

**Chapter  
5****Maintaining Mathematical Proficiency**

Simplify the expression.

1.  $c \cdot c^9$

2.  $\frac{q^{12}}{q^4}$

3.  $\frac{x^3}{x^4 \cdot x^5}$

4.  $\frac{d^2}{d} \cdot 8d^5$

5.  $\left(\frac{4x^3}{2y^4}\right)^2$

6.  $\left(\frac{m^8 \cdot m^3}{n \cdot m}\right)^3$

Solve the literal equation for  $y$ .

7.  $x + y = 1$

8.  $-3y + \frac{1}{2}x = -6$

9.  $24x + 5y = 74$

10.  $6xy + 3y = -72$

11.  $10x - 5xy = 100$

12.  $-\frac{1}{4}x + 8xy = 16$

13. Is  $\left(\frac{x + 3x}{y + 2y}\right)^2 = \left(\frac{x^2 + 3^2x^2}{y^2 + 2^2y^2}\right)$  or is  $\left(\frac{x + 3x}{y + 2y}\right)^2 = \left(\frac{4^2x^2}{3^2y^2}\right)$ ? Explain your reasoning.

**5.1*****n*th Roots and Rational Exponents**

For use with Exploration 5.1

**Essential Question** How can you use a rational exponent to represent a power involving a radical?

**1 EXPLORATION:** Exploring the Definition of a Rational Exponent

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use a calculator to show that each statement is true.

a.  $\sqrt{9} = 9^{1/2}$

b.  $\sqrt{2} = 2^{1/2}$

c.  $\sqrt[3]{8} = 8^{1/3}$

d.  $\sqrt[3]{3} = 3^{1/3}$

e.  $\sqrt[4]{16} = 16^{1/4}$

f.  $\sqrt[4]{12} = 12^{1/4}$

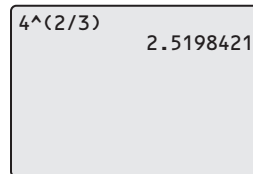
**2 EXPLORATION:** Writing Expressions in Rational Exponent Form

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use the definition of a rational exponent and the properties of exponents to write each expression as a base with a single rational exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

**Sample**

$$\begin{aligned} (\sqrt[3]{4})^2 &= (4^{1/3})^2 \\ &= 4^{2/3} \\ &\approx 2.52 \end{aligned}$$



4^(2/3) 2.5198421

a.  $(\sqrt{5})^3$

b.  $(\sqrt[4]{4})^2$

c.  $(\sqrt[3]{9})^2$

d.  $(\sqrt[5]{10})^4$

e.  $(\sqrt{15})^3$

f.  $(\sqrt[3]{27})^4$

**5.1** *n*th Roots and Rational Exponents (continued)**3** **EXPLORATION:** Writing Expressions in Radical Form

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use the properties of exponents and the definition of a rational exponent to write each expression as a radical raised to an exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

**Sample**  $5^{2/3} = (5^{1/3})^2 = (\sqrt[3]{5})^2 \approx 2.92$

a.  $8^{2/3}$

b.  $6^{5/2}$

c.  $12^{3/4}$

d.  $10^{3/2}$

e.  $16^{3/2}$

f.  $20^{6/5}$

**Communicate Your Answer**

4. How can you use a rational exponent to represent a power involving a radical?

5. Evaluate each expression *without* using a calculator. Explain your reasoning.

a.  $4^{3/2}$

b.  $32^{4/5}$

c.  $625^{3/4}$

d.  $49^{3/2}$

e.  $125^{4/3}$

f.  $100^{6/3}$

**5.1****Notetaking with Vocabulary**

For use after Lesson 5.1

In your own words, write the meaning of each vocabulary term.

$n$ th root of  $a$

index of a radical

**Core Concepts****Real  $n$ th roots of  $a$** 

Let  $n$  be an integer ( $n > 1$ ) and let  $a$  be a real number.

**$n$  is an even integer.**

$a < 0$  No real  $n$ th roots

$a = 0$  One real  $n$ th root:  $\sqrt[n]{0} = 0$

$a > 0$  Two real  $n$ th roots:  $\pm\sqrt[n]{a} = \pm a^{1/n}$

**$n$  is an odd integer.**

$a < 0$  One real  $n$ th root:  $\sqrt[n]{a} = a^{1/n}$

$a = 0$  One real  $n$ th root:  $\sqrt[n]{0} = 0$

$a > 0$  One real  $n$ th root:  $\sqrt[n]{a} = a^{1/n}$

**Notes:**

**5.1** Notetaking with Vocabulary (continued)**Rational Exponents**

Let  $a^{1/n}$  be an  $n$ th root of  $a$ , and let  $m$  be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$$

**Notes:**

**Extra Practice**

In Exercises 1–3, find the indicated real  $n$ th root(s) of  $a$ .

1.  $n = 3, a = -125$

2.  $n = 2, a = -400$

3.  $n = 6, a = 64$

In Exercises 4–11, evaluate the expression without using a calculator.

4.  $64^{1/2}$

5.  $(-27)^{1/3}$

6.  $32^{7/5}$

7.  $49^{-3/2}$

8.  $(-32)^{3/5}$

9.  $1000^{-2/3}$

10.  $81^{3/4}$

11.  $625^{1/4}$

**5.1** Notetaking with Vocabulary (continued)

In Exercises 12–15, match the equivalent expressions. Explain your reasoning.

12.  $(\sqrt{a})^3$

A.  $a^{-1/3}$

13.  $-\sqrt[3]{a}$

B.  $a^{2/3}$

14.  $(\sqrt[3]{a})^2$

C.  $a^{3/2}$

15.  $\frac{1}{\sqrt[3]{a}}$

D.  $-a^{1/3}$

In Exercises 16–19, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

16.  $6x^3 = -6$

17.  $2(x + 5)^4 = 128$

18.  $x^5 - 32 = -64$

19.  $-\frac{1}{10}x^3 + 100 = 0$

20. The volume of a cube is 1728 cubic inches. What are the dimensions of the cube?

**5.2****Properties of Rational Exponents and Radicals**

For use with Exploration 5.2

**Essential Question** How can you use properties of exponents to simplify products and quotients of radicals?

**1 EXPLORATION: Reviewing Properties of Exponents**

**Work with a partner.** Let  $a$  and  $b$  be real numbers. Use the properties of exponents to complete each statement. Then match each completed statement with the property it illustrates.

Statement	Property
a. $a^{-2} = \underline{\hspace{2cm}}$ , $a \neq 0$	A. Product of Powers
b. $(ab)^4 = \underline{\hspace{2cm}}$	B. Power of a Power
c. $(a^3)^4 = \underline{\hspace{2cm}}$	C. Power of a Product
d. $a^3 \cdot a^4 = \underline{\hspace{2cm}}$	D. Negative Exponent
e. $\left(\frac{a}{b}\right)^3 = \underline{\hspace{2cm}}$ , $b \neq 0$	E. Zero Exponent
f. $\frac{a^6}{a^2} = \underline{\hspace{2cm}}$ , $a \neq 0$	F. Quotient of Powers
g. $a^0 = \underline{\hspace{2cm}}$ , $a \neq 0$	G. Power of a Quotient

**2 EXPLORATION: Simplifying Expressions with Rational Exponents**

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Show that you can apply the properties of integer exponents to rational exponents by simplifying each expression. Use a calculator to check your answers.

a.  $5^{2/3} \cdot 5^{4/3}$

b.  $3^{1/5} \cdot 3^{4/5}$

c.  $(4^{2/3})^3$

d.  $(10^{1/2})^4$

e.  $\frac{8^{5/2}}{8^{1/2}}$

f.  $\frac{7^{2/3}}{7^{5/3}}$

**5.2** Properties of Rational Exponents and Radicals (continued)**3** **EXPLORATION:** Simplifying Products and Quotients of Radicals

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use the properties of exponents to write each expression as a single radical. Then evaluate each expression. Use a calculator to check your answers.

a.  $\sqrt{3} \cdot \sqrt{12}$

b.  $\sqrt[3]{5} \cdot \sqrt[3]{25}$

c.  $\sqrt[4]{27} \cdot \sqrt[4]{3}$

d.  $\frac{\sqrt{98}}{\sqrt{2}}$

e.  $\frac{\sqrt[4]{4}}{\sqrt[4]{1024}}$

f.  $\frac{\sqrt[3]{625}}{\sqrt[3]{5}}$

**Communicate Your Answer**

4. How can you use properties of exponents to simplify products and quotients of radicals?

5. Simplify each expression.

a.  $\sqrt{27} \cdot \sqrt{6}$

b.  $\frac{\sqrt[3]{240}}{\sqrt[3]{15}}$

c.  $(5^{1/2} \cdot 16^{1/4})^2$



**5.2****Notetaking with Vocabulary**

For use after Lesson 5.2

In your own words, write the meaning of each vocabulary term.

simplest form of a radical

conjugate

like radicals

**Core Concepts****Properties of Rational Exponents**Let  $a$  and  $b$  be real numbers and let  $m$  and  $n$  be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

**Notes:**

**5.2** Notetaking with Vocabulary (continued)**Properties of Radicals**Let  $a$  and  $b$  be real numbers and let  $n$  be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

**Notes:****Extra Practice**

In Exercises 1–4, use the properties of rational exponents to simplify the expression.

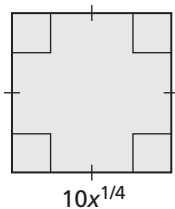
1.  $(2^3 \cdot 3^3)^{-1/3}$

2.  $\frac{10}{10^{-4/5}}$

3.  $\left(\frac{52^5}{4^5}\right)^{1/6}$

4.  $\frac{3^{1/3} \cdot 27^{2/3}}{8^{4/3}}$

5. Find simplified expressions for the perimeter and area of the given figure.



**5.2** Notetaking with Vocabulary (continued)

In Exercises 6–8, use the properties of radicals to simplify the expression.

6.  $\sqrt[6]{25} \cdot \sqrt[6]{625}$

7.  $\frac{\sqrt{343}}{\sqrt{7}}$

8.  $\frac{\sqrt[3]{25} \cdot \sqrt[3]{10}}{\sqrt[3]{2}}$

In Exercises 9–12, write the expression in simplest form.

9.  $\sqrt[7]{384}$

10.  $\sqrt[3]{\frac{5}{9}}$

11.  $\frac{1}{4 - \sqrt{5}}$

12.  $\frac{\sqrt{2}}{1 + \sqrt{6}}$

In Exercises 13–16, write the expression in simplest form. Assume all variables are positive.

13.  $-2\sqrt[3]{5} + 40\sqrt[3]{5}$

14.  $2(1250)^{1/4} - 5(32)^{1/4}$

15.  $\frac{\sqrt[4]{x} \cdot \sqrt[4]{81x}}{\sqrt[4]{16x^{36}}}$

16.  $\frac{21(x^{-3/2})(\sqrt{y})(z^{5/2})}{7^{-1}\sqrt{x}(y^{-1/2})z}$

# 5.3

## Graphing Radical Functions

For use with Exploration 5.3

**Essential Question** How can you identify the domain and range of a radical function?

### 1 EXPLORATION: Identifying Graphs of Radical Functions

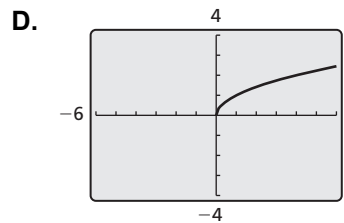
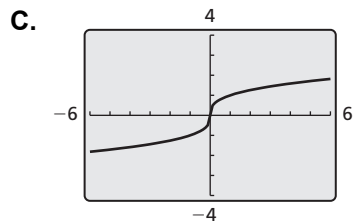
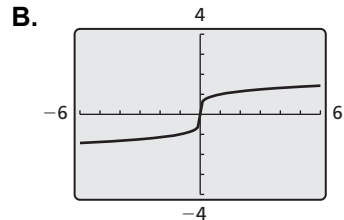
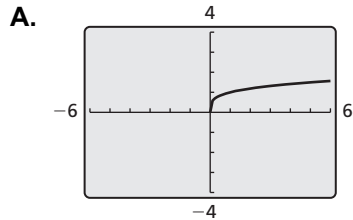
**Work with a partner.** Match each function with its graph. Explain your reasoning. Then identify the domain and range of each function.

a.  $f(x) = \sqrt{x}$

b.  $f(x) = \sqrt[3]{x}$

c.  $f(x) = \sqrt[4]{x}$

d.  $f(x) = \sqrt[5]{x}$



**5.3 Graphing Radical Functions (continued)**

**2 EXPLORATION: Identifying Graphs of Transformations**

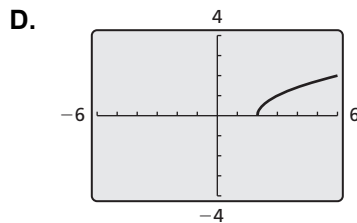
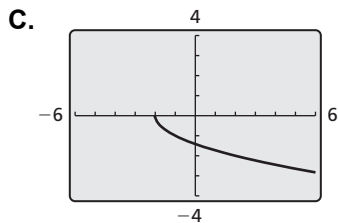
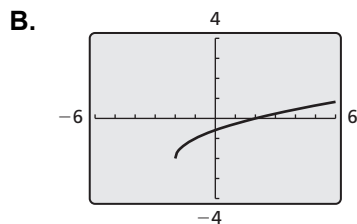
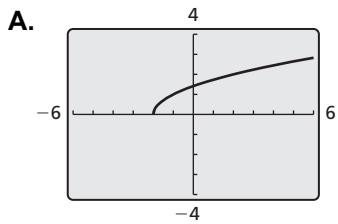
**Work with a partner.** Match each transformation of  $f(x) = \sqrt{x}$  with its graph. Explain your reasoning. Then identify the domain and range of each function.

a.  $g(x) = \sqrt{x + 2}$

b.  $g(x) = \sqrt{x - 2}$

c.  $g(x) = \sqrt{x + 2} - 2$

d.  $g(x) = -\sqrt{x + 2}$



**Communicate Your Answer**

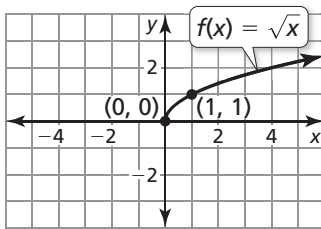
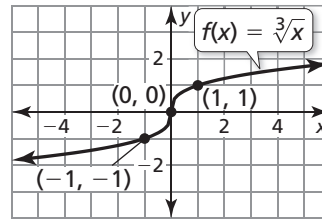
3. How can you identify the domain and range of a radical function?
  
4. Use the results of Exploration 1 to describe how the domain and range of a radical function are related to the index of the radical.

**5.3****Notetaking with Vocabulary**

For use after Lesson 5.3

In your own words, write the meaning of each vocabulary term.

radical function

**Core Concepts****Parent Functions for Square Root and Cube Root Functions**The parent function for the family of square root functions is  $f(x) = \sqrt{x}$ .Domain:  $x \geq 0$ , Range:  $y \geq 0$ The parent function for the family of cube root functions is  $f(x) = \sqrt[3]{x}$ .

Domain and range: All real numbers

**Notes:**

**5.3** Notetaking with Vocabulary (continued)

Transformation	$f(x)$ Notation	Examples
<b>Horizontal Translation</b> Graph shifts left or right.	$f(x - h)$	$g(x) = \sqrt{x - 2}$ 2 units right $g(x) = \sqrt{x + 3}$ 3 units left
<b>Vertical Translation</b> Graph shifts up or down.	$f(x) + k$	$g(x) = \sqrt{x} + 7$ 7 units up $g(x) = \sqrt{x} - 1$ 1 unit down
<b>Reflection</b> Graph flips over $x$ - or $y$ -axis.	$f(-x)$ $-f(x)$	$g(x) = \sqrt{-x}$ in the $y$ -axis $g(x) = -\sqrt{x}$ in the $x$ -axis
<b>Horizontal Stretch or Shrink</b> Graph stretches away from or shrinks toward $y$ -axis.	$f(ax)$	$g(x) = \sqrt{3x}$ shrink by a factor of $\frac{1}{3}$ $g(x) = \sqrt{\frac{1}{2}x}$ stretch by a factor of 2
<b>Vertical Stretch or Shrink</b> Graph stretches away from or shrinks toward $x$ -axis.	$a \cdot f(x)$	$g(x) = 4\sqrt{x}$ stretch by a factor of 4 $g(x) = \frac{1}{5}\sqrt{x}$ shrink by a factor of $\frac{1}{5}$

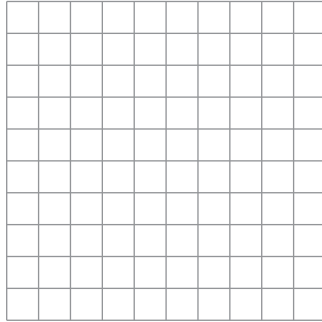
**Notes:**

**5.3** Notetaking with Vocabulary (continued)

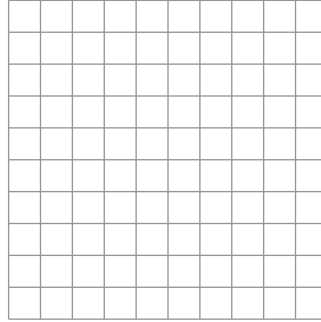
**Extra Practice**

In Exercises 1 and 2, graph the function. Identify the domain and range of each function.

1.  $f(x) = \sqrt[3]{-3x} + 1$



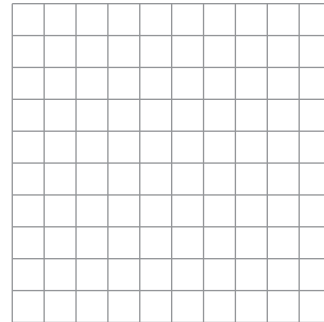
2.  $g(x) = 2(x - 5)^{1/2} - 4$



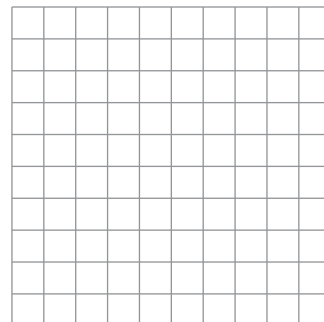
3. Describe the transformation of  $f(x) = \sqrt[4]{2x} + 5$  represented by  $g(x) = -\sqrt[4]{2x} - 5$ .

4. Write a rule for  $g$  if  $g$  is a horizontal shrink by a factor of  $\frac{5}{6}$ , followed by a translation 10 units to the left of the graph of  $f(x) = \sqrt[3]{15x} + 1$ .

5. Use a graphing calculator to graph  $8x = y^2 + 5$ . Identify the vertex and the direction that the parabola opens.



6. Use a graphing calculator to graph  $x^2 = 49 - y^2$ . Identify the radius and the intercepts of the circle.





# 5.4

## Solving Radical Equations and Inequalities

For use with Exploration 5.4

**Essential Question** How can you solve a radical equation?

### 1 EXPLORATION: Solving Radical Equations

**Work with a partner.** Match each radical equation with the graph of its related radical function. Explain your reasoning. Then use the graph to solve the equation, if possible. Check your solutions.

a.  $\sqrt{x-1} - 1 = 0$

b.  $\sqrt{2x+2} - \sqrt{x+4} = 0$

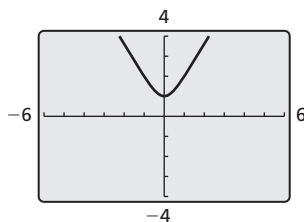
c.  $\sqrt{9-x^2} = 0$

d.  $\sqrt{x+2} - x = 0$

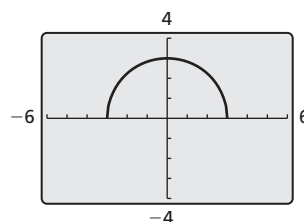
e.  $\sqrt{-x+2} - x = 0$

f.  $\sqrt{3x^2+1} = 0$

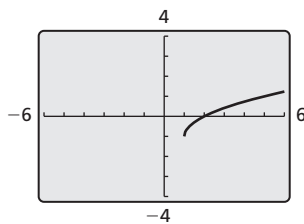
A.



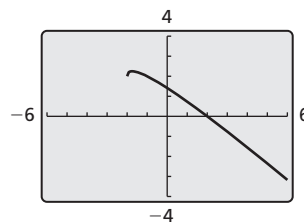
B.



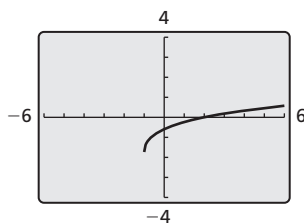
C.



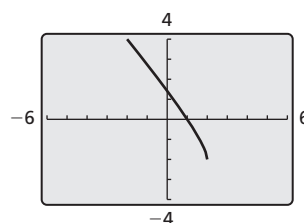
D.



E.



F.



**5.4 Solving Radical Equations and Inequalities (continued)****2 EXPLORATION: Solving Radical Equations**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Look back at the radical equations in Exploration 1. Suppose that you did not know how to solve the equations using a graphical approach.

- a. Show how you could use a *numerical approach* to solve one of the equations. For instance, you might use a spreadsheet to create a table of values.
  
  
  
  
  
  
  
  
  
  
- b. Show how you could use an *analytical approach* to solve one of the equations. For instance, look at the similarities between the equations in Exploration 1. What first step may be necessary so you could square each side to eliminate the radical(s)? How would you proceed to find the solution?

**Communicate Your Answer**

3. How can you solve a radical equation?
  
  
  
  
  
  
  
  
  
  
4. Would you prefer to use a graphical, numerical, or analytical approach to solve the given equation? Explain your reasoning. Then solve the equation.

$$\sqrt{x + 3} - \sqrt{x - 2} = 1$$

## 5.4

# Notetaking with Vocabulary

For use after Lesson 5.4

In your own words, write the meaning of each vocabulary term.

radical equation

extraneous solutions

## Core Concepts

### Solving Radical Equations

To solve a radical equation, follow these steps:

**Step 1** Isolate the radical on one side of the equation, if necessary.

**Step 2** Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

**Step 3** Solve the resulting equation using techniques you learned in previous chapters.  
Check your solution.

**Notes:**

**5.4** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–10, solve the equation. Check your solution(s).

1.  $\sqrt{1-x} = 7$

2.  $\sqrt[3]{5x+1} = -4$

3.  $\frac{1}{4}\sqrt[4]{2x} + 6 = 10$

4.  $2\sqrt[3]{13x-5} = 10$

5.  $x - 7 = \sqrt{x-5}$

6.  $\sqrt[3]{486 - 27x^3} = 3x$

7.  $4\sqrt{x+1} = x+1$

8.  $\sqrt{2x+2} - 3\sqrt{x+1} = 0$

9.  $2 - \sqrt[4]{2x-6} = 14$

10.  $\sqrt{x+7} + 2 = \sqrt{3-x}$

**5.4** Notetaking with Vocabulary (continued)

In Exercises 11 and 12, solve the equation. Check your solution(s).

11.  $\frac{1}{2}x^{5/2} = 16$

12.  $(6x + 10)^{7/3} + 28 = 156$

In Exercises 13–15, solve the inequality.

13.  $-4\sqrt{x-1} + 3 \geq -1$

14.  $\sqrt[3]{\frac{2}{3}x + 1} < 6$

15.  $2\sqrt{\frac{3}{4}x} - 39 \leq -25$

16. In basketball, the term “hang time” is the amount of time that a player is suspended in the air when making a basket. To win a slam-dunk contest, players try to maximize their hang time. A player’s hang time is given by the equation  $t = 0.5\sqrt{h}$ , where  $t$  is the time (in seconds) and  $h$  is the height (in feet) of the jump. The second-place finisher of a slam-dunk contest had a hang time of 1 second, and the winner had a hang time of 1.2 seconds. How many feet higher did the winner jump than the second-place finisher?

**5.5****Performing Function Operations**

For use with Exploration 5.5

**Essential Question** How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to form other functions. For example, the functions  $f(x) = 2x - 3$  and  $g(x) = x^2 - 1$  can be combined to form the sum, difference, product, or quotient of  $f$  and  $g$ .

$$f(x) + g(x) = (2x - 3) + (x^2 - 1) = x^2 + 2x - 4 \quad \text{sum}$$

$$f(x) - g(x) = (2x - 3) - (x^2 - 1) = -x^2 + 2x - 2 \quad \text{difference}$$

$$f(x) \cdot g(x) = (2x - 3)(x^2 - 1) = 2x^3 - 3x^2 - 2x + 3 \quad \text{product}$$

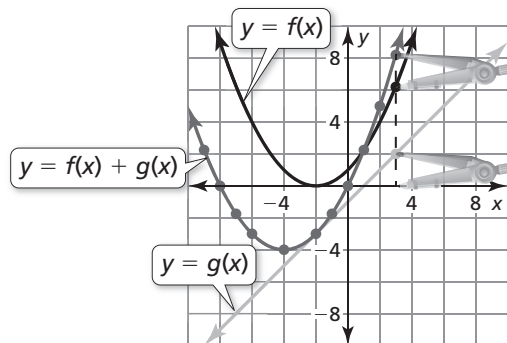
$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, x \neq \pm 1 \quad \text{quotient}$$

**1 EXPLORATION:** Graphing the Sum of Two Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

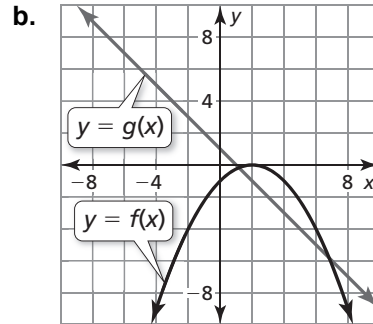
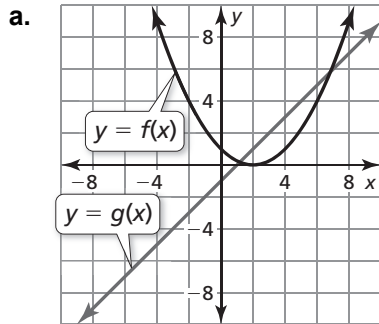
**Work with a partner.** Use the graphs of  $f$  and  $g$  to sketch the graph of  $f + g$ . Explain your steps.

**Sample** Use a compass or a ruler to measure the distance from a point on the graph of  $g$  to the  $x$ -axis. Then add this distance to the point with the same  $x$ -coordinate on the graph of  $f$ . Plot the new point. Repeat this process for several points. Finally, draw a smooth curve through the new points to obtain the graph of  $f + g$ .



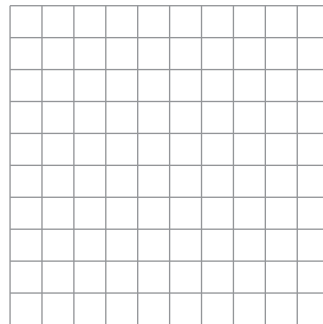
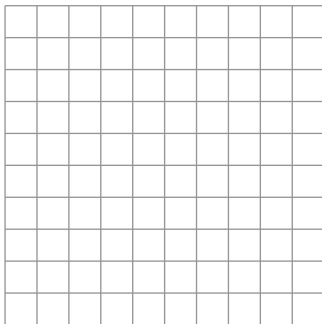
**5.5** Performing Function Operations (continued)

**1** **EXPLORATION:** Graphing the Sum of Two Functions (continued)



**Communicate Your Answer**

2. How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?
  
3. Check your answers in Exploration 1 by writing equations for  $f$  and  $g$ , adding the functions, and graphing the sum.



**5.5****Notetaking with Vocabulary**

For use after Lesson 5.5

In your own words, write the meaning of each vocabulary term.

domain

scientific notation

**Core Concepts****Operations on Functions**

Let  $f$  and  $g$  be any two functions. A new function can be defined by performing any of the four basic operations on  $f$  and  $g$ .

Operation	Definition	Example: $f(x) = 5x$ , $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the  $x$ -values that are in the domains of both  $f$  and  $g$ . Additionally, the domain of the quotient does not include  $x$ -values for which  $g(x) = 0$ .

**Notes:**



**5.5** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–4, find  $(f + g)(x)$  and  $(f - g)(x)$  and state the domain of each. Then evaluate  $f + g$  and  $f - g$  for the given value of  $x$ .

1.  $f(x) = -\frac{1}{2}\sqrt[3]{x}$ ,  $g(x) = \frac{9}{2}\sqrt[3]{x}$ ;  $x = -1000$

2.  $f(x) = -x^2 - 3x + 8$ ,  $g(x) = 6x - 3x^2$ ;  $x = -1$

3.  $f(x) = 4x^3 + 12$ ,  $g(x) = 2x^2 - 3x^3 + 9$ ;  $x = 2$

4.  $f(x) = 5\sqrt[4]{x} + 1$ ,  $g(x) = -3\sqrt[4]{x} - 2$ ;  $x = 1$

In Exercises 5–8, find  $(fg)(x)$  and  $\left(\frac{f}{g}\right)(x)$  and state the domain of each. Then evaluate  $fg$  and  $\frac{f}{g}$  for the given value of  $x$ .

5.  $f(x) = -x^3$ ,  $g(x) = 2\sqrt[3]{x}$ ;  $x = -64$

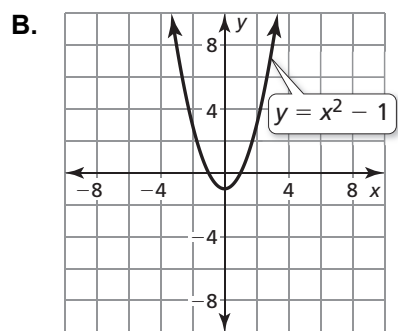
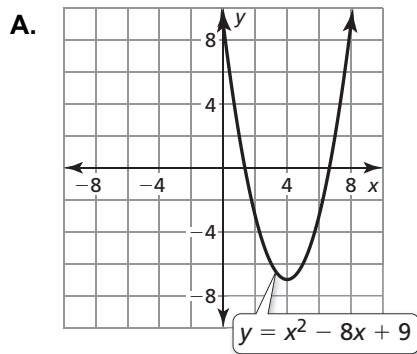
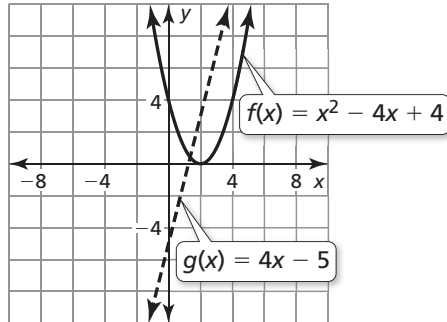
6.  $f(x) = 12x$ ,  $g(x) = 11x^{1/2}$ ;  $x = 4$

7.  $f(x) = 0.25x^{1/3}$ ,  $g(x) = -4x^{3/2}$ ;  $x = 1$

8.  $f(x) = 36x^{7/4}$ ,  $g(x) = 4x^{1/2}$ ;  $x = 16$

**5.5** Notetaking with Vocabulary (continued)

9. The graphs of the functions  $f(x) = x^2 - 4x + 4$  and  $g(x) = 4x - 5$  are shown. Which graph represents the function  $f + g$ ? the function  $f - g$ ? Explain your reasoning.



10. The variable  $x$  represents the number of pages of a textbook to be printed. The cost  $C$  to print the textbook can be modeled by the equation  $C(x) = 0.2x^2 + 10$ . The selling price  $P$  of the textbook can be modeled by the equation  $P(x) = 0.05x^2 + 20$ .

- a. Find  $(P - C)(x)$ .
  
- b. Explain what  $(P - C)(x)$  represents.

**5.6****Inverse of a Function**

For use with Exploration 5.6

**Essential Question** How can you sketch the graph of the inverse of a function?

**1 EXPLORATION:** Graphing Functions and Their Inverses

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Each pair of functions are *inverses* of each other. Use a graphing calculator to graph  $f$  and  $g$  in the same viewing window. What do you notice about the graphs?

a.  $f(x) = 4x + 3$

$$g(x) = \frac{x - 3}{4}$$

b.  $f(x) = x^3 + 1$

$$g(x) = \sqrt[3]{x - 1}$$

c.  $f(x) = \sqrt{x - 3}$

$$g(x) = x^2 + 3, x \geq 0$$

d.  $f(x) = \frac{4x + 4}{x + 5}$

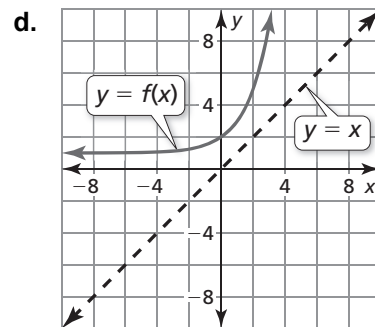
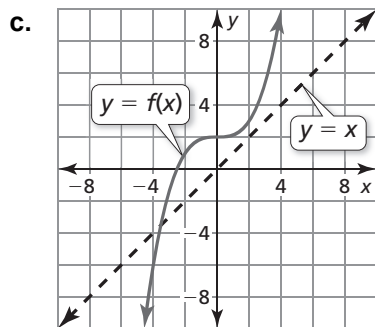
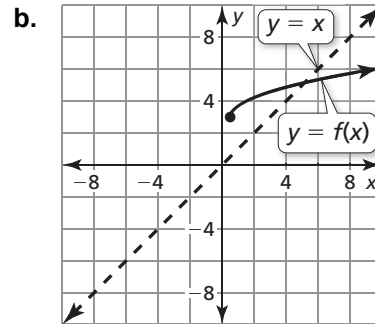
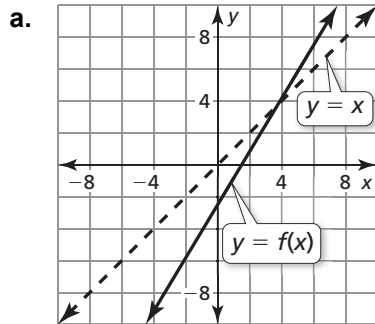
$$g(x) = \frac{4 - 5x}{x - 4}$$

**5.6 Inverse of a Function (continued)**

**2 EXPLORATION: Sketching Graphs of Inverse Functions**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use the graph of  $f$  to sketch the graph of  $g$ , the inverse function of  $f$ , on the same set of coordinate axes. Explain your reasoning.



**Communicate Your Answer**

- How can you sketch the graph of the inverse of a function?
- In Exploration 1, what do you notice about the relationship between the equations of  $f$  and  $g$ ? Use your answer to find  $g$ , the inverse function of

$$f(x) = 2x - 3.$$

Use a graph to check your answer.

**5.6****Notetaking with Vocabulary**

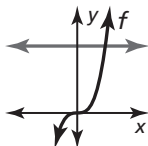
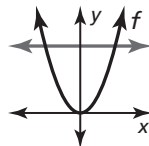
For use after Lesson 5.6

In your own words, write the meaning of each vocabulary term.

inverse functions

**Core Concepts****Horizontal Line Test**

The inverse of a function  $f$  is also a function if and only if no horizontal line intersects the graph of  $f$  more than once.

**Inverse is a function****Inverse is not a function****Notes:**

**5.6** Notetaking with Vocabulary (continued)

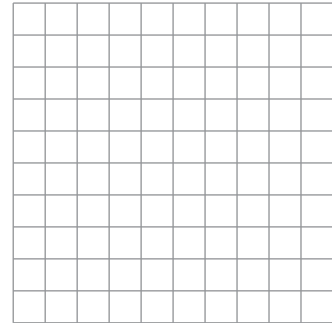
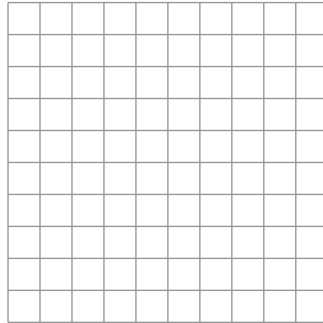
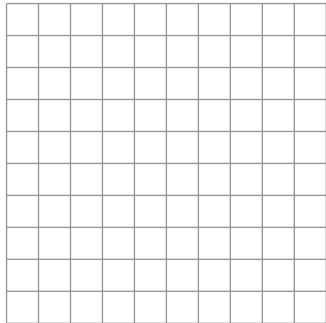
**Extra Practice**

In Exercises 1–3, solve  $y = f(x)$  for  $x$ . Then find the input(s) when the output is  $-6$ .

1.  $f(x) = 2x - 1$                       2.  $f(x) = 1 - x^2$                       3.  $f(x) = (x - 1)^3 + 2$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

4.  $f(x) = 10x$                       5.  $f(x) = -\frac{1}{5}x - 7$                       6.  $f(x) = \frac{3}{4}x + \frac{5}{8}$



In Exercises 7 and 8, determine whether each pair of functions  $f$  and  $g$  are inverses. Explain your reasoning.

7.

$x$	-4	-3	-2	-1	0	1
$f(x)$	17	13	9	5	1	-3

8.

$x$	1	2	3	4	5	6
$f(x)$	-1	-2	-4	-5	-8	-10

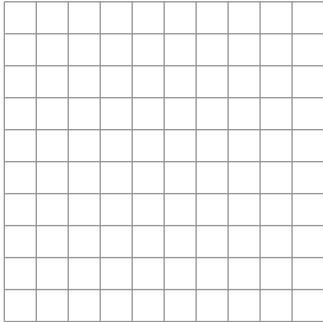
$x$	17	13	9	5	1	-3
$g(x)$	-4	-3	-2	-1	0	-1

$x$	-1	-0.5	-0.25	-0.2	-0.125	-0.1
$g(x)$	1	2	3	4	5	6

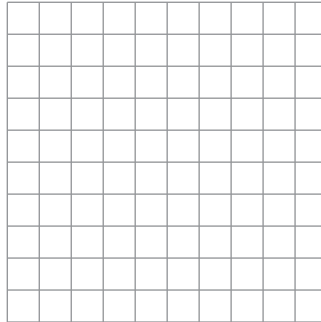
**5.6** Notetaking with Vocabulary (continued)

In Exercises 9 and 10, find the inverse of the function. Then graph the function and its inverse.

9.  $f(x) = (x + 2)^3$

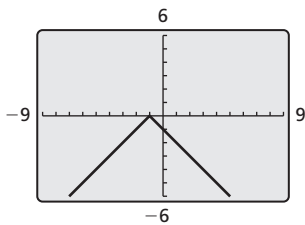


10.  $f(x) = \frac{1}{3}x^4, x \geq 0$

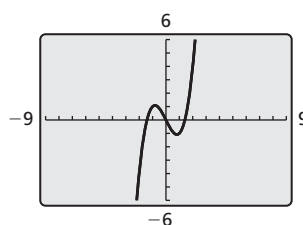


In Exercises 11 and 12, use the graph to determine whether the inverse of  $f$  is a function. Explain your reasoning.

11.



12.



In Exercises 13 and 14, determine whether the functions are inverses.

13.  $f(x) = \frac{4x}{5} - 1, g(x) = \frac{5x + 1}{4}$

14.  $f(x) = -(x - 2)^5 + 6, g(x) = 2 + (6 - x)^{1/5}$