
9 Chapter 9 Quiz 3

SCORE ________

(Lesson 9-6)

1. Graph the set of ordered pairs: {(−3, 3), (−2, 0), (−1, −1), (0, 0), (1, 3)}. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1.



____

2. Determine which model best describes the data. Then write an equation for the function that models the data.

x	−1	0	1	2	3
y	−27	9	−3	1	$-\frac{1}{3}$

2. ______________________

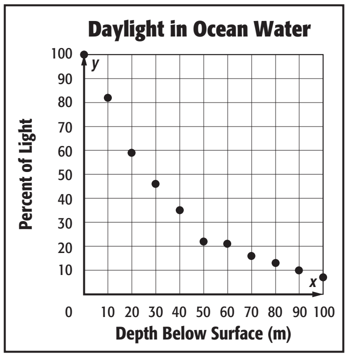
3. MULTIPLE CHOICE Which equation best models the data graphed?

A $y = 0.8x$

B $y = 100 - 2x$

C $y = 100(0.97)^x$

D $y = 0.16x^2 - 2.03x + 98.5$



3. ______________________

NAME ______________________________ DATE ____________ PERIOD ____________

9 Chapter 9 Quiz 4

SCORE ________

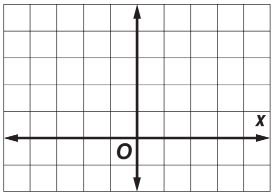
(Lesson 9-7)

1. If $f(x) = 2[\![x - 1]\!]$, find $f(1.5)$.

1. ______________________

2. Graph $f(x) = |2x + 1|$.

2.



3. Determine the domain and range of $f(x) = |2x + 1|$.

3. ______________________

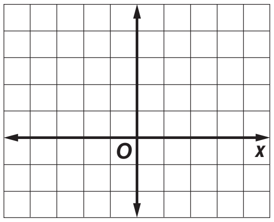
4. MULTIPLE CHOICE Mr. Aronsohn wants to rent a car on vacation. The rate the car rental company charges is $19 per day or any fraction thereof. How much would it cost for Mr. Aronsohn to rent a car for 6.4 days?

A $114.00 **C** $124.00

B $121.60 **D** $133.00

4. ______________________

5. Graph $g(x) = \begin{cases} 2x - 3 & \text{if } x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}$



5. ______________________

NAME ____________________ DATE ________ PERIOD ________

Chapter 9 Mid-Chapter Test

SCORE ______

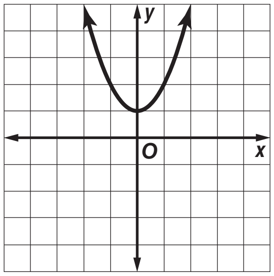
(Lessons 9-1 through 9-5)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Which equation corresponds to the graph shown?

A $y = x^2 - 1$ **C** $y = x^2 + 1$

B $y = -(x - 1)^2$ **D** $y = -(x + 1)^2$ **1.** ________



2. Find the coordinates of the vertex of the graph of $y = x^2 - 8x + 10$. Identify the vertex as a maximum or a minimum.

F (4, −6); minimum **H** (4, 6); maximum

G (−4, 58); maximum **J** (−4, 26); minimum **2.** ________

3. Solve $x^2 - 24x + 144 = 36$ by taking the square root of each side.

A −6, 18 **B** 6, 18 **C** 6, 12 **D** −6, 6 **3.** ________

4. Which equation can be used to solve $5b^2 + 30b - 10 = 0$ by completing the square? **4.** ________

F $(b + 6)^2 = 38$ **G** $(b + 6)^2 = 46$ **H** $(b + 3)^2 = 11$ **J** $(b + 3)^2 = 19$

5. Which step is *not* performed in the process of solving $r^2 + 8r + 5 = 0$ by completing the square? **5.** ________

A Subtract 5 from each side. **C** Add 16 to each side.

B Factor $r^2 + 8r$. **D** Take the square root of each side.

Part II

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

6. $x^2 - 7x - 8 = 0$ **6.** ________________

7. $x^2 + 1 = 5x$ **7.** ________________

For Questions 8 and 9, round to the nearest tenth if necessary.

8. Solve $x^2 + 4x = 20$ by completing the square. **8.** ________________

9. Solve $-2x^2 + 18 = 7x$ by using the Quadratic Formula. **9.** ________________

10. The base of a rectangle is 4 more than the height. The area of the rectangle is 15 square inches. What are the dimensions of the rectangle? **10.** ________________

NAME ______________________ DATE ____________ PERIOD ____________

9 Chapter 9 Vocabulary Test

SCORE ________

- absolute value function
- axis of symmetry
- completing the square
- discriminant
- double root
- greatest integer function
- maximum
- minimum
- nonlinear function
- parabola
- piecewise-defined function
- piecewise-linear function
- Quadratic Formula
- quadratic function
- step function
- transformation
- translation
- vertex

Choose a term from the vocabulary list above to complete each sentence.

1. $\underline{\quad ? \quad}$ is a change in the position of a figure either up, down, or diagonal. **1.** ____________________

2. Symmetry is a geometric property of a(n) $\underline{\quad ? \quad}$. **2.** ____________________

3. The vertical line containing the fold line when a figure is folded in half is called the $\underline{\quad ? \quad}$. **3.** ____________________

4. The function $f(x) = [\![x + 1]\!]$ is a special type of step function called a(n) $\underline{\quad ? \quad}$. **4.** ____________________

5. The $\underline{\quad ? \quad}$ is the result of solving the standard form of the quadratic equation for x. **5.** ____________________

6. The $\underline{\quad ? \quad}$ of a parabola is a minimum or maximum point. **6.** ____________________

7. The graph of a(n) $\underline{\quad ? \quad}$ is a parabola. **7.** ____________________

8. When the vertex of a parabola lies on the x-axis, the related quadratic equation has a(n) $\underline{\quad ? \quad}$. **8.** ____________________

Define each term in your own words.

9. axis of symmetry **9.** ____________________

10. discriminant **10.** ____________________

NAME ____________________ DATE __________ PERIOD __________

9 Chapter 9 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Consider the equation $y = x^2 + 3x - 4$. Determine whether the function has a maximum or minimum value. State the maximum or minimum value. What are the domain and range of the function?

A min.; (0, 0)
D: {all real numbers}
R: {all real numbers}

B max.; (0, 0)
D: {all real numbers}
R: $\{y \mid y \leq 0\}$

C max.; (–1.5, –6.25)
D: $\{x \mid x \leq -1.5\}$
R: $\{y \mid y \geq -6.25\}$

D min.; (–1.5, –6.25)
D: {all real numbers}
R: $\{y \mid y \geq -6.25\}$

1. ____

2. What is the equation of the axis of symmetry of the graph of $y = x^2 + 6x - 7$?

F $x = 6$ **G** $x = -3$ **H** $x = 3$ **J** $x = -6$

2. ____

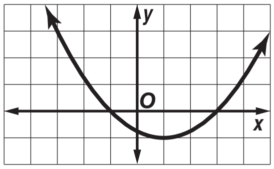
3. Find the coordinates of the vertex of the graph of $y = 4 - x^2$. Identify the vertex as a maximum or a minimum.

A (2, 0); maximum
B (0, 4); minimum
C (0, 4); maximum
D (2, 0); minimum

3. ____

4. What are the roots of the quadratic equation whose related function is graphed at the right?

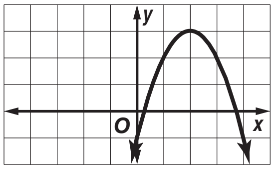
F –1, 3
G –1, 1
H –3, 1
J 1, 3



4. ____

5. One root of the quadratic equation whose related function is graphed lies between which two consecutive integers?

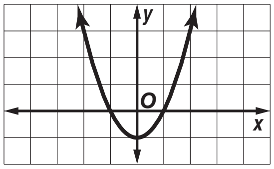
A 1 and 2
B 2 and 3
C 0 and –1
D 0 and 1



5. ____

6. Which equation corresponds to the graph shown?

F $y = x^2 + 1$
G $y = -x^2 - 1$
H $y = x^2 - 1$
J $y = x^2$



6. ____

7. Describe how the graph of the function $g(x) = -3x^2 - 2$ is related to the graph of the function $f(x) = -3x^2$.

A translation of $f(x) = -3x^2$ reflected over the x-axis and down 2 units
B translation of $f(x) = -3x^2$ down 2 units
C translation of $f(x) = -3x^2$ reflected over the x-axis and up 2 units
D translation of $f(x) = -3x^2$ up 2 units

7. ____

8. Find the value of c that makes $x^2 - 5x + c$ a perfect square trinomial.

F –12.25 **G** –6.25 **H** 6.25 **J** 10

8. ____

NAME ____________________ DATE __________ PERIOD __________

9 Chapter 9 Test, Form 1 *(continued)*

9. Which value of c makes $y^2 + 8y + c$ a perfect square trinomial?

A 4 **B** 16 **C** 64 **D** 8 **9.** ________

10. Which equation is equivalent to $x^2 + 2x - 3 = 0$?

F $(x + 1)^2 = 2$ **G** $(x - 1)^2 = 4$ **H** $(x - 1)^2 = 2$ **J** $(x + 1)^2 = 4$ **10.** ________

11. Solve the equation $2x^2 + 3x - 5 = 0$ by using the Quadratic Formula.

A $-2\frac{1}{2}, 1$ **B** $-5, 1$ **C** $-1, 2\frac{1}{2}$ **D** $-1, 5$ **11.** ________

12. State the value of the discriminant for $y = x^2 - 8x + 10$.

F 4.9 **G** 24 **H** 104 **J** 10.2 **12.** ________

13. Determine the number of real solutions of $n^2 - 5n - 6 = 0$.

A 1 real solution **C** infinitely many real solutions

B 2 real solutions **D** no real solutions **13.** ________

14. TREE HOUSE Bob tosses his basketball onto the ground from his tree house. He tosses the basketball with an initial downward velocity of 8 feet per second. The equation $h = -16t^2 - 8t + 20$ represents the height h of the basketball after t seconds. How long does the basketball take to hit the ground?

F 0.9 s **H** 1.0 s **G** 9 s **J** 20 s **14.** ________

15. State the value of the discriminant of $5x^2 + 9x = 3$

A 5 **B** 12 **C** 21 **D** 141 **15.** ________

16. Look for a pattern in the table of values to determine which model best describes the data.

x	0	1	2	3
y	0	2	8	18

F linear **G** quadratic **H** exponential **J** none of these **16.** ________

17. Which function best models the data in Question 16?

A $y = 2x$ **B** $2x + 1$ **C** $y = 2x^2$ **D** $y = 2^x$ **17.** ________

18. What is the domain of $f(x) = \begin{cases} x + 3 & \text{if } x < 0 \\ -2x + 1 & \text{if } x \geq 0 \end{cases}$?

F {all real num.} **G** $\{x \mid x \geq 3\}$ **H** $\{x \mid x < 2\}$ **J** $\left\{x \mid x \leq \frac{1}{2}\right\}$ **18.** ________

19. If $f(x) = 2[\![x]\!]$, find $f\left(-\frac{1}{4}\right)$.

A -2 **B** $-\frac{1}{2}$ **C** 0 **D** $\frac{1}{2}$ **19.** ________

20. What is the range of $y = |3x + 1|$?

F {all real num.} **G** $\{y \mid y \geq 0\}$ **H** $\{y \mid y \geq 1\}$ **J** $\left\{y \mid y \geq \frac{1}{3}\right\}$ **20.** ________

Bonus If $b^2 - 4ac = 0$, determine the number of real solutions of the equation $ax^2 + bx + c = 0$. **B:** ____________________

NAME ______________________ DATE __________ PERIOD __________

9 Chapter 9 Test, Form 2A

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. Consider the equation $y = x^2 + 5x - 6$. Determine whether the function has a maximum or minimum value. State the maximum of minimum value. What are the domain and range of the function?

A min.; (0, 0)
D: {all real numbers}
R: {all real numbers}

B max.; (0, 0)
D: {all real numbers}
R: $\{y \mid y \leq 0\}$

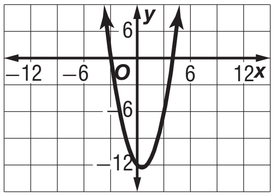
C min.; (−2.5, −12.25)
D: {all real numbers}
R: $\{y \mid y \geq -12.25\}$

D max.; (2.5, −12.25)
D: $x \mid x \leq 2.5$
R: {all real numbers}

1. ________

2. Which equation corresponds to the graph shown?

F $y = x^2 + 7x - 12$ **H** $y = x^2 + 5x + 12$
G $y = x^2 - x - 12$ **J** $y = x^2 + 12x - 1$



2. ________

3. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = 2x^2 - 12x + 6$.

A $x = -3$; (−3, 60) **C** $x = -3$; (−3, 78)
B $x = 3$; (3, −12) **D** $x = 3$; (3, 6)

3. ________

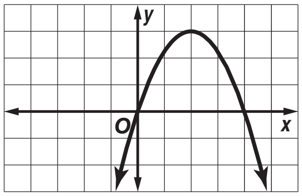
4. Find the coordinates of the vertex of the graph of $y = -2x^2 - 8$. Identify the vertex as a maximum or a minimum.

F (−2, −16); minimum **H** (2, −16); maximum
G (−2, 8); maximum **J** (0, −8); maximum

4. ________

5. What are the root(s) of the quadratic equation whose related function is graphed at the right?

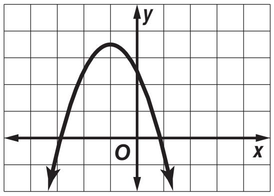
A 2 **C** 0, 4
B 3 **D** −4, 0



5. ________

6. One root of the quadratic equation whose related function is graphed lies between which two consecutive integers?

F −3 and −2 **H** −2 and −1
G 2 and 3 **J** 1 and 2



6. ________

7. How is the graph of $g(x) = x^2 - 3$ related to the graph of $f(x) = x^2$?

A translated down 3 units **C** translated right 3 units
B translated up 3 units **D** translated left 3 units

7. ________

8. Find the value of c that makes $x^2 + 10x + c$ a perfect square trinomial.

F −25 **G** −5 **H** 10 **J** 25

8. ________

NAME ______________________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 2A *(continued)*

9. What value of c makes $3x^2 + 24x + c$ contain a perfect square trinomial?
A 144 **B** 16 **C** 64 **D** 48 **9.** ________

10. Which step is *not* performed in the process of solving $n^2 - 12n - 10 = 0$ by completing the square?
F Add 10 to each side. **H** Factor $n^2 - 12n - 10 = 0$.
G Add 36 to each side. **J** Take the square root of each side. **10.** ________

11. Which equation is equivalent to $2x^2 - 24x - 14 = 0$?
A $(x - 6)^2 = 50$ **B** $(x - 3)^2 = 13$ **C** $(x - 3)^2 = 20$ **D** $(x - 6)^2 = 43$ **11.** ________

12. State the value of the discriminant of $3x^2 + 8x = 2$.
F 3 **G** 40 **H** 88 **J** 100 **12.** ________

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

13. $4x^2 + 11x - 3 = 0$
A $-2.4, -0.3$ **B** $-\frac{1}{4}, 3$ **C** $0.3, 2.4$ **D** $-3, \frac{1}{4}$ **13.** ________

14. $y^2 + 8y = 2$
F $-8.2, 0.2$ **G** $8.2, -0.2$ **H** $0.3, 7.7$ **J** $-7.7, -0.3$ **14.** ________

15. Determine the number of real solutions of $7x^2 - 18x + 12 = 0$.
A 2 **B** infinitely many **C** none **D** 1 **15.** ________

16. Look for a pattern in the table of values to determine which model best describes the data. **16.** ________

x	0	1	2	3
y	1	7	49	343

F linear **H** exponential **G** quadratic **J** none of these

17. Which function best models the data in Question 16?
A $y = 7x$ **B** $y = 7x^2$ **C** $y = 7^x$ **D** $y = 7^x + 1$ **17.** ________

18. If $f(x) = [\![x + 2]\!]$, find $f(1.5)$.
F 0.5 **G** 3 **H** 3.5 **J** 4 **18.** ________

19. Which is *not* true about the graph of $f(x) = |3x + 2|$?
A The range includes all real numbers.
B It includes the point $(-3, 7)$.
C The domain includes all real numbers.
D The graph is "V-shaped." **19.** ________

20. Which point is located on the graph of $f(x) = \begin{cases} \frac{1}{3}x + 2 & \text{if } x \le 1 \\ \frac{1}{2}x + 1 & \text{if } x > 1 \end{cases}$?
F $(-3, 1)$ **G** $(0, 1)$ **H** $(2, 0)$ **J** $(3, 3)$ **20.** ________

Bonus What is the equation of the axis of symmetry of a parabola if its x-intercepts are -3 and 5? **B:** ____________

NAME ______________________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 2B

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. Consider the equation $y = -x^2 - 7x + 12$. Determine whether the function has a maximum or a minimum value. State the maximum or minimum value. What are the domain and range of the function?

A min.; (–7, 0)
D: $\{x \mid x \leq 12\}$
R: {all real numbers}

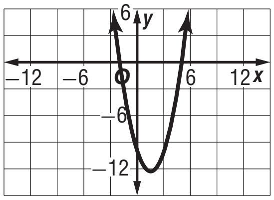
B max.; (–3.5, 24.25)
D: {all real numbers}
R: $\{y \mid y \leq 24.25\}$

C min.; (3.5, –24.25)
D: {all real numbers}
R: $\{y \mid y \geq -24.25\}$

D max.; (12, 0)
D: $\{x \mid x \leq 12\}$
R: {all real numbers}

1. ______

2. Which equation corresponds to the graph shown?



F $y = x^2 - 3x - 10$
G $y = x^2 + 7x + 10$
H $y = x^2 - 10x + 6$
J $y = x^2 - 11x - 10$

2. ______

3. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = -x^2 - 10x + 17$.

A $x = -5$; (–5, –8)
B $x = -5$; (–5, 42)
C $x = 5$; (5, 92)
D $x = 5$; (5, 32)

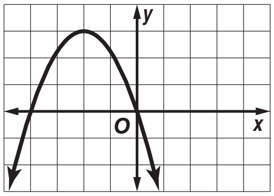
3. ______

4. Find the coordinates of the vertex of the graph of $y = 3x^2 - 6$. Identify the vertex as a maximum or a minimum.

F (0, –6); maximum
G (–6, 0); minimum
H (0, –6); minimum
J (6, 0); minimum

4. ______

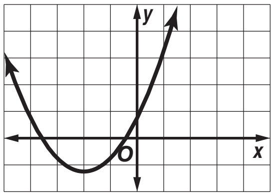
5. What are the root(s) of the quadratic equation whose related function is graphed at the right?



A 4, 0
B –2, 3
C 2, 3
D –4, 0

5. ______

6. One root of the quadratic equation whose related function is graphed lies between which consecutive integers?



F –4 and –3
G 0 and 1
H –5 and –4
J –3 and –2

6. ______

7. Describe how the graph of the function $g(x) = -5x^2 - 2$ is related to the graph of the function $f(x) = 5x^2 + 1$.

A translation of $f(x) = 5x^2 + 1$ reflected over the x-axis and up 3 units
B translation of $f(x) = 5x^2 + 1$ down 2 units
C translation of $f(x) = 5x^2 + 1$ reflected over the x-axis and down 3 units
D translation of $f(x) = 5x^2 + 1$ up 2 units

7. ______

NAME ______________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 2B *(continued)*

8. How is the graph of $f(x) = x^2$ translated to create the graph of $g(x) = x^2 + 4$?
F down 4 units **H** right 4 units **G** up 4 units **J** left 4 units **8.** ______

9. What value of c makes $4x^2 + 24x + c$ a perfect square trinomial?
A 1 **B** 36 **C** 144 **D** 9 **9.** ______

10. Which is *not* performed when solving $r^2 + 12r - 6 = 0$ by completing the square?
F Add 6 to each side. **H** Add 36 to each side.
G Factor $r^2 + 12r - 6$. **J** Take the square root of each side. **10.** ______

11. Which equation is equivalent to $3x^2 + 24x + 15 = 0$?
A $(x + 4)^2 = 11$ **B** $(x + 4)^2 = 1$ **C** $(x + 2)^2 = -1$ **D** $(x + 2)^2 = -11$ **11.** ______

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

12. $3x^2 - 11x - 4 = 0$
F $-4, \frac{1}{3}$ **G** 0.4, 3.3 **H** $-\frac{1}{3}, 4$ **J** −3.3, −0.4 **12.** ______

13. $d^2 + 4 = 6d$
A −5.2, −0.8 **B** −6.6, 0.6 **C** −0.6, 6.6 **D** 0.8, 5.2 **13.** ______

14. Determine the number of real solutions of $6x^2 + 19x + 14 = 0$.
F 2 **G** infinitely many **H** 1 **J** none **14.** ______

15. Determine the number of real solutions of $2x^2 + 2x + 3 = 0$.
A 2 **B** infinitely many **C** 1 **D** none **15.** ______

16. Look for the pattern in the table of values to determine which model best describes the data.

x	0	1	2	3
y	1	5	25	125

F exponential **G** quadratic **H** linear **J** none of these **16.** ______

17. Which function best models the data in Question 16?
A $y = 5^x + 1$ **B** $y = 5^x$ **C** $y = 5x^2$ **D** $y = 5x$ **17.** ______

18. If $f(x) = [\![x - 2]\!]$, find $f(4.5)$.
F 2 **G** 2.5 **H** 3 **J** 6.5 **18.** ______

19. Which is not true about the graph of $f(x) = |2x - 1|$?
A Domain: all real numbers. **C** It includes the point (−2, 5).
B Range: all real numbers. **D** The graph is "V-shaped." **19.** ______

20. Which point is *not* located on the graph of $f(x) = \begin{cases} 2x + 3 \text{ if } x \le -1 \\ 4 + x \text{ if } x > -1 \end{cases}$?
F (−1.5, 0) **G** (−1, 3) **H** (0, 4) **J** (4, 8) **20.** ______

Bonus Find all values of c that make $x^2 + cx + 64$ a perfect square trinomial. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 2C

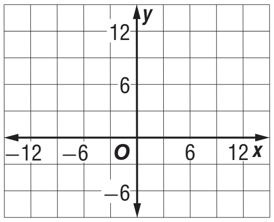
SCORE ________

Assessment

Use a table of values to graph each function.

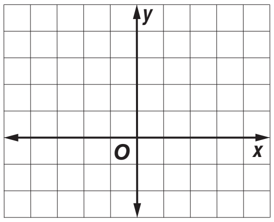
1. $y = -x^2 + 3x + 10$

1.



2. $y = 2x^2 - 3x$

2.

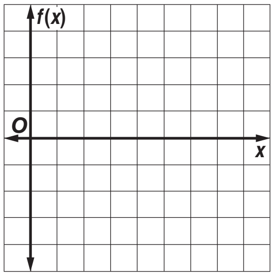


3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = 2x^2 - 8x + 3$. Identify the vertex as a maximum or a minimum.

3. ____________________

4. Solve $x^2 - 6x + 4 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4.



____

5. **NUMBER THEORY** Two numbers have a sum of 1 and a product of -6. Use the quadratic equation $-n^2 + n + 6 = 0$ to determine the two numbers.

5. ____________________

Solve each equation by factoring.

6. $2x^2 + x = 10$

7. $r^2 - 11r + 28 = 0$

6. ____________________

7. ____________________

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = -2 + x^2$

9. $h(x) = -x^2 + 6$

8. ____________________

9. ____________________

NAME ______________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 2C *(continued)*

SCORE ________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

10. $2x^2 + 3x = 20$

11. $p^2 - 12p + 9 = 0$

10. ______________

11. ______________

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

12. $15n^2 - 3 = 4n$

13. $r^2 + 16r + 21 = 0$

12. ______________

13. ______________

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

14. $7m^2 + 8m = 3$

15. $4p^2 = 4p - 1$

14. ______________

15. ______________

16. The length of a rectangle is 5 inches more than the width. The area is 33 square inches. Find the length and width. Round to nearest tenth if necessary.

16. ______________

Determine an equation that models the data.

17.

x	1	2	3	4
y	4	16	36	64

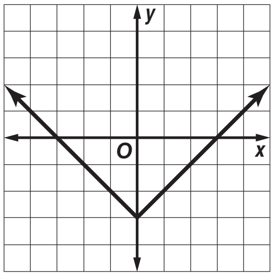
17. ______________

18.

x	0	1	2	3	4
y	2	6	18	54	162

18. ______________

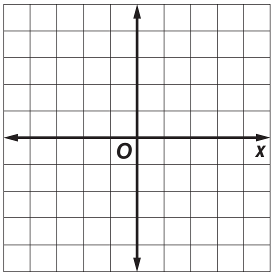
19. Determine the domain and range for the graph shown.



19. ______________

20. Graph $g(x) = \begin{cases} x + 2 \text{ if } x > 1 \\ 2x - 1 \text{ if } x \leq 1 \end{cases}$.

20.



Bonus Without graphing, determine the x-intercepts of the graph of $f(x) = 7x^2 + 9x + 1$.

B: ______________

NAME ______________________ DATE ____________ PERIOD ____________

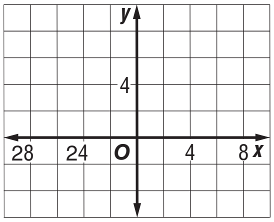
9 Chapter 9 Test, Form 2D

SCORE ________

Use a table of values to graph each function.

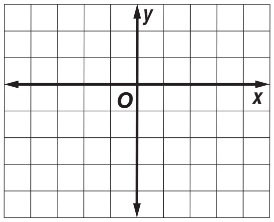
1. $y = x^2 - 7x + 12$

1.



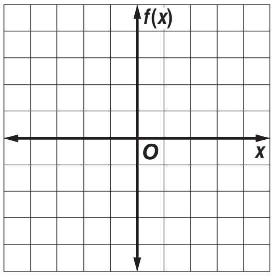
2. $y = 4x^2 - 8x$

2.



3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = -2x^2 + 4x - 5$. Identify the vertex as a maximum or a minimum.

3. ______________________



4. Solve $x^2 - 2x - 1 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4. ______________________

5. **NUMBER THEORY** Two numbers have a sum of 4 and a product of −5. Use the quadratic equation $-n^2 + 4n + 5 = 0$ to determine the two numbers.

5. ______________________

Solve each equation by factoring.

6. $x^2 + 6x = 27$

7. $k^2 - 13k + 36 = 0$

6. ______________________

7. ______________________

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = 5 + x^2$

9. $g(x) = -x^2 - 3$

8. ______________________

9. ______________________

9 Chapter 9 Test, Form 2D *(continued)*

Solve each equation by completing the square. Round to the nearest tenth if necessary.

10. $2x^2 + 9x = 18$

11. $t^2 + 15t + 11 = 0$

10. ____________

11. ____________

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

12. $12v^2 - 6 = -v$

13. $d^2 - 14d - 22 = 0$

12. ____________

13. ____________

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

14. $3b^2 + 10 = -8b$

15. $9a^2 = 6a - 1$

14. ____________

15. ____________

16. The width of a rectangle is 3 inches less than the length. The area is 50 square inches. Find the length and width. Round to nearest tenth if necessary.

16. ____________

Determine which model best describes the data. Then write an equation that models the data.

17.

x	1	2	3	4
y	2	5	8	11

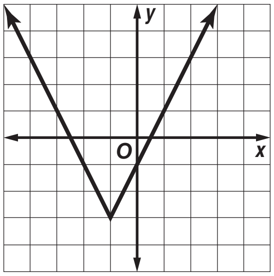
17. ____________

18.

x	1	2	3	4
y	14	28	56	112

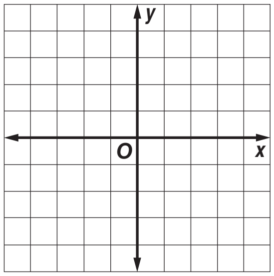
18. ____________

19. Determine the domain and range for the graph shown.



19. ____________

20. Graph $g(x) = \begin{cases} 2x + 1 \text{ if } x \leq 2 \\ x - 2 \text{ if } x > 2 \end{cases}$.



20.

Bonus Find the value of a so that the equation $ax^2 + 8x + 32 = 0$ has 1 real root.

B: ____________

NAME ______________________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 3

SCORE ________

Use a table of values to graph each function.

1. $y = -2x^2 - 3x + 3$

2. $y = 3x^2 - 5x - 2$

3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y - 1 = 5x^2 - 25x + 3$. Identify the vertex as a maximum or a minimum.

4. Two points on a parabola are $(-3, 5)$ and $(11, 5)$. What is the equation of the axis of symmetry?

5. **NUMBER THEORY** Two numbers have a sum of -6 and a product of 9. Use the quadratic equation $n^2 - 6n + 9 = 0$ to determine the two numbers.

Solve each equation by factoring.

6. $12x^2 - 5x = 2$

7. $18y^2 + 39y - 15 = 0$

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = 4 - 2x^2$

9. $g(x) = -\frac{1}{6}x^2 + \frac{5}{6}$

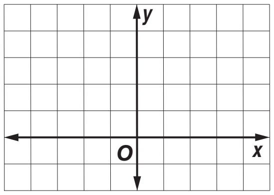
Solve each equation by graphing. If integral solutions cannot be found, estimate the solutions by stating the consecutive integers between which the solutions lie.

10. $\frac{1}{2}x^2 + x - 4 = 0$

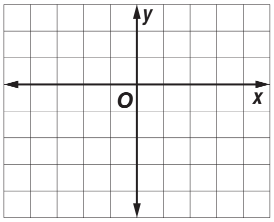
11. $x^2 + 7 = 6x$

12. $4x^2 + 6x + 5 = 0$

1.



2.



3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

NAME ______________________ DATE ____________ PERIOD ____________

9 Chapter 9 Test, Form 3 *(continued)*

13. Solve $21u^2 = u + 10$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

13. ____________

14. Solve $-2x^2 = -(5x + 4)$ by using the Quadratic Formula. Round to nearest tenth if necessary.

14. ____________

15. **PHYSICAL SCIENCE** A projectile is shot vertically from ground level. Its height h, in feet, after t seconds is given by $h = 88t - 16t^2$. Will the projectile have a height of 125 feet 0, 1, or 2 times after being shot?

15. ____________

16. Find the values for c so that $t^2 + ct + 81 = 0$ has one real root.

16. ____________

Determine which model best describes the data. Then write an equation that models the data.

17.

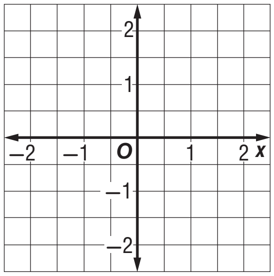
x	0	1	2	3	4
y	0	–4	–16	–36	–64

17. ____________

Graph each function.

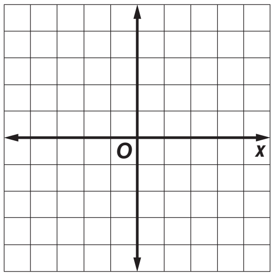
18. $f(x) = [\![2x + 1]\!]$

18.



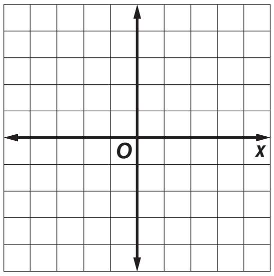
19. $g(x) = |-x - 1| + 1$

19.



20. $h(x) = \begin{cases} 2x + 1 \text{ if } x \leq 2 \\ x - 2 \text{ if } x > 2 \end{cases}$

20.



Bonus Solve the equation $ax^2 + 7x - 4 = 0$ for x by using the Quadratic Formula.

B: ____________

NAME ______________________________ DATE ____________ PERIOD ____________

9 Chapter 9 Extended-Response Test

SCORE ________

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Write a quadratic function whose graph opens upward. Write an exponential function. Graph both functions on the same coordinate plane. Compare and contrast the domains and ranges of the two functions and any symmetry of the two graphs.

2. a. Write a quadratic equation that has no real roots, and find its discriminant. Explain how the discriminant shows that a quadratic equation has no real roots.

b. Write a quadratic equation that has one real root, and find its discriminant. Explain why the Quadratic Formula yields only one solution when the discriminant of a quadratic equation is equal to zero.

c. Write a quadratic equation that has two real roots. Determine whether completing the square, graphing, or factoring would be the better method to use to solve your quadratic equation.

3. Curt's Appliance Service charges $125 for the first half hour of each home visit plus $40 for each additional half-hour block of work.

a. Write and graph a step-function to represent the total charges for every hour h of work.

b. How much is charged for 76 minutes of work?

c. Laurie was charged $365. How long did it take the work to get completed?

NAME ______________________ DATE ____________ PERIOD ____________

9 Standardized Test Practice

(Chapters 1–9)

SCORE ________

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Armando collects baseball cards. He currently has 48 cards, and buys 5 new cards a week. How many cards will Armando have in 10 weeks? (Lesson 3-5)

A 102 **B** 98 **C** 94 **D** 100

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Wesley carves whistles out of wood and sells them at a gift shop. The equation $C = 2n + 15$ models his weekly cost C of making n whistles. The equation $R = 5n$ models his weekly revenue R from selling n whistles. How many whistles must Wesley make and sell for his cost and revenue to be equal? (Lesson 6-2)

F 5 **G** 4 **H** 25 **J** 10

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Solve $2(5x - 4) \geq 7(x - 2)$. (Lesson 5-3)

A $x \geq -2$ **B** $x \leq 2$ **C** $x \geq \frac{2}{3}$ **D** $x \leq 2$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Find $3mt(2m^2 - 3mt + t^2)$. (Lesson 8-2)

F $5m^3t - 6m^2t + 3mt^3$

G $6m^2t - 3mt + 3mt^2$

H $6m^3t - 9m^2t^2 + 3mt^3$

J $2m^2 + t^2$

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Which binomial is a factor of $6ut - 9u + 8yt - 12y$? (Lesson 8-5)

A $2t + 3$ **B** $3u + 4$ **C** $-2t + 3$ **D** $3u + 4y$

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Factor $16 - n^4$. (Lesson 8-8)

F $(4 - n^2)(4 + n^2)$

G $(n + 2)(n - 2)(4 + n^2)$

H $(2 - n)(2 + n)(n^2 + 4)$

J $(2 - n)(2 + n)(2 - n)(2 + n)$

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Solve $2x^2 + 72 = 24x$. (Lesson 8-9)

A $\{2, 6\}$ **B** $\{-6, 2\}$ **C** $\{6\}$ **D** $\{-6\}$

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Find the coordinates of the vertex of the graph of $y = 3x^2 + 2x$. (Lesson 9-1)

F $\left(\frac{1}{3}, 1\right)$ **G** $\left(-\frac{1}{3}, -\frac{1}{3}\right)$ **H** $\left(\frac{1}{3}, -\frac{1}{3}\right)$ **J** $\left(-\frac{1}{3}, 1\right)$

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Which equation is equivalent to $2r^2 - 28r + 38 = 0$? (Lesson 9-4)

A $2(r - 7)^2 = 49$

B $(r - 14)^2 = 60$

C $(r - 14)^2 = 177$

D $(r - 7)^2 = 30$

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. What number should be inserted into the table for the data to display exponential behavior? (Lesson 9-6)

x	2	3	4	5
y	8	24	72	

F 152 **G** 186 **H** 248 **J** 216

10. Ⓕ Ⓖ Ⓗ Ⓙ

NAME ______________________________ DATE ____________ PERIOD ____________

9 Standardized Test Practice *(continued)*

11. A watch is on sale for 30% off the original price. If the original price of the watch is \$14, what is the discounted price? (Lesson 2-7)
A \$9.80 **B** \$4.20 **C** \$13.70 **D** \$9.33 **11.** Ⓐ Ⓑ Ⓒ Ⓓ

12. Solve the proportion $\frac{n}{500} = \frac{2}{40}$. (Lesson 2-6)
F 125 **G** 25 **H** 16 **J** 80 **12.** Ⓕ Ⓖ Ⓗ Ⓙ

13. Determine the x-intercept and y-intercept of $4x - 2y = 10$. (Lesson 3-1)
A 0, 0 **B** 2.5, 0 **C** 0, –5 **D** 2.5, –5 **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. Write a direct variation equation that relates x and y if $y = 10$ when $x = 12$. (Lesson 3-4)
F $y = \frac{6}{5}x$ **G** $y = \frac{5}{6}x$ **H** $y = 12x$ **J** $10y = 12x$ **14.** Ⓕ Ⓖ Ⓗ Ⓙ

15. Write the slope-intercept form of an equation for the line that passes through (4, 0) and is parallel to the graph of $3y - 6x = 4$. (Lesson 4-4)
A $2y = -x$ **C** $y = 2x + 4$
B $y = 2x - 8$ **D** $y = -2x + 8$ **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Which ordered pair is not a solution of $4x - 8y \geq 24$? (Lesson 5-6)
F (–5, –4) **G** (2, –2) **H** (7, –1) **J** (–8, –8) **16.** Ⓕ Ⓖ Ⓗ Ⓙ

17. Which binomial is a factor of $6x^2 + x - 12$? (Lesson 8-7)
A $(3x - 4)$ **B** $(2x + 6)$ **C** $(3x + 4)$ **D** $(2x - 3)$ **17.** Ⓐ Ⓑ Ⓒ Ⓓ

Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

18. If $3x + 2y = 9$ and $3x + 3y = 6$, what is the value of x? (Lesson 6-3)

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

19. State the value of the discriminant for $4x^2 + 17x + 18 = 0$. (Lesson 9-5)

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

NAME ______________________ DATE ____________ PERIOD ____________

9 Standardized Test Practice *(continued)*

(Chapters 1–9)

Part 3: Short Response

Instructions: Write your answers in the space provided.

20. Write an equation of the line that passes through (2, 4) and (1, −2) in slope-intercept form. (Lesson 4-2)

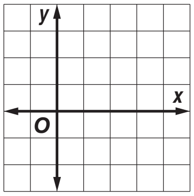
20. ____________

21. Solve $4y - 3(7y - 2) \leq -14 - 13y$. (Lesson 5-3)

21. ____________

22. Solve the system of inequalities by graphing. (Lesson 5-6)
$x + y \leq 4$
$y \leq 2x - 4$

22.



23. Simplify $(4xy + 3x^2y - 5y^2) - (3y^2 - 5xy + 7x^2y)$. (Lesson 8-1)

23. ____________

24. Find $(3a^2 + 2)(3a^2 - 2)$. (Lesson 8-4)

24. ____________

25. Factor $x^2 + 12x + 35$. (Lesson 8-6)

25. ____________

26. Factor $2m^2 + 11m + 15$. (Lesson 8-7)

26. ____________

27. Solve $36 - \frac{1}{4}y^2 = 0$ by factoring. (Lesson 8-8)

27. ____________

28. Use the formula $h = -16t^2 + 250t$ to model the height h in feet of a model rocket t seconds after it is launched. Determine when the rocket will reach a height of 900 feet. (Lesson 9-5)

28. ____________

29. The population of North Carolina has been increasing at an annual rate of 1.7%. If the population of North Carolina was 7,650,789 in 1999, predict its population in 2015. (Lesson 7-6)

29. ____________

30. Consider $y = -x^2 - 2x + 2$. (Lesson 9-1)

a. Find the equation for the axis of symmetry.

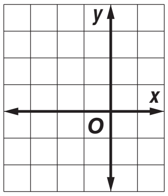
30a. ____________

b. Find the coordinates of the vertex and determine if it is a maximum or minimum.

30b. ____________

c. Graph the equation.

30c.



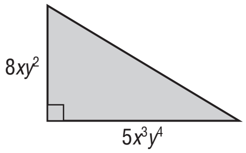
NAME ______________________ DATE ____________ PERIOD ____________

9 Unit 3 Test

(Chapters 7–9)

SCORE ________

1. Express the area of the triangle as a monomial.



1. ____________

2. Simplify $\frac{(3b^{-4}g^3)^{-2}}{(4b)^2}$

2. ____________

3. Solve $12^{3x-1} = 144$.

3. ____________

4. Solve $(3.4 \times 10^5)(7.5 \times 10^{-9})$. Write your answer in both standard and scientific notation.

4. ____________

5. The population of Las Vegas, Nevada has been increasing at an annual rate of 7.0%. If the population of Las Vegas was 478,434 in 1999, predict its population in 2015.

5. ____________

6. A new motor home costs $89,000. It is expected to depreciate 9% each year. Find the value of the motor home in 4 years.

6. ____________

7. Find the total amount of an investment if $1200 is invested at an interest rate of 3.5% compounded quarterly for 7 years.

7. ____________

8. Write an equation for the nth term of the geometric sequence $-5, 15, -45, 135, \ldots$.

8. ____________

9. Write a recursive formula for $14, 9, 4, -1, \ldots$.

9. ____________

10. Find $(4a - 2b + c) - (5c + a - 4b)$.

10. ____________

11. Solve $x(x - 3) = x(x + 5) - 16$.

11. ____________

Find each product.

12. $(p - 3)(p + 5)$

12. ____________

13. $(a + 2)(a^2 - 3a - 7)$

13. ____________

14. $(5t + r)^2$

14. ____________

15. $(3k - 1)(3k + 1)$

15. ____________

9 Unit 3 Test *(continued)*

Factor each polynomial.

16. $4p^2r - 16pr + 8pr^2$ **16.** ____________

17. $m^2 - 11m + 18$ **17.** ____________

18. $3a^2 - 13a + 12$ **18.** ____________

19. $27r^2 - 12t^2$ **19.** ____________

20. The area of a square is $4p^2 - 4p + 1$ square inches. What is the length of the side of the square? **20.** ____________

Solve each equation.

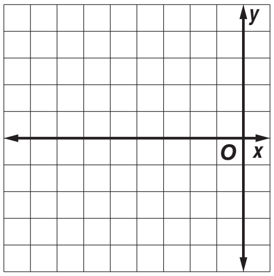
21. $22y^2 = -11y$ **21.** ____________

22. $h^2 - 13h + 36 = 0$ **22.** ____________

23. $5a^2 - 33a = 14$ **23.** ____________

24. $m^2 + 81 = -18m$ **24.** ____________

25. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = x^2 + 8x + 12$. Then graph $y = x^2 + 8x + 12$. **25.** ____________



26. Find the value of c that makes $x^2 - 20x + c$ a perfect square trinomial. **26.** ____________

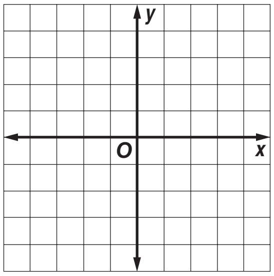
27. State the value of the discriminant for $3x^2 + 2x + 1 = 0$. **27.** ____________

28. Solve $6 = 3t^2 + 2t$ by using the Quadratic Formula. Round to the nearest tenth if necessary. **28.** ____________

29. Look for a pattern in the table of values to determine which model best describes the data. Then write an equation of the function that models the data. **29.** ____________

x	0	1	2	3
y	6	12	24	48

30. Graph $y = |2x + 1|$. **30.**



NAME ______________________ DATE ____________ PERIOD ____________

9 Anticipation Guide

Quadratic Functions and Equations

Step 1 *Before you begin Chapter 9*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, o NS	Statement	STEP 2 A o D
	1. The graph of a quadratic function is a parabola.	A
	2. The graph of $y = 4x^2 - 2x + 7$ will be a parabola opening downward since the coefficient of x^2 is positive.	D
	3. A quadratic function's axis of symmetry is either the x-axis or the y-axis.	D
	4. The graph of a quadratic function opening upward has no maximum value.	A
	5. The x-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.	A
	6. All quadratic equations have two real solutions.	D
	7. Any quadratic expression can be written as a perfect square by a method called *completing the square.*	A
	8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.	D
	9. The graph of a step function is a series of disjointed line segments.	A
	10. It is not possible to identify data as linear based on patterns of behavior of their y-values.	A

Step 2 *After you complete Chapter 9*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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9-1 Study Guide and Intervention

Graphing Quadratic Functions

Characteristics of Quadratic Functions

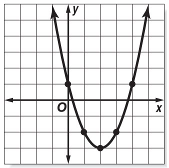
Quadratic Function	a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$	**Example:** $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic functions have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a is positive, and a parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1

a. Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
−1	6
0	1
1	−2
2	−3
3	−2
4	1



Graph the ordered pairs in the table and connect them with a smooth curve.

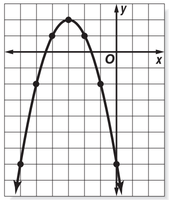
b. What are the domain and range of this function?

The domain is all real numbers. The range is all real numbers greater than or equal to −3, which is the minimum.

Example 2

a. Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
−6	−7
−5	−2
−4	1
−3	2
−2	1
−1	−2
0	−7



Graph the ordered pairs in the table and connect them with a smooth curve.

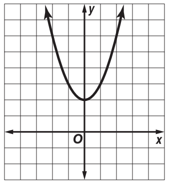
b. What are the domain and range of this function?

The domain is all real numbers. The range is all real numbers less than or equal to 2, which is the maximum.

Exercises

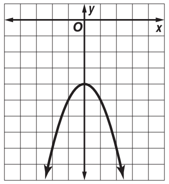
Use a table of values to graph each function. Determine the domain and range.

1. $y = x^2 + 2$



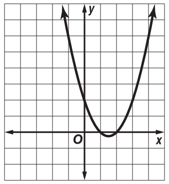
D: {all real numbers}
R: $\{y \mid y \geq 2\}$

2. $y = -x^2 - 4$



D: {all real numbers}
R: $\{y \mid y \leq -4\}$

3. $y = x^2 - 3x + 2$



D: {all real numbers}
R: $\left\{y \mid y \geq -\frac{1}{4}\right\}$

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9-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Symmetry and Vertices Parabolas have a geometric property called **symmetry**. That is, if the figure is folded in half, each half will match the other half exactly. The vertical line containing the fold line is called the **axis of symmetry**. The axis of symmetry contains the minimum or maximum point of the parabola, the **vertex**.

Axis of Symmetry	For the parabola $y = ax^2 + bx + c$, where $a \neq 0$, the line $x = -\frac{b}{2a}$ is the axis of symmetry.	**Example:** The axis of symmetry of $y = x^2 + 2x + 5$ is the line $x = -1$.

Example **Consider the graph of $y = 2x^2 + 4x + 1$.**

a. Write the equation of the axis of symmetry.

In $y = 2x^2 + 4x + 1$, $a = 2$ and $b = 4$. Substitute these values into the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$

$$x = -\frac{4}{2(2)} = -1$$

The axis of symmetry is $x = -1$.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate of the vertex is -1.

$y = 2x^2 + 4x + 1$ Original equation

$y = 2(-1)^2 + 4(-1) + 1$ Substitute.

$y = 2(1) - 4 + 1$ Simplify.

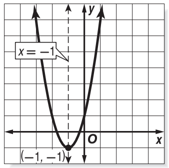
$y = -1$

The vertex is at $(-1, -1)$.

c. Identify the vertex as a maximum or a minimum.

Since the coefficient of the x^2-term is positive, the parabola opens upward, and the vertex is a minimum point.

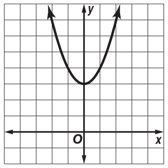
d. Graph the function.



Exercises

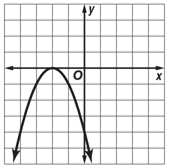
Consider each equation. Determine whether the function has *maximum* or *minimum* value. State the maximum or minimum value and the domain and range of the function. Find the equation of the axis of symmetry. Graph the function.

1. $y = x^2 + 3$



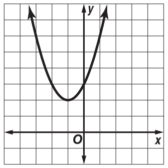
min.; (0, 3);
D: {all real numbers},
R: $\{y \mid y \geq 3\}$; $x = 0$

2. $y = -x^2 - 4x - 4$



max.; (−2, 0);
D: {all real numbers},
R: $\{y \mid y \leq 0\}$; $x = -2$

3. $y = x^2 + 2x + 3$



min.; (−1, 2);
D: {all real numbers},
R: $\{y \mid y \geq 2\}$; $x = -1$

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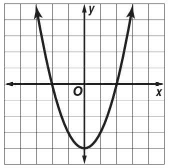
9-1 Skills Practice

Graphing Quadratic Functions

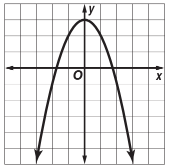
Use a table of values to graph each function. State the domain and the range.

1. $y = x^2 - 4$



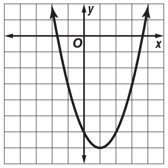
D = {all real numbers}
R = $\{y \mid y \geq -4\}$

2. $y = -x^2 + 3$



D = {all real numbers}
R = $\{y \mid y \leq 3\}$

3. $y = x^2 - 2x - 6$



D = {all real numbers}
R = $\{y \mid y \geq -7\}$

Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each function.

4. $y = 2x^2 - 8x + 6$

(2, −2); $x = 2$; (0, 6)

5. $y = x^2 + 4x + 6$

(−2, 2); $x = -2$; (0, 6)

6. $y = -3x^2 - 12x + 3$

(−2, 15); $x = -2$; (0, 3)

Consider each equation.

a. Determine whether the function has a *maximum* or a *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

7. $y = 2x^2$

minimum; (0, 0);
D = {all real numbers},
R = $\{y \mid y \geq 0\}$

8. $y = x^2 - 2x - 5$

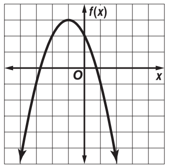
minimum; (1, −6);
D = {all real numbers},
R = $\{y \mid y \geq -6\}$

9. $y = -x^2 + 4x - 1$

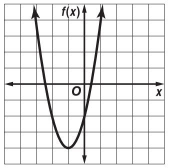
maximum; (2, 3);
D = {all real numbers},
R = $\{y \mid y \leq 3\}$

Graph each function.

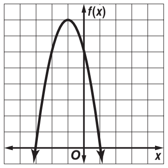
10. $f(x) = -x^2 - 2x + 2$



11. $f(x) = 2x^2 + 4x - 2$



12. $f(x) = -2x^2 - 4x + 6$



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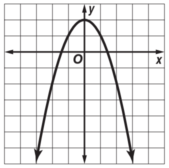
9-1 Practice

Graphing Quadratic Functions

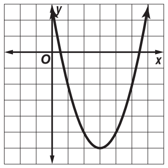
Use a table of values to graph each function. Determine the domain and range.

1. $y = -x^2 + 2$



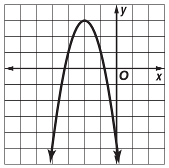
D: {all real numbers}
R: {y | y ≤ 2}

2. $y = x^2 - 6x + 3$



D: {all real numbers}
R: {y | y ≥ −6}

3. $y = -2x^2 - 8x - 5$



D: {all real numbers}
R: {y | y ≤ 3}

Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each function.

4. $y = x^2 - 9$
(0, −9); x = 0; (0, −9)

5. $y = -2x^2 + 8x - 5$
(2, 3); x = 2; (0, −5)

6. $y = 4x^2 - 4x + 1$
(0.5, 0); x = 0.5; (0, 1)

Consider each equation. Determine whether the function has a *maximum* or a *minimum* value. State the maximum or minimum value. What are the domain and range of the function?

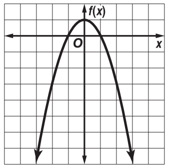
7. $y = 5x^2 - 2x + 2$
min.; (0.2, 1.8);
D: {all real numbers},
R: {y | y ≥ 1.8}

8. $y = -x^2 + 5x - 10$
max.; (2.5, −3.75);
D: {all real numbers},
R: {y | y ≤ −3.75}

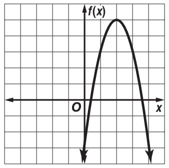
9. $y = \frac{3}{2}x^2 + 4x - 9$
min.; $\left(-1\frac{1}{3}, -11\frac{2}{3}\right)$
D: {all real numbers},
R: $\left\{y \mid y \geq -11\frac{2}{3}\right\}$

Graph each function.

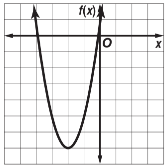
10. $f(x) = -x^2 + 1$



11. $f(x) = -2x^2 + 8x - 3$



12. $f(x) = 2x^2 + 8x + 1$



13. **BASEBALL** The equation $h = -0.005x^2 + x + 3$ describes the path of a baseball hit into the outfield, where h is the height and x is the horizontal distance the ball travels.

a. What is the equation of the axis of symmetry? **x = 100**

b. What is the maximum height reached by the baseball? **53 ft**

c. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it? **200 ft**

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9-1 Word Problem Practice

Graphing Quadratic Functions

1. **OLYMPICS** Olympics were held in 1896 and have been held every four years except 1916, 1940, and 1944. The winning height y in men's pole vault at any number Olympiad x can be approximated by the equation $y = 0.37x^2 + 4.3x + 126$. Complete the table to estimate the pole vault heights in each of the Olympic Games. Round your answers to the nearest tenth.

Year	Olympiad (x)	Height (y inches)
1896	1	**130.7**
1900	2	**136.1**
1924	7	**174.2**
1936	10	**206.0**
1964	15	**273.8**
2008	26	**487.9**

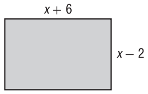
Source: National Security Agency

2. **PHYSICS** Mrs. Capwell's physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that $y = -x^2 + 6x$ accurately predicts the ball's position y after rolling x seconds. On the graph of the equation, what would be the y value when $x = 4$?
8

3. **ARCHITECTURE** A hotel's main entrance is in the shape of a parabolic arch. The equation $y = -x^2 + 10x$ models the arch height y for any distance x from one side of the arch. Use a graph to determine its maximum height.
25 ft

4. **SOFTBALL** Olympic softball gold medalist Michele Smith pitches a curveball with a speed of 64 feet per second. If she throws the ball straight upward at this speed, the ball's height h in feet after t seconds is given by $h = -16t^2 + 64t$. Find the coordinates of the vertex of the graph of the ball's height and interpret its meaning.
(2, 64); After 2 seconds, the ball reaches its highest point, 64 feet above the ground.

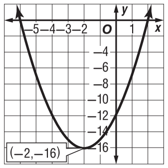
5. **GEOMETRY** Teddy is building the rectangular deck shown below.



a. Write an equation representing the area of the deck y.
y = (x − 2)(x + 6) or
y = x² + 4x − 12

b. What is the equation of the axis of symmetry? **x = −2**

c. Graph the equation and label its vertex.



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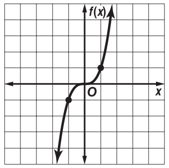
9-1 Enrichment

Graphing Cubic Functions

A **cubic function** is a polynomial written in the form of $f(x) = ax^3 + bx^2 + cx + n$, where $a \neq 0$. Cubic functions do not have absolute minimum and maximum values like quadratic functions do, but they can have a **local minimum** and a **local maximum** point.

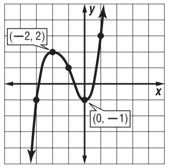
Parent Function: $f(x) = x^3$

Domain: {all real numbers}
Range: {all real numbers}



Example **Use a table of values to graph $y = x^3 + 3x^2 - 1$. Then use the graph to estimate the locations of the local minimum and local maximum points.**

x	–3	–2	–1	0	1
y	–1	2	1	–1	2



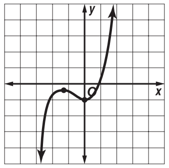
Graph the ordered pairs, and connect them to create a smooth curve. The end behavior of the "S" shaped curve shows that as x increases, y increases, and as x decreases, y decreases.

The local minimum is located at (0, –1). The local maximum is located at (–2, 2).

Exercises

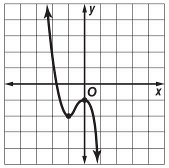
Use a table of values to graph each equation. Then use the graph to estimate the locations of the local minimum and local maximum points.

1. $y = 0.5x^3 + x^2 - 1$



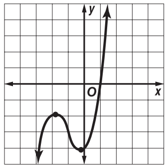
local maximum: (−1.3, −0.4); local minimum: (0, 1)

2. $y = -2x^3 - 3x^2 - 1$



local maximum: (0, −1); local minimum: (−1, −2)

3. $y = x^3 + 3x^2 + x - 4$



local maximum: (−1.8, −1.9); local minimum: (−0.2, −4.1)

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9-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

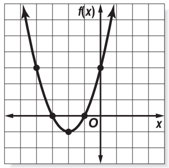
Solve by Graphing

Quadratic Equation	an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by graphing the related quadratic function $f(x) = ax^2 + bx + c$ and finding the x-intercepts or **zeros** of the function.

Example 1 **Solve $x^2 + 4x + 3 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + 4x + 3$. The equation of the axis of symmetry is $x = -\frac{4}{2(1)}$ or –2. The vertex is at (–2, –1). Graph the vertex and several other points on either side of the axis of symmetry.

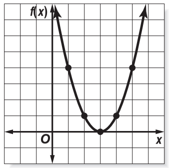


To solve $x^2 + 4x + 3 = 0$, you need to know where $f(x) = 0$. This occurs at the x-intercepts, –3 and –1.

The solutions are –3 and –1.

Example 2 **Solve $x^2 - 6x + 9 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - 6x + 9$. The equation of the axis of symmetry is $x = \frac{6}{2(1)}$ or 3. The vertex is at (3, 0). Graph the vertex and several other points on either side of the axis of symmetry.

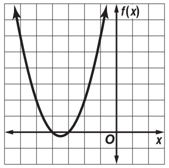


To solve $x^2 - 6x + 9 = 0$, you need to know where $f(x) = 0$. The vertex of the parabola is the x-intercept. Thus, the only solution is 3.

Exercises

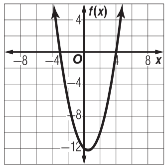
Solve each equation by graphing.

1. $x^2 + 7x + 12 = 0$



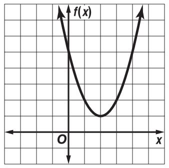
−3, −4

2. $x^2 - x - 12 = 0$



4, −3

3. $x^2 - 4x + 5 = 0$



no real roots

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9-2 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Graphing

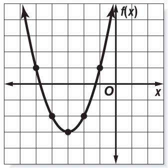
Estimate Solutions The roots of a quadratic equation may not be integers. If exact roots cannot be found, they can be estimated by finding the consecutive integers between which the roots lie.

Example **Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(x) = x^2 + 6x + 6$.

x	f(x)
−5	1
−4	−2
−3	−3
−2	−2
−1	1

Notice that the value of the function changes from negative to positive between the x-values of −5 and −4 and between −2 and −1.

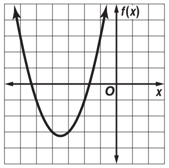


The x-intercepts of the graph are between −5 and −4 and between −2 and −1. So one root is between −5 and −4, and the other root is between −2 and −1.

Exercises

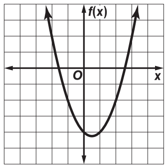
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

1. $x^2 + 7x + 9 = 0$



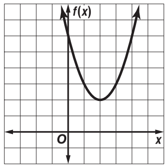
−6 < x < −5, −2 < x < −1

2. $x^2 - x - 4 = 0$



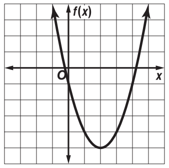
−2 < x < −1, 2 < x < 3

3. $x^2 - 4x + 6 = 0$



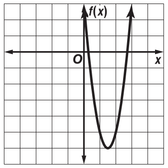
no real roots

4. $x^2 - 4x - 1 = 0$



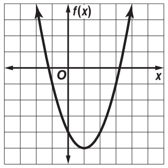
−1 < x < 0, 4 < x < 5

5. $4x^2 - 12x + 3 = 0$



0 < x < 1, 2 < x < 3

6. $x^2 - 2x - 4 = 0$



−2 < x < −1, 3 < x < 4

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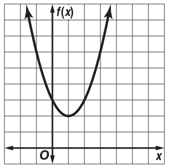
9-2 Skills Practice

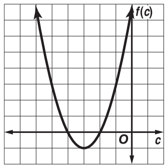
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

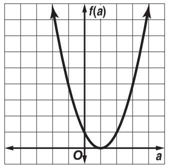
1. $x^2 - 2x + 3 = 0$ **∅**



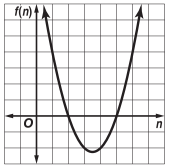
2. $c^2 + 6c + 8 = 0$ **−4, −2**



3. $a^2 - 2a = -1$ **1**

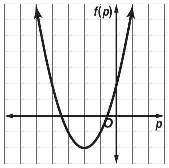


4. $n^2 - 7n = -10$ **2, 5**



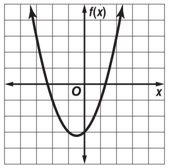
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

5. $p^2 + 4p + 2 = 0$



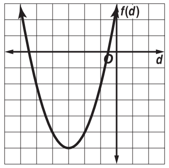
−3.4, −0.6

6. $x^2 + x - 3 = 0$



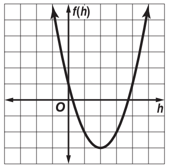
−2.3, 1.3

7. $d^2 + 6d = -3$



−5.4, −0.6

8. $h^2 + 1 = 4h$



0.3, 3.7

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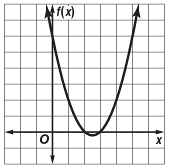
9-2 Practice

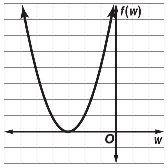
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

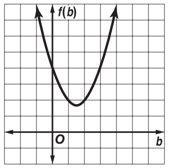
1. $x^2 - 5x + 6 = 0$ **2, 3**



2. $w^2 + 6w + 9 = 0$ **−3**

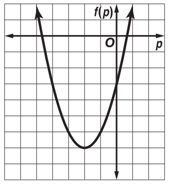


3. $b^2 - 3b + 4 = 0$ **∅**



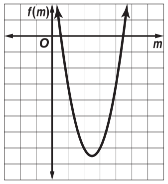
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

4. $p^2 + 4p = 3$



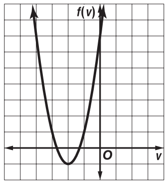
−4.6, 0.6

5. $2m^2 + 5 = 10m$



0.6, 4.4

6. $2v^2 + 8v = -7$

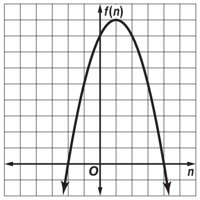


−2.7, −1.3

7. **NUMBER THEORY** Two numbers have a sum of 2 and a product of −8. The quadratic equation $-n^2 + 2n + 8 = 0$ can be used to determine the two numbers.

   a. Graph the related function $f(n) = -n^2 + 2n + 8$ and determine its x-intercepts. **−2, 4**

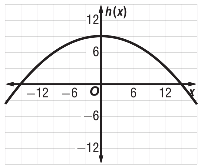
   b. What are the two numbers? **−2 and 4**



8. **DESIGN** A footbridge is suspended from a parabolic support. The function $h(x) = -\frac{1}{25}x^2 + 9$ represents the height in feet of the support above the walkway, where $x = 0$ represents the midpoint of the bridge.

   a. Graph the function and determine its x-intercepts. **−15, 15**

   b. What is the length of the walkway between the two supports? **30 ft**



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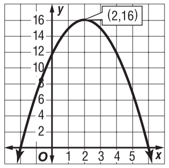
9-2 Word Problem Practice

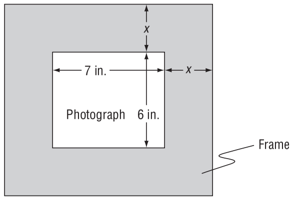
Solving Quadratic Equations by Graphing

1. **FARMING** In order for Mr. Moore to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. His crop yield y depends on the amount of fertilizer he applies to his fields x according to the equation $y = -x^2 + 4x + 12$. Graph the function, and find the point at which Mr. Moore gets the highest yield possible.



2. **LIGHT** Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the parabola and find that the equation $y = x^2 - 3x - 10$ matches the shape of the light beam. Determine the zeros of the function. **−2 and 5**

3. **FRAMING** A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using a material that is x inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material? **3 in.**

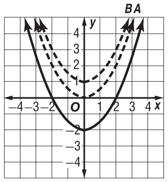


4. **WRAPPING PAPER** Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (*Hint:* Let length = 30 − w.) Explain. **Yes; solving the equation (30 − w)w = 81 gives w = 3 or 27. A 3 in. by 27 in. sheet of paper has area 81 in² and perimeter 60 in.**

5. **ENGINEERING** The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation.

$$0.5x^2 = 2 + y$$

   a. Approximate the zeros of this function by graphing. **−2 and 2**



   b. On the coordinate plane above, translate the parabola so that there is only one zero. Label this curve A. **See students' work.**

   c. Translate the parabola so that there are no zeros. Label this curve B. **See students' work.**

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9-2 Enrichment

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points $(0, -2)$, $(3, 0)$, and $(5, 2)$.

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y, you get three equations.

$(0, -2)$: $-2 = c$
$(3, 0)$: $0 = 9a + 3b + c$
$(5, 2)$: $2 = 25a + 5b + c$

First, substitute -2 for c in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by -3.

$0 = 9a + 3b - 2$ Multiply by 5. $0 = 45a + 15b - 10$
$2 = 25a + 5b - 2$ Multiply by -3. $-6 = -75a - 15b + 6$

$$-6 = -30a \quad - 4$$
$$a = \frac{1}{15}$$

To find b, substitute $\frac{1}{15}$ for a in either the second or third equation.

$$0 = 9\left(\frac{1}{15}\right) + 3b - 2$$
$$b = \frac{7}{15}$$

The equation of a parabola through the three points is

$$y = \frac{1}{15}x^2 + \frac{7}{15}x - 2.$$

Find the equation of a parabola through each set of three points.

1. $(1, 5)$, $(0, 6)$, $(2, 3)$ $\mathbf{y = -\frac{1}{2}x^2 - \frac{1}{2}x + 6}$	**2.** $(-5, 0)$, $(0, 0)$, $(8, 100)$ $\mathbf{y = \frac{25}{26}x^2 + \frac{125}{26}x}$
3. $(4, -4)$, $(0, 1)$, $(3, -2)$ $\mathbf{y = -\frac{1}{4}x^2 - \frac{1}{4}x + 1}$	**4.** $(1, 3)$, $(6, 0)$, $(0, 0)$ $\mathbf{y = -\frac{3}{5}x^2 + \frac{18}{5}x}$
5. $(2, 2)$, $(5, -3)$, $(0, -1)$ $\mathbf{y = -\frac{19}{30}x^2 + \frac{83}{30}x - 1}$	**6.** $(0, 4)$, $(4, 0)$, $(-4, 4)$ $\mathbf{y = -\frac{1}{8}x^2 - \frac{1}{2}x + 4}$

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9-3 Study Guide and Intervention

Transformations of Quadratic Functions

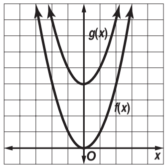
Translations A **translation** is a change in the position of a figure either up, down, left, right, or diagonal. Adding or subtracting constants in the equations of functions translates the graphs of the functions.

> The graph of $g(x) = x^2 + k$ translates the graph of $f(x) = x^2$ vertically.
> If $k > 0$, the graph of $f(x) = x^2$ is translated k units up.
> If $k < 0$, the graph of $f(x) = x^2$ is translated $|k|$ units down.

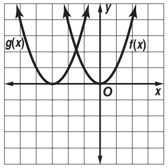
> The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.
> If $h > 0$, the graph of $f(x) = x^2$ is translated h units to the right.
> If $h < 0$, the graph of $f(x) = x^2$ is translated $|h|$ units to the left.

Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

a. $\mathbf{g(x) = x^2 + 4}$
The value of k is 4, and $4 > 0$. Therefore, the graph of $g(x) = x^2 + 4$ is a translation of the graph of $f(x) = x^2$ up 4 units.



b. $\mathbf{g(x) = (x + 3)^2}$
The value of h is –3, and $-3 < 0$. Thus, the graph of $g(x) = (x + 3)^2$ is a translation of the graph of $f(x) = x^2$ to the left 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 1$ **Translation of $f(x) = x^2$ up 1 unit.**	**2.** $g(x) = (x - 6)^2$ **Translation of $f(x) = x^2$ to the right 6 units.**	**3.** $g(x) = (x + 1)^2$ **Translation of $f(x) = x^2$ to the left 1 unit.**
4. $g(x) = 20 + x^2$ **Translation of $f(x) = x^2$ up 20 units.**	**5.** $g(x) = (-2 + x)^2$ **Translation of $f(x) = x^2$ to the right 2 units.**	**6.** $g(x) = -\frac{1}{2} + x^2$ **Translation of $f(x) = x^2$ down $\frac{1}{2}$ unit.**
7. $g(x) = x^2 + \frac{8}{9}$ **Translation of $f(x) = x^2$ up $\frac{8}{9}$ unit.**	**8.** $g(x) = x^2 - 0.3$ **Translation of $f(x) = x^2$ down 0.3 unit.**	**9.** $g(x) = (x + 4)^2$ **Translation of $f(x) = x^2$ to the left 4 units.**

Lesson 9-3

Answers

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9-3 Study Guide and Intervention *(continued)*

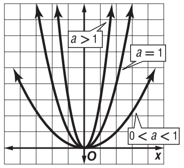
Transformations of Quadratic Functions

Dilations and Reflections A **dilation** is a transformation that makes the graph narrower or wider than the parent graph. A **reflection** flips a figure over the x- or y-axis.

The graph of $f(x) = ax^2$ stretches or compresses the graph of $f(x) = x^2$.

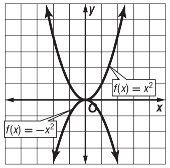
If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



The graph of the function $-f(x)$ flips the graph of $f(x) = x^2$ across the x-axis.

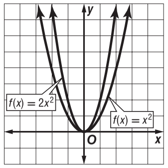
The graph of the function $f(-x)$ flips the graph of $f(x) = x^2$ across the y-axis.



Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

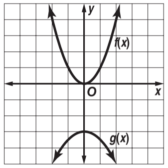
a. $g(x) = 2x^2$

The function can be written as $f(x) = ax^2$ where $a = 2$. Because $|a| > 1$, the graph of $y = 2x^2$ is the graph of $y = x^2$ that is stretched vertically.



b. $g(x) = -\frac{1}{2}x^2 - 3$

The negative sign causes a reflection across the x-axis. Then a dilation occurs in which $a = \frac{1}{2}$ and a translation in which $k = -3$. So the graph of $g(x) = -\frac{1}{2}x^2 - 3$ is reflected across the x-axis, dilated wider than the graph of $f(x) = x^2$, and translated down 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = -5x^2$
Compression of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$ reflected over the x-axis.

2. $g(x) = -(x + 1)^2$
Translation of $f(x) = x^2$ to the left 1 unit and reflected over the x-axis.

3. $g(x) = -\frac{1}{4}x^2 - 1$
Dilation of $f(x) = x^2$ wider than the graph of $f(x) = x^2$ reflected over the x-axis translated down 1 unit.

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9-3 Skills Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 2$
Translation of $f(x) = x^2$ up 2 units

2. $g(x) = (x - 1)^2$
Translation of $f(x) = x^2$ to the right 1 unit

3. $g(x) = x^2 - 8$
Translation of $f(x) = x^2$ down 8 units

4. $g(x) = 7x^2$
Stretch of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$

5. $g(x) = \frac{1}{5}x^2$
Compression of $f(x) = x^2$ wider than the graph of $f(x) = x^2$

6. $g(x) = -6x^2$
Stretch of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$ reflected over the x-axis

7. $g(x) = -x^2 + 3$
Reflection of $f(x) = x^2$ over the x-axis translated up 3 units

8. $g(x) = 5 - \frac{1}{2}x^2$
Compression of $f(x) = x^2$ wider than the graph of $f(x) = x^2$ reflected over the x-axis, and translated up 5 units

9. $g(x) = 4(x - 1)^2$
Stretch of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$ translated to the right 1 unit

Match each equation to its graph.

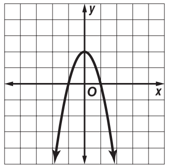
10. $y = 2x^2 - 2$ **B**

11. $y = \frac{1}{2}x^2 - 2$ **D**

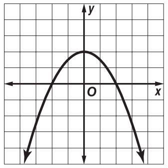
12. $y = -\frac{1}{2}x^2 + 2$ **C**

13. $y = -2x^2 + 2$ **A**

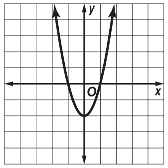
A.



C.



B.



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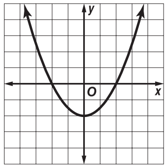

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9-3 Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = (10 + x)^2$
   Translation of $f(x) = x^2$ to the left 10 units.

2. $g(x) = -\frac{2}{5} + x^2$
   Translation of $f(x) = x^2$ down $\frac{2}{5}$ unit.

3. $g(x) = 9 - x^2$
   Reflection of $f(x) = x^2$ across the x-axis translated up 9 units.

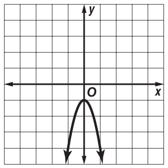
4. $g(x) = 2x^2 + 2$
   Stretch of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$ translated up 2 units.

5. $g(x) = -\frac{3}{4}x^2 - \frac{1}{2}$
   Compression of $f(x) = x^2$ wider than the graph of $f(x) = x^2$, reflected over the x-axis, translated down $\frac{1}{2}$ unit.

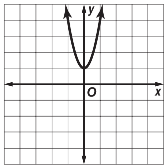
6. $g(x) = -3(x + 4)^2$
   Stretch of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$, reflected over the x-axis translated to the left 4 units.

Match each equation to its graph.

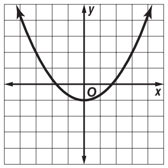
A.



B.



C.



7. $y = -3x^2 - 1$ **A**

8. $y = \frac{1}{3}x^2 - 1$ **C**

9. $y = 3x^2 + 1$ **B**

List the functions in order from the most vertically stretched to the least vertically stretched graph.

10. $f(x) = 3x^2, g(x) = \frac{1}{2}x^2, h(x) = -2x^2$
    $f(x)$, $h(x)$, $g(x)$

11. $f(x) = \frac{1}{2}x^2, g(x) = -\frac{1}{6}x^2, h(x) = 4x^2$
    $h(x)$, $f(x)$, $g(x)$

12. **PARACHUTING** Two parachutists jump at the same time from two different planes as part of an aerial show. The height h_1 of the first parachutist in feet after t seconds is modeled by the function $h_1 = -16t^2 + 5000$. The height h_2 of the second parachutist in feet after t seconds is modeled by the function $h_2 = -16t^2 + 4000$.

    a. What is the parent function of the two functions given? **$h = t^2$**

    b. Describe the transformations needed to obtain the graph of h_1 from the parent function. **Stretch of $y = x^2$ narrower than the graph of $f(x) = x^2$, reflected over the x-axis, translated up 5000 units.**

    c. Which parachutist will reach the ground first? **the second parachutist**

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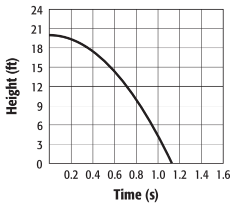
9-3 Word Problem Practice

Transformations of Quadratic Functions

1. **SPRINGS** The potential energy stored in a spring is given by $U_s = \frac{1}{2}kx^2$ where k is a constant known as the spring constant, and x is the distance the spring is stretched or compressed from its initial position. How is the graph of the function for a spring where $k = 2$ newtons/meter related to the graph of the function for a spring where $k = 10$ newtons/meter?
   The graph of $U_s = \frac{1}{2}(10)x^2$ is a stretch of the other graph.

2. **BLUEPRINTS** The bottom left corner of a rectangular park is located at (–3, 5) on a construction blueprint. The park is 8 units long and has an area of 48 units² on the blueprint. Suppose the park is translated on the blueprint so that the top right corner is now located at the origin. Describe the translation of the graph.
   Translated 5 units left and 11 units down

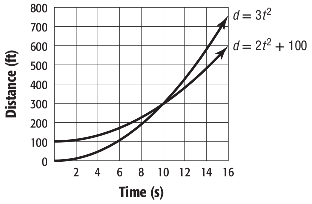
3. **PHYSICS** A ball is dropped from a height of 20 feet. The function $h = -16t^2 + 20$ models the height of the ball in feet after t seconds. Graph the function and compare this graph to the graph of its parent function.



$h = -16t^2 + 20$ is a stretch of $h = t^2$ reflected over the x-axis and translated up 20 units (feet).

4. **ACCELERATION** The distance d in feet a car accelerating at 6 ft/s² travels after t seconds is modeled by the function $d = 3t^2$. Suppose that at the same time the first car begins accelerating, a second car begins accelerating at 4 ft/s² exactly 100 feet down the road from the first car. The distance traveled by second car is modeled by the function $d = 2t^2 + 100$.

   a. Graph and label each function on the same coordinate plane.



   b. Explain how each graph is related to the graph of $d = t^2$.
   $d = 3t^2$ is a stretch of $d = t^2$; $d = 2t^2 + 100$ is a stretch of $d = t^2$ translated up 100 units (feet).

   c. After how many seconds will the first car pass the second car?
   10 seconds

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9-3 Enrichment

Graphing Polynomial Functions

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable.

Polynomial Function

If n is a nonnegative integer, $a_0, a_1, a_2, \dots, a_{n-1}, a_n$ are real numbers, and $a_n \neq 0$, then

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$$

is a polynomial function of degree n.

Notice that a quadratic function is a polynomial function of degree 2.

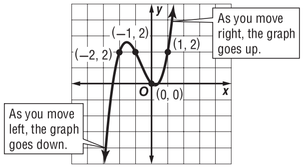
Example **Create a table of values and a graph for $y = x^3 + 2x^2 - x$. Then describe its end behavior.**

Create a table of values, and graph the ordered pairs. Connect the points with a smooth curve. Find and plot additional points to better approximate the curve's shape.

x	$x^3 + 2x^2 - x$	y
−4	$(-4)^3 + 2(-4)^2 - (-4)$	−28
−3	$(-3)^3 + 2(-3)^2 - (-3)$	−6
−2	$(-2)^3 + 2(-2)^2 - (-2)$	2
−1	$(-1)^3 + 2(-1)^2 - (-1)$	2
0	$(0)^3 + 2(0)^2 - 0$	0
1	$1^3 + 2(1)^2 - 1$	2
2	$2^3 + 2(2)^2 - 2$	14
3	$3^3 + 2(3)^2 - 3$	42

As x decreases, y decreases

As x increases, y increases.



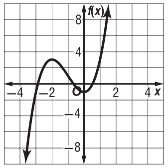
From the table and the graph we see that as x decreases, y decreases and as x increases, y increases.

Exercises

Create a table of values and a graph for each function. Then describe its end behavior.

1. $f(x) = x^3 + 3x^2 - 1$

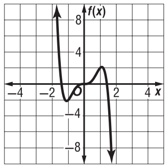
x	$f(x)$
−4	**−17**
−3	**−1**
−2	**3**
−1	**1**
0	**−1**
1	**3**



As x decreases, y decreases and, as x increases, y increases.

2. $f(x) = -2x^5 + 4x^3$

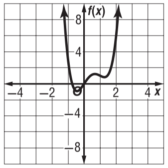
x	$f(x)$
−2	**32**
−1	**−2**
−0.5	**−0.4**
0	**0**
0.5	**−0.4**
1	**2**
2	**−32**



As x decreases, y increases, as x increases, y decreases.

3. $f(x) = 2x^4 - 4x^3 + 3x$

x	$f(x)$
−2	**58**
−1	**3**
−0.5	**−0.1**
0	**0**
0.5	**1.1**
1	**1**
2	**6**



As x decreases, y increases, as x increases, y increases.

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9-4 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Complete the Square Perfect square trinomials can be solved quickly by taking the square root of both sides of the equation. A quadratic equation that is not in perfect square form can be made into a perfect square by a method called **completing the square**.

Completing the Square

To complete the square for any quadratic equation of the form $x^2 + bx$:

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example **Find the value of c that makes $x^2 + 2x + c$ a perfect square trinomial.**

Step 1 Find $\frac{1}{2}$ of 2. $\frac{2}{2} = 1$

Step 2 Square the result of Step 1. $1^2 = 1$

Step 3 Add the result of Step 2 to $x^2 + 2x$. $x^2 + 2x + 1$

Thus, $c = 1$. Notice that $x^2 + 2x + 1$ equals $(x + 1)^2$.

Exercises

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 10x + c$ **25**

2. $x^2 + 14x + c$ **49**

3. $x^2 - 4x + c$ **4**

4. $x^2 - 8x + c$ **16**

5. $x^2 + 5x + c$ $\mathbf{\frac{25}{4}}$

6. $x^2 + 9x + c$ $\mathbf{\frac{81}{4}}$

7. $x^2 - 3x + c$ $\mathbf{\frac{9}{4}}$

8. $x^2 - 15x + c$ $\mathbf{\frac{225}{4}}$

9. $x^2 + 28x + c$ **196**

10. $x^2 + 22x + c$ **121**

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9-4 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Solve by Completing the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$ Original equation

$x^2 + 6x + 3 - 3 = 10 - 3$ Subtract 3 from each side.

$x^2 + 6x = 7$ Simplify.

$x^2 + 6x + 9 = 7 + 9$ Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.

$(x + 3)^2 = 16$ Factor $x^2 + 6x + 9$.

$x + 3 = \pm 4$ Take the square root of each side.

$x = -3 \pm 4$ Simplify.

$x = -3 + 4$ or $x = -3 - 4$

$= 1$ $= -7$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $x^2 - 4x + 3 = 0$ **1, 3**

2. $x^2 + 10x = -9$ **−1, −9**

3. $x^2 - 8x - 9 = 0$ **−1, 9**

4. $x^2 - 6x = 16$ **−2, 8**

5. $x^2 - 4x - 5 = 0$ **−1, 5**

6. $x^2 - 12x = 9$ **−0.7, 12.7**

7. $x^2 + 8x = 20$ **−10, 2**

8. $x^2 = 2x + 1$ **−0.4, 2.4**

9. $x^2 + 20x + 11 = -8$ **−19, −1**

10. $x^2 - 1 = 5x$ **−0.2, 5.2**

11. $x^2 = 22x + 23$ **−1, 23**

12. $x^2 - 8x = -7$ **1, 7**

13. $x^2 + 10x = 24$ **−12, 2**

14. $x^2 - 18x = 19$ **−1, 19**

15. $x^2 + 16x = -16$ **−14.9, −1.1**

16. $4x^2 = 24 + 4x$ **−2, 3**

17. $2x^2 + 4x + 2 = 8$ **−3, 1**

18. $4x^2 = 40x + 44$ **−1, 11**

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9-4 Skills Practice

Solving Quadratic Equations by Completing the Square

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 6x + c$ **9**

2. $x^2 + 4x + c$ **4**

3. $x^2 - 14x + c$ **49**

4. $x^2 - 2x + c$ **1**

5. $x^2 - 18x + c$ **81**

6. $x^2 + 20x + c$ **100**

7. $x^2 + 5x + c$ **6.25**

8. $x^2 - 70x + c$ **1225**

9. $x^2 - 11x + c$ **30.25**

10. $x^2 + 9x + c$ **20.25**

Solve each equation by completing the square. Round to the nearest tenth if necessary.

11. $x^2 + 4x - 12 = 0$ **2, −6**

12. $x^2 - 8x + 15 = 0$ **3, 5**

13. $x^2 + 6x = 7$ **−7, 1**

14. $x^2 - 2x = 15$ **−3, 5**

15. $x^2 - 14x + 30 = 6$ **2, 12**

16. $x^2 + 12x + 21 = 10$ **−11, −1**

17. $x^2 - 4x + 1 = 0$ **0.3, 3.7**

18. $x^2 - 6x + 4 = 0$ **0.8, 5.2**

19. $x^2 - 8x + 10 = 0$ **1.6, 6.4**

20. $x^2 - 2x = 5$ **−1.4, 3.4**

21. $2x^2 + 20x = -2$ **−9.9, −0.1**

22. $0.5x^2 + 8x = -7$ **−15.1, −0.9**

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9-4 Practice

Solving Quadratic Equations by Completing the Square

Find the value of *c* that makes each trinomial a perfect square.

1. $x^2 - 24x + c$ **144**

2. $x^2 + 28x + c$ **196**

3. $x^2 + 40x + c$ **400**

4. $x^2 + 3x + c$ $\frac{9}{4}$

5. $x^2 - 9x + c$ $\frac{81}{4}$

6. $x^2 - x + c$ $\frac{1}{4}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. $x^2 - 14x + 24 = 0$ **2, 12**

8. $x^2 + 12x = 13$ **−13, 1**

9. $x^2 - 30x + 56 = -25$ **3, 27**

10. $x^2 + 8x + 9 = 0$ **−6.6, −1.4**

11. $x^2 - 10x + 6 = -7$ **1.5, 8.5**

12. $x^2 + 18x + 50 = 9$ **−15.3, −2.7**

13. $3x^2 + 15x - 3 = 0$ **−5.2, 0.2**

14. $4x^2 - 72 = 24x$ **−2.2, 8.2**

15. $0.9x^2 + 5.4x - 4 = 0$ $-6\frac{2}{3}, \frac{2}{3}$

16. $0.4x^2 + 0.8x = 0.2$ **−2.2, 0.2**

17. $\frac{1}{2}x^2 - x - 10 = 0$ **−3.6, 5.6**

18. $\frac{1}{4}x^2 + x - 2 = 0$ **−5.5, 1.5**

19. NUMBER THEORY The product of two consecutive even integers is 728. Find the integers.
26, 28

20. BUSINESS Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Jaime has made in month x for the first two years of his business.

a. Write an equation representing the month in which Jaime's profit is $2400.
$x^2 + 50x + 1800 = 2400$

b. Use completing the square to find out in which month Jaime's profit is $2400.
the tenth month

21. PHYSICS From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height H of the rock t seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace H with 0.) **5.1 s**

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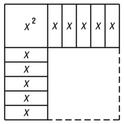
9-4 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. INTERIOR DESIGN Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 foot by 1 foot squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below. **25; 5**



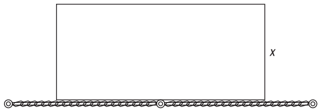
$x^2 + 10x +$ _____ $= (x +$ _____$)^2$

2. FALLING OBJECTS Keisha throws a rock down an old well. The distance d in feet the rock falls after t seconds can be represented by $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water?
1 second

3. MARS On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will hit the ground in about 5.4 seconds. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. How much longer will it take for the golf ball hit on Mars to reach the ground? Round your answer to the nearest tenth.
about 8.3 seconds

4. FROGS A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h in feet is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground?
$2 + \sqrt{10}$ **or about 5.16 ft**

5. GARDENING Peg is planning a rectangular vegetable garden using 200 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



a. Let x = the width of the rectangle. Write an equation to represent the area A of the garden if Peg uses all the fencing material. $A = x(200 - 2x)$

b. For what widths would the area of Peg's garden equal 4800 square feet if she uses all the fencing material?
The area of the garden will be 4800 ft² for a width of 40 ft and a width of 60 ft.

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9-4 Enrichment

Factoring Quartic Polynomials

Completing the square is a useful tool for factoring quadratic expressions. You can utilize a similar technique to factor simple **quartic polynomials** of the form $x^4 + c$.

Example **Factor the quartic polynomial $x^4 + 64$.**

Step 1 Find the value of the middle term needed to complete the square.
This value is $\left(2\sqrt{64}\right)(x^2)$, or $16x^2$.

Step 2 Rewrite the original polynomial in factorable form.
$\left(x^4 + 16x^2 + \left(\frac{16}{2}\right)^2\right) - 16x^2$

Step 3 Factor the polynomials. $(x^2 + 8)^2 - (4x)^2$

Step 4 Rewrite using the difference of two squares.
$(x^2 + 8 + 4x)(x^2 + 8 - 4x)$

The factored form of $x^4 + 64$ is $(x^2 + 4x + 8)(x^2 - 4x + 8)$. This could then be factored further, if needed, to find the solutions to a quartic equation.

Exercises

Factor each quartic polynomial.

1. $x^4 + 4$
$\mathbf{(x^2 + 2x + 2)(x^2 - 2x + 2)}$

2. $x^4 + 324$
$\mathbf{(x^2 + 6x + 18)(x^2 - 6x + 18)}$

3. $x^4 + 2500$
$\mathbf{(x^2 + 10x + 50)(x^2 - 10x + 50)}$

4. $x^4 + 9604$
$\mathbf{(x^2 + 14x + 98)(x^2 - 14x + 98)}$

5. $x^4 + 1024$
$\mathbf{(x^2 + 8x + 32)(x^2 - 8x + 32)}$

6. $x^4 + 5184$
$\mathbf{(x^2 + 12x + 72)(x^2 - 12x + 72)}$

7. $x^4 + 484$
$\mathbf{(x^2 + x\sqrt{44} + 22)(x^2 - x\sqrt{44} + 22)}$

8. $x^4 + 9$
$\mathbf{(x^2 + x\sqrt{6} + 3)(x^2 - x\sqrt{6} + 3)}$

9. $x^4 + 144$
$\mathbf{(x^2 + 2x\sqrt{6} + 12)(x^2 - 2x\sqrt{6} + 12)}$

10. $x^8 + 16{,}384$
$\mathbf{(x^4 + 16x^2 + 128)(x^4 - 16x^2 + 128)}$

11. Factor $x^4 + c$ to come up with a general rule for factoring quartic polynomials.
$\mathbf{\left(x^2 + x\sqrt{2\sqrt{c}} + \sqrt{c}\right) + \left(x^2 + x\sqrt{2\sqrt{c}} + \sqrt{c}\right)}$

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9-5 Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation
$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.
$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{16}}{2}$$
$$x = \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2}$$
$$= 1 \qquad\qquad = -3$$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{6 + \sqrt{44}}{2}$$
$$x = \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2}$$
$$\approx 6.3 \qquad\qquad \approx -0.3$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$ **1, 2**

2. $x^2 - 8x = -16$ **4**

3. $16x^2 - 8x = -1$ $\mathbf{\frac{1}{4}}$

4. $x^2 + 5x = 6$ **−6, 1**

5. $3x^2 + 2x = 8$ $\mathbf{-2, \frac{4}{3}}$

6. $8x^2 - 8x - 5 = 0$ **−0.4, 1.4**

7. $-4x^2 + 19x = 21$ $\mathbf{\frac{7}{4}, 3}$

8. $2x^2 + 6x = 5$ **−3.7, 0.7**

9. $48x^2 + 22x - 15 = 0$ $\mathbf{-\frac{5}{6}, \frac{3}{8}}$

10. $8x^2 - 4x = 24$ $\mathbf{-\frac{3}{2}, 2}$

11. $2x^2 + 5x = 8$ **−3.6, 1.1**

12. $8x^2 + 9x - 4 = 0$ **−1.5, 0.3**

13. $2x^2 + 9x + 4 = 0$ $\mathbf{-4, -\frac{1}{2}}$

14. $8x^2 + 17x + 2 = 0$ $\mathbf{-2, -\frac{1}{8}}$

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9-5 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real solutions for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	no real solutions
Case 2: $b^2 - 4ac = 0$	one real solution
Case 3: $b^2 - 4ac > 0$	two real solutions

Example **State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.**

a. $\mathbf{12x^2 + 5x = 4}$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real solutions.

b. $\mathbf{2x^2 + 3x = -4}$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

Find the discriminant.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real solutions.

Exercises

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

1. $3x^2 + 2x - 3 = 0$ **40, 2 real solutions**

2. $3x^2 - 7x - 8 = 0$ **145, 2 real solutions**

3. $2x^2 - 10x - 9 = 0$ **172, 2 real solutions**

4. $4x^2 = x + 4$ **65, 2 real solutions**

5. $3x^2 - 13x = 10$ **289, 2 real solutions**

6. $6x^2 - 10x + 10 = 0$ **–140, no real solutions**

7. $2x^2 - 20 = -x$ **161, 2 real solutions**

8. $6x^2 = -11x - 40$ **–839, no real solutions**

9. $9 - 18x + 9x^2 = 0$ **0, 1 real solution**

10. $12x^2 + 9 = -6x$ **–396, no real solutions**

11. $9x^2 = 81$ **2916, 2 real solutions**

12. $16x^2 + 16x + 4 = 0$ **0, 1 real solution**

13. $8x^2 + 9x = 2$ **145, 2 real solutions**

14. $4x^2 - 4x + 4 = 3$ **0, 1 real solution**

15. $3x^2 - 18x = -14$ **156, 2 real solutions**

NAME ______________ DATE ________ PERIOD ________

9-5 Skills Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 49 = 0$ **–7, 7**

2. $x^2 - x - 20 = 0$ **–4, 5**

3. $x^2 - 5x - 36 = 0$ **–4, 9**

4. $x^2 + 11x + 30 = 0$ **–6, –5**

5. $x^2 - 7x = -3$ **0.5, 6.5**

6. $x^2 + 4x = -1$ **–3.7, –0.3**

7. $x^2 - 9x + 22 = 0$ $\varnothing$

8. $x^2 + 6x + 3 = 0$ **–5.4, –0.6**

9. $2x^2 + 5x - 7 = 0$ $\mathbf{-3\frac{1}{2}, 1}$

10. $2x^2 - 3x = -1$ $\mathbf{\frac{1}{2}, 1}$

11. $2x^2 + 5x + 4 = 0$ $\varnothing$

12. $2x^2 + 7x = 9$ $\mathbf{-4\frac{1}{2}, 1}$

13. $3x^2 + 2x - 3 = 0$ **–1.4, 0.7**

14. $3x^2 - 7x - 6 = 0$ $\mathbf{-\frac{2}{3}, 3}$

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

15. $x^2 + 4x + 3 = 0$ **4; 2 real solutions**

16. $x^2 + 2x + 1 = 0$ **0; 1 real solution**

17. $x^2 - 4x + 10 = 0$ **–24; no real solutions**

18. $x^2 - 6x + 7 = 0$ **8; 2 real solutions**

19. $x^2 - 2x - 7 = 0$ **32; 2 real solutions**

20. $x^2 - 10x + 25 = 0$ **0; 1 real solution**

21. $2x^2 + 5x - 8 = 0$ **89; 2 real solutions**

22. $2x^2 + 6x + 12 = 0$ **–60; no real solutions**

23. $2x^2 - 4x + 10 = 0$ **–64; no real solutions**

24. $3x^2 + 7x + 3 = 0$ **13; 2 real solutions**

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9-5 Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 + 2x - 3 = 0$ **−3, 1**

2. $x^2 + 8x + 7 = 0$ **−7, −1**

3. $x^2 - 4x + 6 = 0$ ∅

4. $x^2 - 6x + 7 = 0$ **1.6, 4.4**

5. $2x^2 + 9x - 5 = 0$ **$-5, \frac{1}{2}$**

6. $2x^2 + 12x + 10 = 0$ **−5, −1**

7. $2x^2 - 9x = -12$ ∅

8. $2x^2 - 5x = 12$ **$-1\frac{1}{2}, 4$**

9. $3x^2 + x = 4$ **$-1\frac{1}{3}, 1$**

10. $3x^2 - 1 = -8x$ **−2.8, 0.1**

11. $4x^2 + 7x = 15$ **$-3, 1\frac{1}{4}$**

12. $1.6x^2 + 2x + 2.5 = 0$ ∅

13. $4.5x^2 + 4x - 1.5 = 0$
−1.2, 0.3

14. $\frac{1}{2}x^2 + 2x + \frac{3}{2} = 0$
−3, −1

15. $3x^2 - \frac{3}{4}x = \frac{1}{2}$
−0.3, 0.6

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

16. $x^2 + 8x + 16 = 0$
0; 1 real solution

17. $x^2 + 3x + 12 = 0$
−39; no real solutions

18. $2x^2 + 12x = -7$
88; 2 real solutions

19. $2x^2 + 15x = -30$
−15; no real solutions

20. $4x^2 + 9 = 12x$
0; 1 real solution

21. $3x^2 - 2x = 3.5$
46; 2 real solutions

22. $2.5x^2 + 3x - 0.5 = 0$
14; 2 real solutions

23. $\frac{3}{4}x^2 - 3x = -4$
−3; no real solutions

24. $\frac{1}{4}x^2 = -x - 1$
0; 1 real solution

25. CONSTRUCTION A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

a. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint*: Since the object is thrown down, the initial velocity is negative.) **$H = -16t^2 - 10t + 30$**

b. How long does it take the tile to hit the ground? **about 1.1 s**

26. PHYSICS Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint:* Substitute 25 for h and use the discriminant.) **No; the discriminant, −316, is negative, so there is no real solution.**

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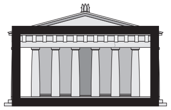
9-5 Word Problem Practice

Solving Quadratic Equations by Using the Quadratic Formula

1. BUSINESS Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit?
6 meals

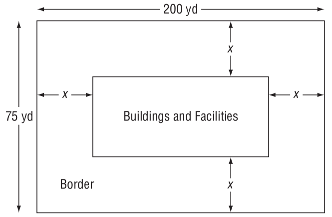
2. AERONAUTICS At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth.
25.8 seconds

3. ARCHITECTURE The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth.
25.9 in.



4. CRAFTS Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x be the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.
$\left(\frac{60 - x}{4}\right)^2 + \left(\frac{x}{4}\right)^2 = 117;$
24 in. and 36 in.

5. SITE DESIGN The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x represent the width of the border.



a. Use an equation similar to $A = \ell \times w$ to represent the situation.
10,000 = (200 − 2x)(75 − 2x)

b. Write the equation in standard quadratic form.
$4x^2 - 550x + 5000 = 0$

c. What should be the width of the border? Round your answer to the nearest tenth.
9.8 yd

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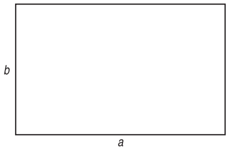
9-5 Enrichment

Golden Rectangles



A **golden rectangle** has the property that its sides satisfy the following proportion. $\frac{a+b}{a} = \frac{a}{b}$

Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.

1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation.

$b^2 + b - 1 = 0$

2. Solve the equation in Exercise 1 for b.

$b = \frac{-1 \pm \sqrt{5}}{2}$

3. In the proportion, let $b = 1$. Write a quadratic equation in a.

$a^2 - a - 1 = 0$

4. Solve the equation in Exercise 3 for a.

$a = \frac{1 \pm \sqrt{5}}{2}$

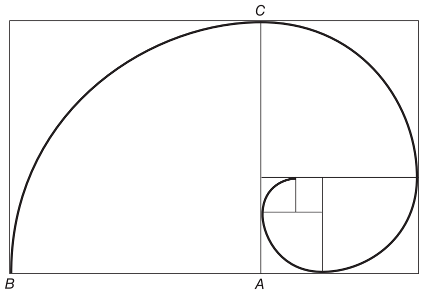
5. Explain why $\frac{1}{2}(\sqrt{5} + 1)$ and $\frac{1}{2}(\sqrt{5} - 1)$ are called golden ratios.

They are the ratios of the sides in a golden rectangle. The first is the ratio of the long side to the short side; the second is short side: long side.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarters of circles.

For example, the arc from point B to point C is created by putting the point of a compass at point A. The radius of the arc is the length BA.



6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

The short side should be about $6\frac{3}{16}$ inches.

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9-6 Study Guide and Intervention

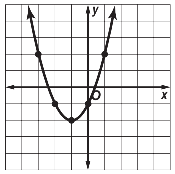
Analyzing Functions with Successive Differences and Ratios

Identify Functions Linear functions, quadratic functions, and exponential functions can all be used to model data. The general forms of the equations are listed at the right.

Linear Function	$y = mx + b$
Quadratic Function	$y = ax^2 + bx + c$
Exponential Function	$y = ab^x$

You can also identify data as linear, quadratic, or exponential based on patterns of behavior of their y-values.

Example 1 **Graph the set of ordered pairs {(–3, 2), (–2, –1), (–1, –2), (0, –1), (1, 2)}. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.**



The ordered pairs appear to represent a quadratic function.

Example 2 **Look for a pattern in the table to determine which model best describes the data.**

x	–2	–1	0	1	2
y	4	2	1	0.5	0.25

Start by comparing the first differences.

$4 \xrightarrow[-2]{} 2 \xrightarrow[-1]{} 1 \xrightarrow[-0.5]{} 0.5 \xrightarrow[-0.25]{} 0.25$

The first differences are not all equal. The table does not represent a linear function. Find the second differences and compare.

$-2 \xrightarrow[+1]{} -1 \xrightarrow[+0.5]{} -0.5 \xrightarrow[+0.25]{} -0.25$

The table does not represent a quadratic function. Find the ratios of the y-values.

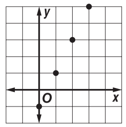
$4 \xrightarrow[\times 0.5]{} 2 \xrightarrow[\times 0.5]{} 1 \xrightarrow[\times 0.5]{} 0.5 \xrightarrow[\times 0.5]{} 0.25$

The ratios are equal. Therefore, the table can be modeled by an exponential function.

Exercises

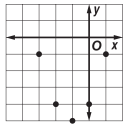
Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (0, –1), (1, 1), (2, 3), (3, 5)



linear

2. (–3, –1), (–2, –4), (–1, –5), (0, –4), (1, –1)



quadratic

Look for a pattern in each table to determine which model best describes the data.

3.

x	–2	–1	0	1	2
y	6	5	4	3	2

linear

4.

x	–2	–1	0	1	2
y	6.25	2.5	1	0.4	0.16

exponential

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9-6 Study Guide and Intervention *(continued)*

Analyzing Functions with Successive Differences and Ratios

Write Equations Once you find the model that best describes the data, you can write an equation for the function.

Basic Forms	Linear Function	$y = mx + b$
	Quadratic Function	$y = ax^2$
	Exponential Function	$y = ab^x$

Example **Determine which model best describes the data. Then write an equation for the function that models the data.**

x	0	1	2	3	4
y	3	6	12	24	48

Step 1 Determine whether the data is modeled by a linear, quadratic, or exponential function.

First differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24 \xrightarrow[+24]{} 48$

Second differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24$

y-value ratios: $3 \xrightarrow[\times 2]{} 6 \xrightarrow[\times 2]{} 12 \xrightarrow[\times 2]{} 24 \xrightarrow[\times 2]{} 48$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

Step 2 Write an equation for the function that models the data. The equation has the form $y = ab^x$. The y-value ratio is 2, so this is the value of the base.

$y = ab^x$ Equation for exponential function

$3 = a(2)^0$ $x = 0$, $y = 3$, and $b = 2$

$3 = a$ Simplify.

An equation that models the data is $y = 3 \cdot 2^x$. To check the results, you can verify that the other ordered pairs satisfy the function.

Exercises

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

1.

x	−2	−1	0	1	2
y	12	3	0	3	12

quadratic; $y = 3x^2$

2.

x	−1	0	1	2	3
y	−2	1	4	7	10

linear; $y = 3x + 1$

3.

x	−1	0	1	2	3
y	0.75	3	12	48	192

exponential; $y = 3 \cdot 4^x$

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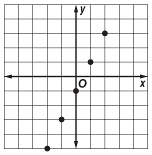
9-6 Skills Practice

Analyzing Functions with Successive Differences and Ratios

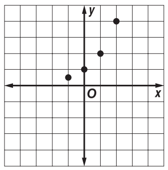
Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (2, 3), (1, 1), (0, −1), (−1, −3), (−3, −5)



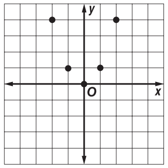
linear

2. (−1, 0.5), (0, 1), (1, 2), (2, 4)



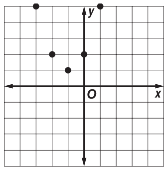
exponential

3. (−2, 4), (−1, 1), (0, 0), (1, 1), (2, 4)



quadratic

4. (−3, 5), (−2, 2), (−1, 1), (0, 2), (1, 5)



quadratic

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5.

x	−3	−2	−1	0	1	2
y	32	16	8	4	2	1

exponential; $y = 4 \cdot (0.5)^x$

6.

x	−1	0	1	2	3
y	7	3	−1	−5	−9

linear; $y = -4x + 3$

7.

x	−3	−2	−1	0	1
y	−27	−12	−3	0	−3

quadratic; $y = -3x^2$

8.

x	0	1	2	3	4
y	0.5	1.5	4.5	13.5	40.5

exponential; $y = \frac{1}{2} \cdot 3^x$

9.

x	−2	−1	0	1	2
y	−8	−4	0	4	8

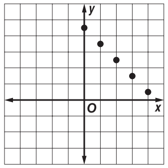
linear; $y = 4x$

9-6 Practice

Analyzing Functions with Successive Differences and Ratios

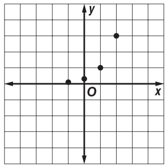
Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (4, 0.5), (3, 1.5), (2, 2.5), (1, 3.5), (0, 4.5)



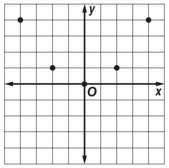
linear

2. $\left(-1, \frac{1}{9}\right), \left(0, \frac{1}{3}\right)$, (1, 1), (2, 3)



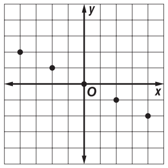
exponential

3. (–4, 4), (–2, 1), (0, 0), (2, 1), (4, 4)



quadratic

4. (–4, 2), (–2, 1), (0, 0), (2, –1), (4, –2)



linear

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5.

x	–3	–1	1	3	5
y	–5	–2	1	4	7

linear; $y = \frac{3}{2}x - \frac{1}{2}$

6.

x	–2	–1	0	1	2
y	0.02	0.2	2	20	200

exponential; $y = 2 \cdot 10^x$

7.

x	–1	0	1	2	3
y	6	0	6	24	54

quadratic; $y = 6x^2$

8.

x	–2	–1	0	1	2
y	18	9	0	–9	–18

linear; $y = -9x$

9. INSECTS The local zoo keeps track of the number of dragonflies breeding in their insect exhibit each day.

Day	1	2	3	4	5
Dragonflies	9	18	36	72	144

a. Determine which function best models the data. **exponential**

b. Write an equation for the function that models the data. **$y = 9 \cdot 2^x$**

c. Use your equation to determine the number of dragonflies that will be breeding after 9 days. **4608 dragonflies**

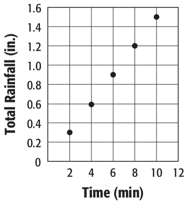
9-6 Word Problem Practice

Analyzing Functions with Successive Differences and Ratios

1. WEATHER The San Mateo weather station records the amount of rainfall since the beginning of a thunderstorm. Data for a storm is recorded as a series of ordered pairs shown below, where the *x* value is the time in minutes since the start of the storm, and the *y* value is the amount of rain in inches that has fallen since the start of the storm.

(2, 0.3), (4, 0.6), (6, 0.9), (8, 1.2), (10, 1.5)

Graph the ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.



linear

2. INVESTING The value of a certain parcel of land has been increasing in value ever since it was purchased. The table shows the value of the land parcel over time.

Year Since Purchasing	0	1	2	3	4
Land Value (thousands $)	$1.05	$2.10	$4.20	$8.40	$16.80

Look for a pattern in the table of values to determine which model best describes the data. Then write an equation for the function that models the data.

exponential; $y = 1.05 \cdot 2^n$

3. BOATS The value of a boat typically depreciates over time. The table shows the value of a boat over a period of time.

Years	0	1	2	3	4
Boat Value ($)	8250	6930	5821.20	4889.81	4107.44

Write an equation for the function that models the data. Then use the equation to determine how much the boat is worth after 9 years.

$y = 8250 \cdot (0.84)^x$; $1717.78

4. NUCLEAR WASTE Radioactive material slowly decays over time. The amount of time needed for an amount of radioactive material to decay to half its initial quantity is known as its half-life. Consider a 20-gram sample of a radioactive isotope.

Half-Lives Elapsed	0	1	2	3	4
Amount of Isotope Remaining (grams)	20	10	5	2.5	1.25

a. Is radioactive decay a *linear* decay, *quadratic* decay, or an *exponential* decay? **exponential**

b. Write an equation to determine how many grams *y* of a radioactive isotope will be remaining after *x* half-lives. **$y = 20 \cdot (0.5)^x$**

c. How many grams of the isotope will remain after 11 half-lives? **0.0098 g**

d. Plutonium-238 is one of the most dangerous waste products of nuclear power plants. If the half-life of plutonium-238 is 87.7 years, how long would it take for a 20-gram sample of plutonium-238 to decay to 0.078 gram? **701.8 years**

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9-6 Enrichment

Sierpinski Triangle

Sierpinski Triangle is an example of a fractal that changes exponentially. Start with an equilateral triangle and find the midpoints of each side. Then connect the midpoints to form a smaller triangle. Remove this smaller triangle from the larger one.

Repeat the process to create the next triangle in the sequence. Find the midpoints of the sides of the three remaining triangles and connect them to form smaller triangles to be removed.

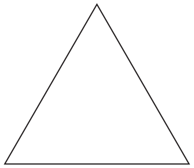

figure 1

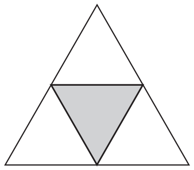

figure 2
Cut out: $\frac{1}{4}$
Area = $1 - \frac{1}{4}$ or $\frac{3}{4}$

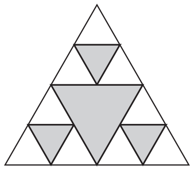

figure 3
Cut out: $\frac{1}{4} + \frac{3}{16}$ or $\frac{7}{16}$
Area = $1 - \frac{7}{16}$ or $\frac{9}{16}$

1. Find the next triangle in the sequence. How much has been cut out? What is the area of the fourth figure in the sequence?

See students' triangles; $\frac{1}{4} + \frac{3}{16} + \frac{9}{64}$ or $\frac{37}{64}$; $1 - \frac{37}{64}$ or $\frac{27}{64}$

2. Make a conjecture as to what you need to multiply the previous amount cut by to find the new amount cut. **$\frac{3}{4}$**

3. Fill in the chart to represent the amount cut and the area remaining for each triangle in the sequence.

Figure	1	2	3	4	5	6
Amount Cut	0	$\frac{1}{4}$	$\frac{7}{16}$	**$\frac{37}{64}$**	**$\frac{175}{256}$**	**$\frac{781}{1024}$**
Area Remaining	1	$\frac{3}{4}$	$\frac{9}{16}$	**$\frac{27}{64}$**	**$\frac{81}{256}$**	**$\frac{243}{1024}$**

4. Write an equation to represent the area that is left in the nth triangle in the sequence.

$A = \left(\frac{3}{4}\right)^n$

5. If this process is continued, make a conjecture as to the remaining area.
The remaining area gets very close to 0.

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9-7 Study Guide and Intervention

Special Functions

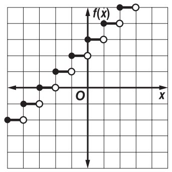
Step Functions The graph of a **step function** is a series of disjointed line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function.**

One example of a step function is the greatest integer function, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x.

Example **Graph $f(x) = [\![x + 3]\!]$.**

Make a table of values using integer and noninteger values. On the graph, dots represent included points, and circles represent points that are excluded.

x	$x + 3$	$[\![x + 3]\!]$
−5	−2	−2
−3.5	−0.5	−1
−2	1	1
−0.5	2.5	2
1	4	4
2.5	5.5	5



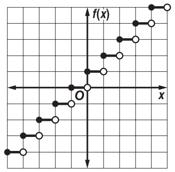
Because the dots and circles overlap, the domain is all real numbers. The range is all integers.

Exercises

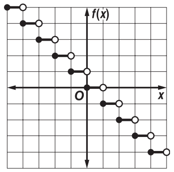
Graph each function. State the domain and range.

1–6. D = {all real numbers}; R = {all integers}

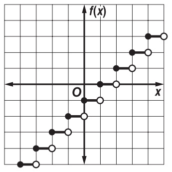
1. $f(x) = [\![x + 1]\!]$



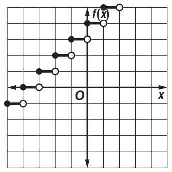
2. $f(x) = -[\![x]\!]$



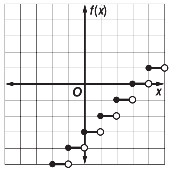
3. $f(x) = [\![x - 1]\!]$



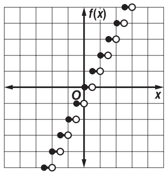
4. $f(x) = [\![x]\!] + 4$



5. $f(x) = [\![x]\!] - 3$



6. $f(x) = [\![2x]\!]$



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9-7 Study Guide and Intervention *(continued)*

Special Functions

Absolute Value Functions Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

The absolute value function is called a **piecewise-defined function** because it can be written using two or more expressions.

Example 1 **Graph $f(x) = |x + 2|$. State the domain and range.**

$f(x)$ cannot be negative, so the minimum point is $f(x) = 0$.

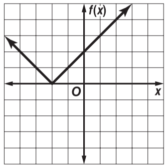
$f(x) = |x + 2|$ Original function

$0 = x + 2$ Replace f(x) with 0.

$-2 = x$ Subtract 2 from each side.

Make a table. Include values for $x > -2$ and $x < -2$.

x	f(x)
–5	3
–4	2
–3	1
–2	0
–1	1
0	2
1	3
2	4

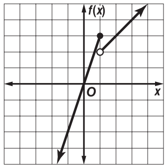


The domain is all real numbers. The range is all real numbers greater than or equal to 0.

Example 2 **Graph $f(x) = \begin{cases} x + 1 \text{ if } x > 1 \\ 3x \text{ if } x \le 1 \end{cases}$. State the domain and range.**

Graph the first expression. When $x > 1$, $f(x) = x + 1$. Since $x \neq 1$, place an open circle at (1, 2).

Next, graph the second expression. When $x \le 1$, $f(x) = 3x$. Since $x = 1$, place a closed circle at (1, 3).

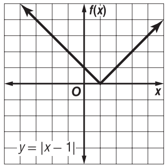


The domain and range are both all real numbers.

Exercises

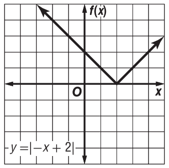
Graph each function. State the domain and range.

1. $f(x) = |x - 1|$



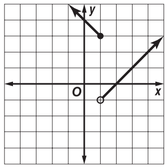
D = {all real numbers};
R = {y | y > 0}

2. $f(x) = |-x + 2|$



D = {all real numbers};
R = {y | y > 0}

3. $f(x) = \begin{cases} -x + 4 & \text{if } x \le 1 \\ x - 2 & \text{if } x > 1 \end{cases}$



D = {all real numbers};
R = {y | y > –1}

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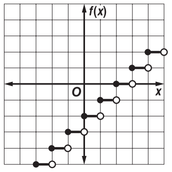
9-7 Skills Practice

Special Functions

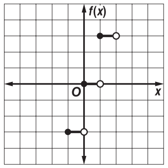
Graph each function. State the domain and range.

1. $f(x) = [\![x - 2]\!]$



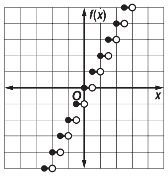
D = {all real numbers};
R = {all integers}

2. $f(x) = 3[\![x]\!]$



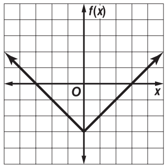
D = {all real numbers};
R = {all integers evenly divisible by 3}

3. $f(x) = [\![2x]\!]$



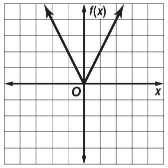
D = {all real numbers};
R = {all integers}

4. $f(x) = |x| - 3$



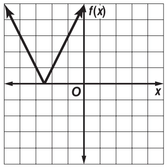
D = {all real numbers};
R = {y | y ≥ –3}

5. $f(x) = |2x|$



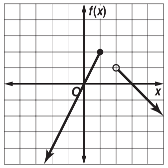
D = {all real numbers};
R = {y | y ≥ 0}

6. $f(x) = |2x + 5|$



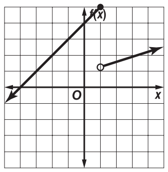
D = {all real numbers};
R = {y | y ≥ 0}

7. $f(x) = \begin{cases} 2x & \text{if } x \le 1 \\ -x + 3 & \text{if } x > 2 \end{cases}$



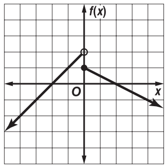
D = {x | x ≤ 1, x > 2};
R = {y | y ≤ 2}

8. $f(x) = \begin{cases} x + 4 & \text{if } x \le 1 \\ 0.25x + 1 & \text{if } x > 1 \end{cases}$



D = {all real numbers};
R = {all real numbers}

9. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ -0.5x + 1 & \text{if } x \ge 0 \end{cases}$



D = {all real numbers};
R = {y | y < 2}

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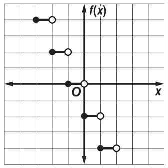
9-7 Practice

Special Functions

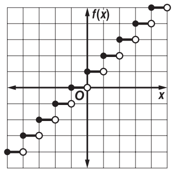
Graph each function. State the domain and range.

1. $f(x) = -2[\![x + 1]\!]$



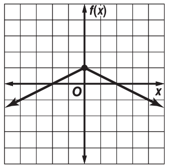
D = {all real numsbers};
R = {all multiples of 2}

2. $f(x) = [\![x + 3]\!] - 2$



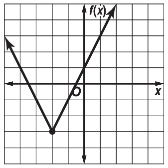
D = {all real numsbers};
R = {all integers}

3. $f(x) = -\left|\frac{1}{2}x\right| + 1$



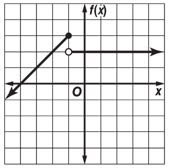
D = {all real numbers};
R = {f(x) | f(x) ≤ 1}

4. $f(x) = |2x + 4| - 3$



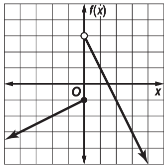
D = {all real numbers};
R = {f(x) | f(x) ≥ −3}

5. $f(x) = \begin{cases} 2 & \text{if } x > -1 \\ x + 4 & \text{if } x \le -1 \end{cases}$



D = {all real numbers};
R = {f(x) | f(x) ≤ 3}

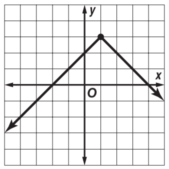
6. $f(x) = \begin{cases} -2x + 3 & \text{if } x > 0 \\ \frac{1}{2}x - 1 & \text{if } x \le 0 \end{cases}$



D = {all real numbers};
R = {f(x) | f(x) < 3}

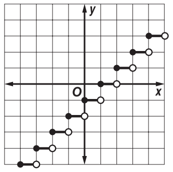
Determine the domain and range of each function.

7.



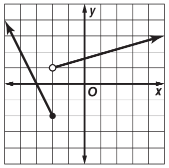
D = {all real numbers};
R = {y | y ≤ 3}

8.



D = {all real numbers};
R = {all integers}

9.

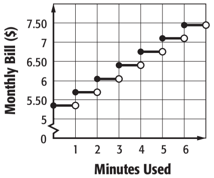


D = {all real numbers};
R = {y | y ≥ −2}

10. **CELL PHONES** Jacob's cell phone service costs \$5 each month plus \$0.35 for each minute he uses. Every fraction of a minute is rounded up to the next minute.

a. Draw a graph to represent the cost of using the cell phone.

b. What is Jacob's monthly bill if he uses 124.8 minutes?
\$48.75



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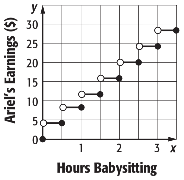
9-7 Word Problem Practice

Special Functions

1. **BABYSITTING** Ariel charges \$4 per hour as a babysitter. She rounds every fraction of an hour up to the next half-hour. Draw a graph to represent Ariel's total earnings y after x hours.



2. **SHIPPING** A package delivery service determines rates for express shipping by the weight of a package, with every fraction of a pound rounded up to the next pound. The table shows the cost of express shipping for packages between 1 and 9 pounds. Write a piecewise-linear function representing the cost to ship a package between 1 and 9 pounds. State the domain and range.

Weight (pounds)	Rate (dollars)
1	17.40
2	19.30
3	22.40
4	25.50
5	28.60
6	31.70
7	34.80
8	37.90
9	41.00

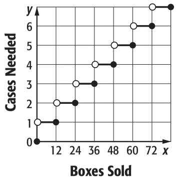
f(x) = 3.1[\![x−1]\!] + 16.2;

D = {x | 1 < x < 9}

R = {f(x) | 17.40 ≤ f(x) ≤ 41.00}

3. **DISEASE PREVENTION** Body Mass Index (BMI) is used by doctors to determine weight categories that may lead to health problems. According to the Centers for Disease Control and Prevention, 21.7 is a healthy BMI for adults. If the BMI differs from the desired 21.7 by more than x, there may be health risk involved. Write an equation that can be used to find the highest and lowest BMI for a healthy adult. Then solve if $x = 3.2$.
$y = |x - 21.7|$; 18.5

4. **FUNDRAISING** Students are selling boxes of cookies at a fund-raiser. The boxes of cookies can only be ordered by the case, with 12 boxes per case. Draw a graph to represent the number of cases needed y when x boxes of cookies are sold.



5. **WAGES** Kelly earns \$8 per hour the first 8 hours she works in a day and \$11.50 per hour each hour thereafter.

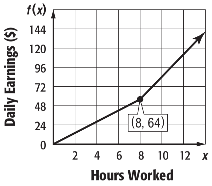
a. Organize the information into a table. Include a column for hours worked x, and a column for daily earnings $f(x)$.

x	0	4	8	12	16
f(x)	0	32	64	110	156

b. Write the piecewise equation describing Kelly's daily earnings $f(x)$ for x hours.

$$f(x) = \begin{cases} 8x \text{ if } x \le 8 \\ 64 + 11.5(x - 8) \text{ if } x > 8 \end{cases}$$

c. Draw a graph to represent Kelly's daily earnings.



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9-7 Enrichment

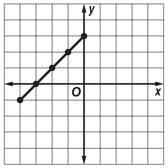
Parametric Equations

A **parametric equation** is a pair of functions $x = f(t)$ and $y = g(t)$ that describe both the x- and y-coordinates for the graph as a whole. Parametric functions allow the drawing of many complex curves and figures.

Example **Graph the parametric function given by $x = t - 2$ and $y = t + 1$ for $-2 \le t \le 2$.**

Step 1 Create a table of values for $-2 \le t \le 2$ by evaluating x and y for each value of t.

t	–2	–1	0	1	2
x	–4	–3	–2	–1	0
y	–1	0	1	2	3



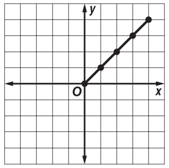
Step 2 Plot the points.

Step 3 Draw a line to connect the points between $-2 \le t \le 2$.

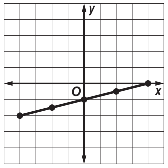
Exercises

Graph each pair of parametric equations over the given range of t values.

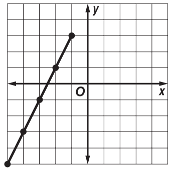
1. $x = t + 2$
$y = t + 2$
$-2 \le t \le 2$



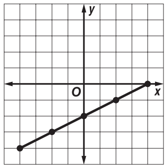
2. $x = 2t$
$y = \frac{1}{2}t - 1$
$-2 \le t \le 2$



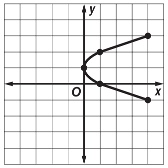
3. $x = t - 3$
$y = 2t - 1$
$-2 \le t \le 2$



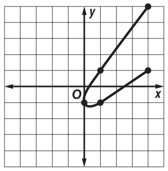
4. $x = 4t$
$y = 2t - 2$
$-1 \le t \le 1$



5. $x = t^2$
$y = t + 1$
$-2 \le t \le 2$



6. $x = t^2$
$y = t^2 - t - 1$
$-2 \le t \le 2$



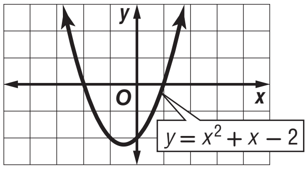
Chapter 9 Assessment Answer Key

Quiz 1 (Lessons 9-1 through 9-3)
Page 49

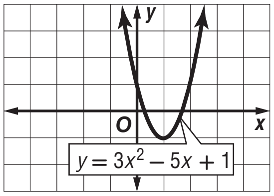
1. D = {all real numbers}, R = $\{y \mid y \geq -4\}$

2. $x = -2$; $(-2, 9)$; maximum

3. $-2, 1$



4. $0 < x < 1, 1 < x < 2$



5. C

Quiz 2 (Lessons 9-4 and 9-5)
Page 49

1. $\frac{81}{4}$

2. $2, -10$

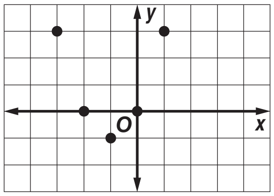
3. $3.4, -2.7$

4. D

5. -55, no real solutions

Quiz 3 (Lesson 9-6)
Page 50

1.



quadratic

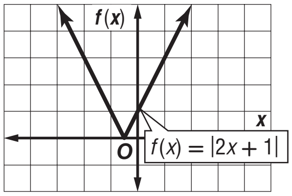
2. exponential; $y = 9 \cdot \left(\frac{1}{3}\right)^x$

3. C

Quiz 4 (Lesson 9-7)
Page 50

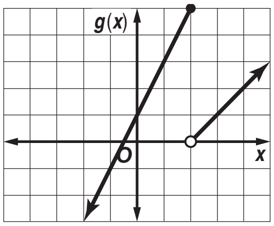
1. 0

2.



3. D: {all real numbers}; R: $\{y|y \geq 0\}$

4. D



5.

Mid-Chapter Test
Page 51

1. C

2. F

3. B

4. H

5. B

6. $-1, 8$

7. $0 < x < 1$, $4 < x < 5$

8. $-2 \pm 2\sqrt{6}$

9. $\frac{-7 \pm \sqrt{193}}{4}$

10. height is 2.4 in.; base is 6.4 in.

Chapter 9 Assessment Answer Key

Vocabulary Test
Page 52

1. translation
2. parabola
3. axis of symmetry
4. greatest integer function
5. Quadratic Formula
6. vertex
7. quadratic function
8. double root
9. Sample answer: The axis of symmetry of a parabola is the line that divides a parabola into two matching halves.
10. Sample answer: The discriminant is the expression, $b^2 - 4ac$, that is under the radical sign in the quadratic formula.

Form 1
Page 53

1. D
2. G
3. C
4. F
5. D
6. H
7. B
8. H

Page 54

9. B
10. J
11. A
12. G
13. B
14. F
15. D
16. H
17. C
18. F
19. A
20. G

B: 1 real solution

Chapter 9 Assessment Answer Key

Form 2A

Page 55

1. C
2. G
3. B
4. J
5. C
6. F
7. A
8. J

Page 56

9. D
10. H
11. D
12. H
13. A
14. H
15. D
16. F
17. C
18. G
19. A
20. F

B: $x = 1$

Form 2B

Page 57

1. B
2. F
3. B
4. H
5. D
6. F
7. C

Page 58

8. G
9. B
10. G
11. A
12. H
13. D
14. F
15. D
16. F
17. B
18. F
19. C
20. G

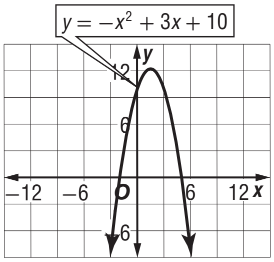
B: $-16, 16$

Chapter 9 Assessment Answer Key

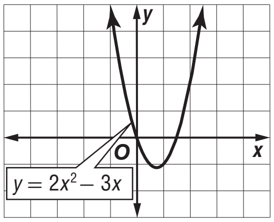
Form 2C

Page 59

1.

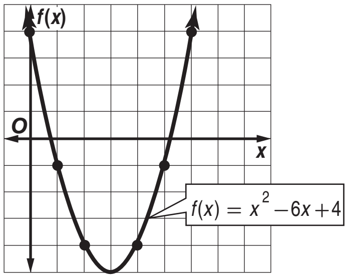


2.



3. $x = 2$; $(2, -5)$; minimum

4.



$0 < x < 1, 5 < x < 6$

5. −2, 3

6. 2, −2.5

7. 4, 7

8. shifted 2 units down

9. reflected about the *x*-axis and shifted 6 units up

Page 60

10. 2.5, −4

11. 0.8, 11.2

12. 0.6, −0.3

13. −9.4, −14.6

14. 148; 2 real solutions

15. 0; 1 real solution

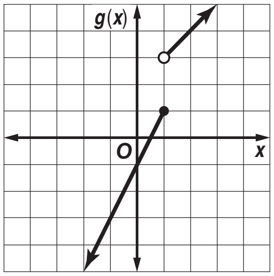
16. 3.8; 8.8

17. quadratic; $y = 4x^2$

18. exponential; $y = 2(3)^x$

19. D: {all real numbers}; R: $\{y \mid y \geq -3\}$

20.



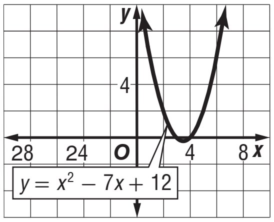
B: about −1.2 and −0.1

Chapter 9 Assessment Answer Key

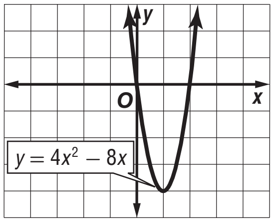
Form 2D

Page 61

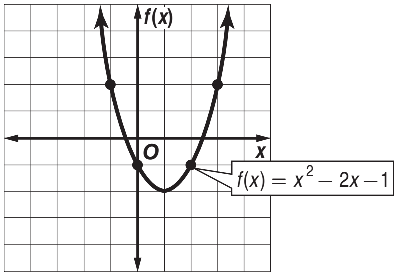
1.



2.



3. $x = 1$; $(1, -3)$; maximum



4. $-1 < x < 0$, $2 < x < 3$

5. −1, 5

6. 3, −9

7. 4, 9

8. translated up 5 units

9. reflected across the *x*-axis and translated down 3 units

Page 62

10. $-6, \frac{3}{2}$

11. −14.2, −0.8

12. $-\frac{3}{4}, \frac{2}{3}$

13. −1.4, 15.4

14. −56; no real solutions

15. 0; 1 real solution

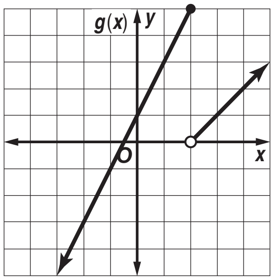
16. length is 8.7 in.; width is 5.7 in.

17. linear, $y = 3x - 1$

18. exponential, $y = 7 \cdot 2^n$

19. D: {all real numbers}; R: $\{y \mid y \geq -3\}$

20.



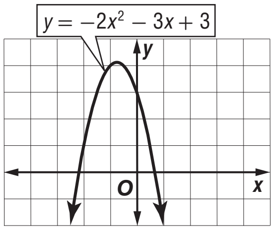
B: $\frac{1}{2}$

Chapter 9 Assessment Answer Key

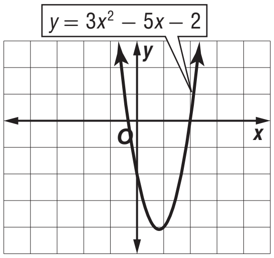
Form 3

Page 63

1.



2.



3. $x = \frac{5}{2}$; $\left(\frac{5}{2}, -\frac{109}{4}\right)$; minimum

4. $x = 4$

5. −3, −3

6. $-\frac{1}{4}, \frac{2}{3}$

7. $\frac{1}{3}, -\frac{5}{2}$

8. compression of $f(x) = x^2$, reflected over the x-axis, translated up 4 units

9. dilation of $f(x) = x^2$, reflected over the x-axis, translated up $\frac{5}{6}$ unit

10. −4, −2

11. −0.3, −9.7

12. no real solutions

Page 64

13. $-\frac{2}{3}, \frac{5}{7}$ or −0.7, 0.7

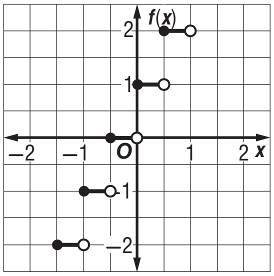
14. −0.6, 3.1

15. 0 times

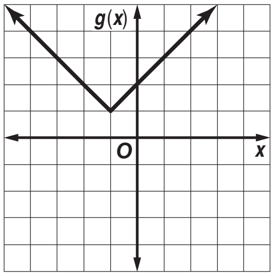
16. −18 or 18

17. 4

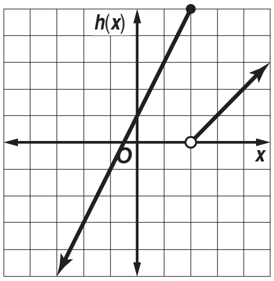
18.



19.



20.



B: $\frac{-7 \pm \sqrt{49 + 16a}}{2a}$

Chapter 9 Assessment Answer Key

Page 65, Extended-Response Test
Scoring Rubric

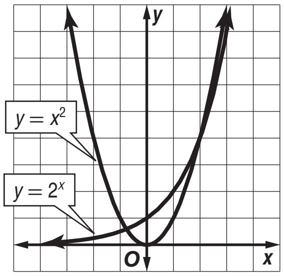
Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 9 Assessment Answer Key

Page 65, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A29, the following sample answers may be used as guidance in evaluating extended-response assessment items.

1. Sample answer: $f(x) = x^2$; $f(x) = 2^x$



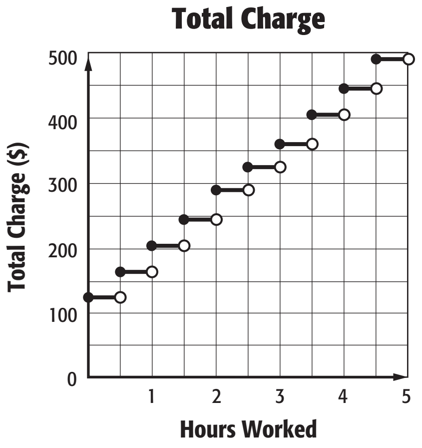
The domains of the two functions are the same, all real numbers. The range of $f(x) = x^2$ is $y \geq 0$, while the range of $f(x) = 2^x$ is $y > 0$.
The quadratic function has a vertical line of symmetry, $x = 0$, and is symmetrical about that line. However, the exponential function has no symmetry.

2a. Sample answer: $x^2 + x + 1 = 0$; -3; The student should explain that the discriminant is the expression under the radical sign in the Quadratic Formula. Since there is no real square root of a negative number, there are no real roots of an equation whose discriminant is negative.

2b. Sample answer: $x^2 + 2x + 1 = 0$; 0; The student should explain that zero has only one square root. Thus, when the determinant is zero the Quadratic Formula yields only one value when you simplify the numerator of the formula.

2c. Sample answer: $x^2 + x - 1 = 0$; Since this quadratic equation does not factor and the roots are not integers, completing the square would be the better method to use to solve the equation.

3a. $C(h) = 125 + 40[\![h]\!]$



3b. \$205

3c. $3 \leq h < 3.5$

Chapter 9 Assessment Answer Key

Standardized Test Practice

Page 66

1. Ⓐ ● Ⓒ Ⓓ

2. ● Ⓖ Ⓗ Ⓙ

3. ● Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ ● Ⓙ

5. Ⓐ Ⓑ Ⓒ ●

6. Ⓕ Ⓖ ● Ⓙ

7. Ⓐ Ⓑ ● Ⓓ

8. Ⓕ ● Ⓗ Ⓙ

9. Ⓐ Ⓑ Ⓒ ●

10. Ⓕ Ⓖ Ⓗ ●

Page 67

11. ● Ⓑ Ⓒ Ⓓ

12. Ⓕ ● Ⓗ Ⓙ

13. Ⓐ Ⓑ Ⓒ ●

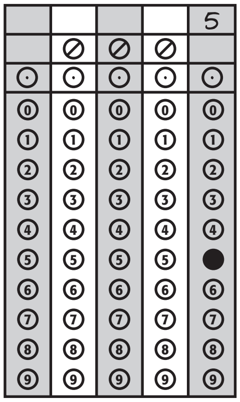
14. Ⓕ ● Ⓗ Ⓙ

15. Ⓐ ● Ⓒ Ⓓ

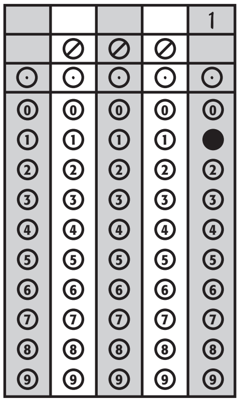
16. ● Ⓖ Ⓗ Ⓙ

17. ● Ⓑ Ⓒ Ⓓ

18.



19.



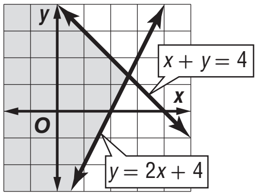
Answers

Standardized Test Practice
Page 68

20. $y = 6x - 8$

21. $\{y \mid y \geq 5\}$

22.



23. $-4x^2y + 9xy - 8y^2$

24. $9a^4 - 4$

25. $(x + 7)(x + 5)$

26. $(2m + 5)(m + 3)$

27. $\{-12, 12\}$

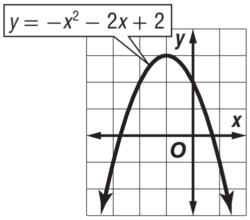
28. 10 s and about 5.6 s

29. about 10,019,394 people

30a. $x = -1$

30b. $(-1, 3)$; maximum

30c.



Chapter 9 Assessment Answer Key

Unit 3 Test

Page 69

1. $20x^4y^6$

2. $\frac{b^6}{144g^6}$

3. 1

4. 0.00255, 2.55×10^{-3}

5. about 1,412,416 people

6. about $61,032

7. $1531.51

8. $a_n = -5 \cdot (-3)^{n-1}$

9. $a_1 = 14$,
$a_n = a_{n-1} - 5$,
$n \geq 2$

10. $3a + 2b - 4c$

11. 2

12. $p^2 + 2p - 15$

13. $a^3 - a^2 - 13a - 14$

14. $25t^2 + 10tr + r^2$

15. $9k^2 - 1$

Page 70

16. $4pr(p - 4 + 2r)$

17. $(m - 2)(m - 9)$

18. $(3a - 4)(a - 3)$

19. $3(3r - 2t)(3r + 2t)$

20. $2p - 1$

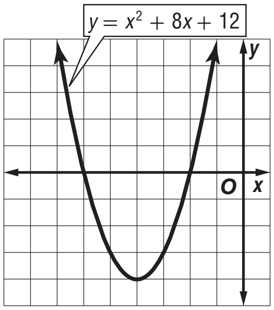
21. $\left\{-\frac{1}{2}, 0\right\}$

22. $\{4, 9\}$

23. $\left\{-\frac{2}{5}, 7\right\}$

24. $\{-9\}$

25. $x = -4$; $(-4, -4)$



26. 100

27. -8

28. $\{-1.8, 1.1\}$

29. exponential,
$y = 6 \cdot 2^x$

30.