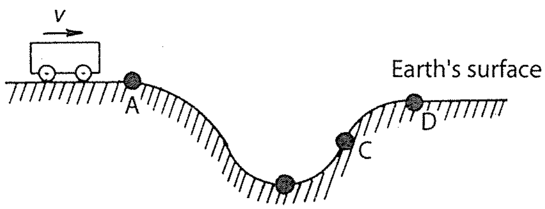
31. As an object slides across a horizontal surface, the gravitational potential energy of the object
(1) decreases (2) increases (3) remains the same

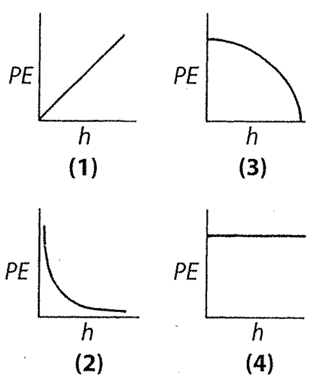
32. The following diagram represents a cart traveling with initial speed *v* from left to right along a frictionless surface.



At which point is the gravitational potential energy of the cart least? (1) A (2) B (3) C (4) D

33. Determine the gain in potential energy of a 5.00-kilogram mass as it is raised 2.00 meters from the surface of Earth.

34. Which graph best represents the relationship between potential energy *PE* and height above the ground *h* for a freely falling object released from rest?

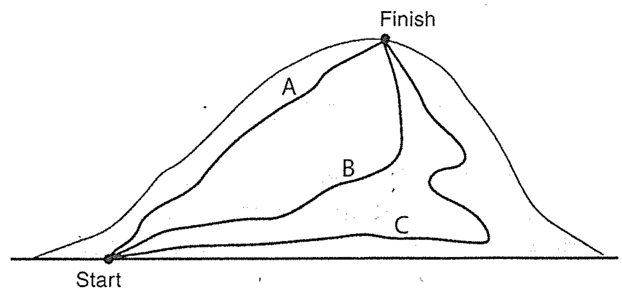


35. At the top of an incline, a 0.50-kilogram sphere has a potential energy of 6.0 joules. After rolling halfway down the incline, the sphere's potential energy is
(1) 0.0 J (2) 6.0 J (3) 3.0 J (4) 12 J

36. A ball is thrown upward from Earth's surface. While the ball is rising, its gravitational potential energy is
(1) decreasing (2) increasing (3) remaining the same

37. When a 5-kilogram mass is lifted from the ground to a height of 10 meters, the gravitational potential energy of the mass is increased by approximately
(1) 0.5 J (2) 2 J (3) 50 J (4) 500 J

38. Three people of equal mass climb a mountain using paths A, B, and C shown in the following diagram.



Along which path(s) does a person gain the greatest amount of gravitational potential energy from start to finish? (1) A only (2) B only (3) C only
(4) The gain is the same along all paths.

Elastic Potential Energy

The energy stored in a spring, when work is done in compressing or stretching it, is called **elastic potential energy.** The **compression** or **elongation** of a spring is the change in spring length from its equilibrium position when a force is applied to it. Provided the elastic limit of the spring is not exceeded, the compression or elongation of a spring is directly proportional to the applied force. This relationship, called Hooke's law, is given by the following equation:

$$F_s = kx$$

In the equation, k is the **spring constant,** the constant of proportionality between the applied force F_s and the compression or elongation x of the spring. If F_s is in newtons and x is in meters, then k is in newtons per meter. The SI unit for the spring constant is the newton/meter, N/m.

A common laboratory activity is to vary the force applied to a spring and measure the resulting elongation or compression. Force is the independent variable and change in spring length is the dependent variable. However, force is often indicated on the vertical axis and change in spring length on the horizontal axis when the data from the experiment is graphed. If a graph of F_s versus x is plotted for the data collected for a given spring, the slope of the line of best fit is equal to the spring constant for that spring. For an ideal spring, the line is straight and passes through the origin. A stiff spring has a larger value of k than a weak spring.

SAMPLE PROBLEM

In an experiment, a student varied the force applied to a spring and measured the resulting elongation. The table shows the average elongation for three trials with each force.

Force (N)	**Average Elongation** (m)
0.00	0.000
1.00	0.040
2.00	0.075
3.00	0.120
4.00	0.165
5.00	0.200

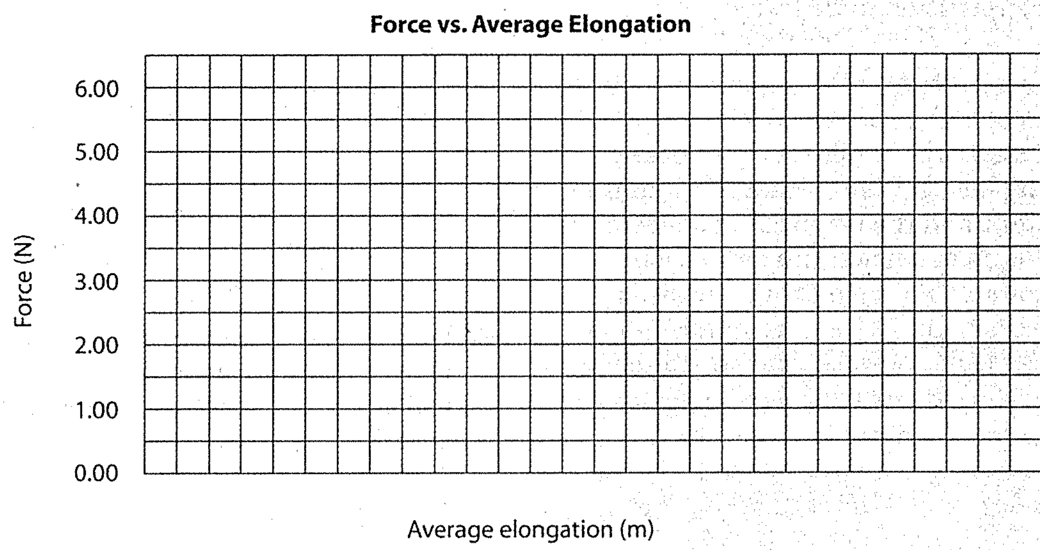
Using the information in the data table and the grid provided, complete (1) through (4).

(1) Mark an appropriate scale on the axis labeled "Average elongation (m)".

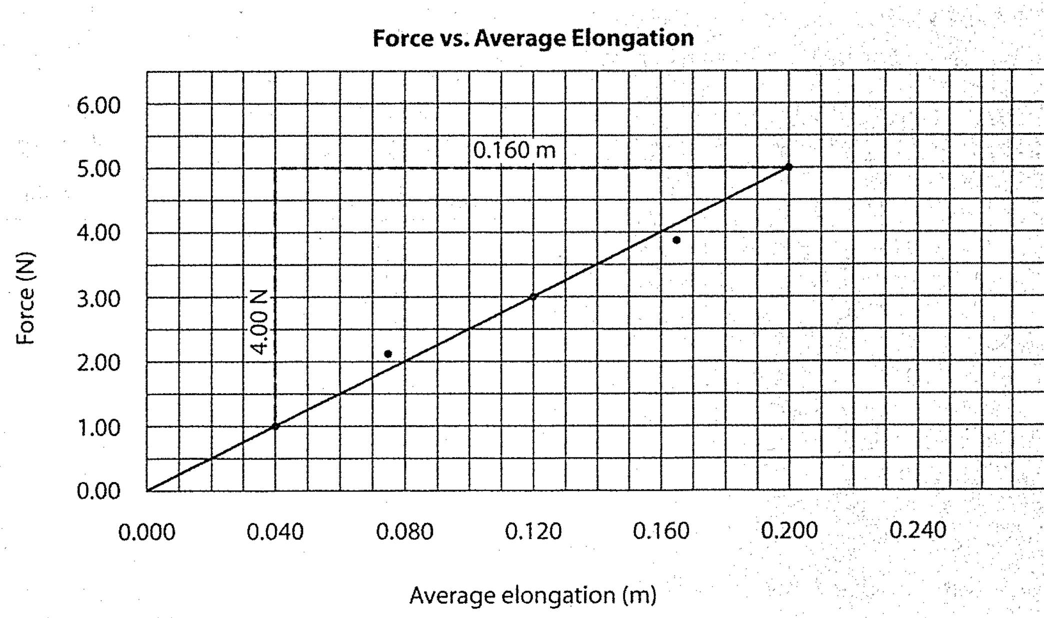
(2) Plot the data points.

(3) Draw the line of best fit.

(4) Use your line to determine the spring constant k.



Solution: The spring constant k is the slope of the line.



$$k = \frac{\Delta F}{\Delta x} = \frac{4.00\text{ N}}{0.160\text{ m}} = 25.0\text{ N/m}$$

Potential Energy of a Spring

When no force is applied to a spring, there is no change in spring length from the equilibrium position. That is, when $F_s = 0$ N, $x = 0$ m. According to Hooke's law, as F_s increases, x increases. Because F_s increases uniformly from 0 to kx, the *average* applied force equals $\frac{1}{2}kx$. The work done in stretching the spring is equal to the product of the *average* force $\overline{F}_s$ and the elongation x.

$$W = \overline{F}_s x = \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2$$

Because the work done on the spring is equal to the spring's elastic potential energy PE_s, the equation can be rewritten in this way:

$$PE_s = \frac{1}{2}kx^2$$

The spring constant k is in newtons per meter, the change in spring length from the equilibrium position x is in meters, and the potential energy stored in the spring PE_s is in newton · meters, or joules. As the following Sample Problem shows, the area under an F_s versus x curve yields a number equal to the number of joules of work done in stretching the spring, and thus, the potential energy stored in the spring.

SAMPLE PROBLEM

Determine the potential energy stored in the spring in the previous Sample Problem when a force of 2.50 newtons is applied to it.

Solution: Identify the known and unknown values.

Known	*Unknown*
$F_s = 2.50$ N	$PE_s = ?$ J
$k = 25.0$ N/m	

Find A_Δ, the area under the curve in the previous Sample Problem. At $F = 2.50$ N the area is a triangle with height h equal to 2.50 N and base b equal to 0.100 m. Write the formula for the area of a triangle.

$$A_\Delta = \frac{1}{2}bh$$

Substitute the known values and solve.

$$A_\Delta = PE_s = \frac{1}{2}(0.100 \text{ m})(2.50 \text{ N})$$

$$PE_s = 0.125 \text{ J}$$

An alternative solution is to use the relationship

$$F_s = kx$$

Solve the equation for x.

$$x = \frac{F_s}{k}$$

Substitute the known values and solve.

$$x = \frac{2.50 \text{ N}}{25.0 \text{ N/m}} = 0.100 \text{ m}$$

Write the formula that relates PE_s and x.

$$PE_s = \frac{1}{2}kx^2$$

Substitute the known values and solve.

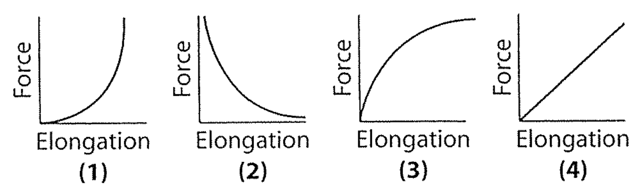
$$PE_s = \frac{1}{2}(25.0 \text{ N})(0.100 \text{ m})^2$$

$$PE_s = 0.125 \text{ J}$$

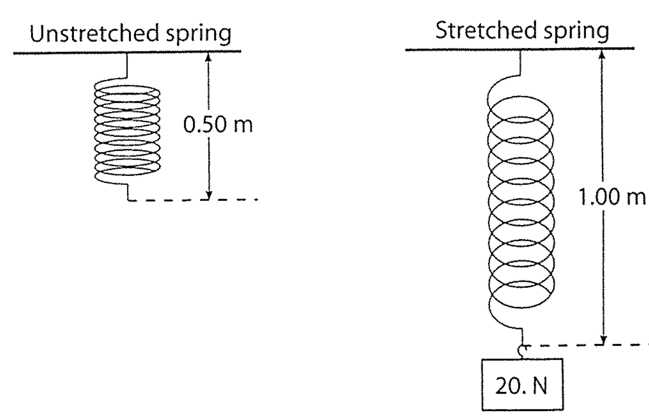
Review Questions

39. A spring has a spring constant of 25 newtons per meter. Determine the magnitude of the minimum force required to stretch the spring 0.25 meter from its equilibrium position.

40. Which graph best represents the relationship between the force applied to a spring and the elongation of the spring? (Assume the spring's elastic limit has not been reached.)



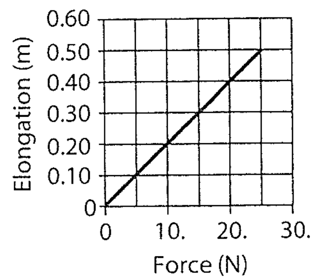
41. A 20.-newton weight is attached to a spring causing it to stretch, as shown in the following diagram.



What is the spring constant of this spring?
(1) 0.050 N/m (2) 0.25 N/m (3) 20. N/m (4) 40. N/m

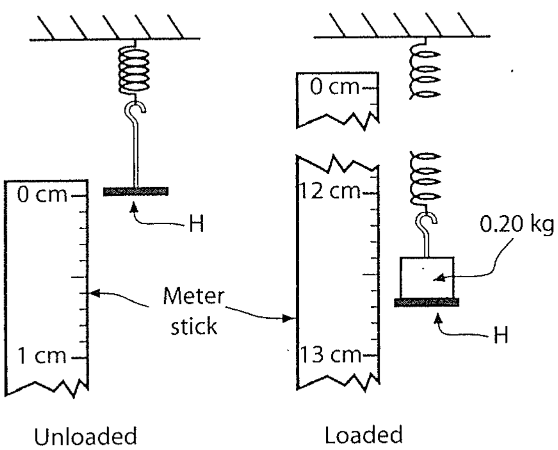
42. The graph that follows shows the relationship between the elongation of a spring and the force applied to the spring causing it to stretch.

Elongation vs. Applied Force



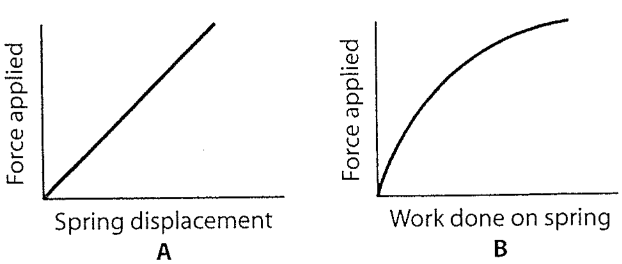
What is the spring constant for this spring?
(1) 0.020 N/m (2) 2.0 N/m (3) 25 N/m (4) 50. N/m

43. A mass hanger is attached to a spring, as shown in the following diagrams.



What is the magnitude of the displacement of the mass hanger H after a 0.20-kilogram mass is loaded on it? (Assume the hanger is at rest in both positions.)

44. Graphs A and B represent the results of applying an increasing force to stretch a spring. The spring did not exceed its elastic limit.



The spring constant can be represented by the (1) slope of graph A (2) slope of graph B (3) reciprocal of the slope of graph A (4) reciprocal of the slope of graph B

45. Force F is applied to a spring causing it to stretch a distance x. If force $2F$ is applied to the spring and the elasticity of the spring is not exceeded, the spring will stretch a distance (1) x (2) $2x$ (3) $\frac{x}{2}$ (4) $\frac{x}{4}$

46. A spring having a spring constant k is cut in half. Each of the newly formed springs has a spring constant that is equal to (1) k (2) $2k$ (3) $\frac{k}{2}$ (4) $4k$

47. A force is applied to a spring causing it to stretch. If the applied force is halved, the potential energy stored in the spring will be (1) halved (2) doubled (3) quartered (4) quadrupled

48. If the distance a spring is stretched is doubled and the elastic limit is not exceeded, the potential energy stored in the spring is (1) halved (2) doubled (3) quartered (4) quadrupled

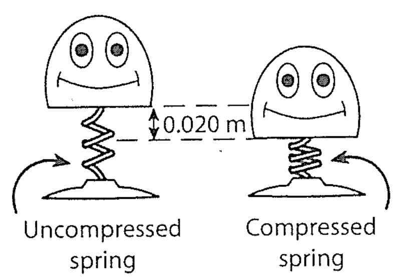
49. When a spring is stretched 0.200 meter from its equilibrium position, it possesses a potential energy of 10.0 joules. What is the spring constant for this spring? (1) 100. N/m (2) 125 N/m (3) 250. N/m (4) 500. N/m

50. A spring has a spring constant of 120 newtons per meter. Determine the potential energy stored in the spring as it is stretched 2.0 centimeters.

51. A force of 0.2 newton is needed to compress a spring a distance of 0.02 meter. The potential energy stored in this compressed spring is (1) 8×10^{-5} J (2) 2×10^{-3} J (3) 2×10^{-5} J (4) 4×10^{-5} J

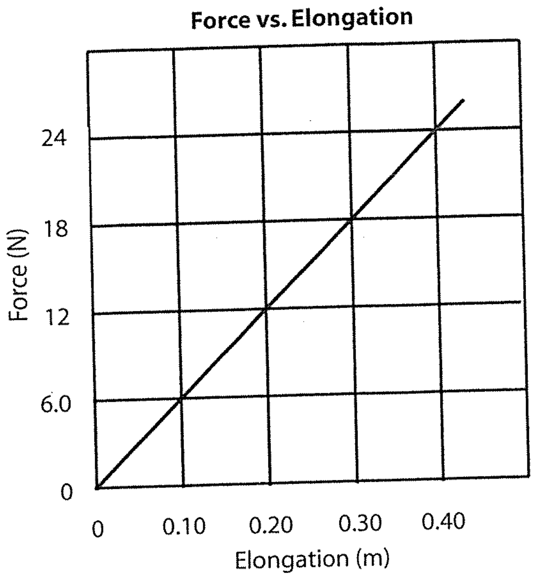
52. A spring of negligible mass with a spring constant of 2.0×10^2 newtons per meter is stretched 0.20 meter. How much potential energy is stored in the spring? (1) 8 J (2) 8.0 J (3) 4 J (4) 4.0 J

53. In the diagram below, a child compresses the spring in a pop-up toy 0.020 meter.



If the spring has a spring constant of 340 newtons per meter, how much energy is being stored in the spring? (1) 0.068 J (2) 0.14 J (3) 3.4 J (4) 6.8 J

Base your answers to questions 54 through 56 on the following graph, which represents the relationship between the force applied to a spring and its elongation.



54. How much work must be done to stretch the spring 0.40 meter? (1) 4.8 J (2) 6.0 J (3) 9.8 J (4) 24 J

55. Determine the spring constant k for the spring.

56. On the grid, sketch a line that represents the relationship between applied force and elongation for a stiffer spring.

Kinetic Energy

When a moving object strikes another object and displaces it, the moving object exerts a force on the second object and does work on it. The moving object possesses energy due to its motion. The energy an object possesses due to its motion is called **kinetic energy.** The equation for kinetic energy is $KE = \frac{1}{2}mv^2$ and can be derived from the definition of work and Newton's second law.

$W = Fd$ and $F = ma$

$W = mad$ where $a = \frac{v}{t}$ from rest, $d = \bar{v}t$, and

$\bar{v} = \frac{v}{2}$ from rest.

$$W = m \cdot \frac{v}{t} \cdot \bar{v}t = m \cdot \frac{v}{t} \cdot \frac{v}{2} \cdot t$$

$$W = \frac{1}{2}mv^2$$

The net work done in accelerating an object from rest to some speed is equal to the kinetic energy of the object. The following equation describes the relationship:

$$KE = \frac{1}{2}mv^2$$

Mass m is in kilograms, velocity or speed v is in meters per second, and kinetic energy KE is in kilogram·meter^2/second^2 or joules.

SAMPLE PROBLEM

How much kinetic energy is possessed by a 2.7-kilogram cart traveling at 1.5 meter per second?

Solution: Identify the known and unknown values.

Known	*Unknown*
$m = 2.7$ kg	$KE = ?$ J
$v = 1.5$ m/s	

Write the equation for kinetic energy.

$$KE = \frac{1}{2}mv^2$$

Substitute the known values and solve.

$$KE = \frac{1}{2}(2.7\text{ kg})(1.5\text{ m/s})^2 = 3.0\text{ J}$$

Note: If the weight of the cart had been given, it would have been necessary to use the formula

$g = \frac{F_g}{m}$ to determine the cart's mass.

Review Questions

57. If the speed of a car is doubled, its kinetic energy is (1) halved (2) doubled (3) quartered (4) quadrupled

58. A 1.0×10^3-kilogram car is moving at a constant speed of 4.0 meters per second. What is the kinetic energy of the car? (1) 1.6×10^3 J (2) 2.0×10^4 J (3) 8.0×10^3 J (4) 4.0×10^3 J

59. A 3.0-kilogram cart possesses 96 joules of kinetic energy. Determine the speed of the car.

60. A cart of mass m traveling at speed v has kinetic energy KE. If the mass of the cart is doubled and the speed is halved, the kinetic energy of the cart will be (1) half as great (2) twice as great (3) one-fourth as great (4) four times as great