Properties of Rational Exponents and Radicals

Core Concept

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$$

Core Concept

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When n is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When n is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

Name:____ Exponents (day 2)

Write the expression using a radical sign and no negative exponents.

1. $x^{\frac{2}{3}}$
2. $3y^{\frac{2}{5}}$
3. $(3y)^{\frac{2}{5}}$

Write the expression using positive rational exponents.

4. $\sqrt{x^5}$
5. $\left(\sqrt[6]{2a}\right)^5$
6. $\sqrt{64x^3}$

Simplify the expression.

7. $\left(\frac{9}{25}\right)^{\frac{1}{2}}$
8. $\left(\frac{9}{25}\right)^{-\frac{1}{2}}$
9. $\left(16^{-\frac{3}{5}}\right)^{\frac{5}{4}}$
10. $\left(81^{\frac{1}{2}} - 9^{\frac{1}{2}}\right)^2$

11. $(8a^{-6})^{-\frac{2}{3}}$

12. $(4x^{-3})^{-\frac{1}{2}} \cdot 4x^{\frac{1}{2}}$

13. $a^{\frac{1}{2}}\left(a^{\frac{3}{2}} - 2a^{\frac{1}{2}}\right)$

14. $x^{-\frac{1}{2}}\left(x^{\frac{5}{2}} - 2x^{\frac{3}{2}}\right)$

Simplify:

1) $\sqrt{42x^{6}y^{9}}$

2) $\sqrt[3]{27x^{2}y^{6}}$

3) $\sqrt[4]{32xy^{6}z^{8}}$

4) $\sqrt[3]{-32x^{5}y^{8}}$

Name ______________________________ Date __________

5.1 Practice A

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = 125$
2. $n = 2, a = 49$
3. $n = 4, a = 81$

In Exercises 4–9, evaluate the expression without using a calculator.

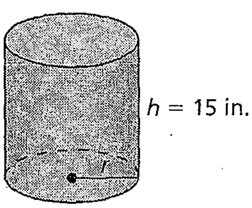
4. $27^{1/3}$
5. $16^{1/4}$
6. $4^{3/2}$
7. $625^{3/4}$
8. $(-1000)^{2/3}$
9. $32^{1/5}$

In Exercises 10–15, evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

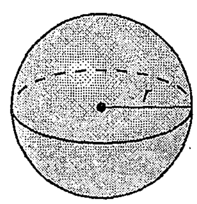
10. $\sqrt[5]{16{,}807}$
11. $\sqrt[6]{15{,}625}$
12. $12^{-1/3}$
13. $92^{1/5}$
14. $6561^{5/4}$
15. $113^{-3/4}$

In Exercises 16 and 17, find the radius of the figure with the given volume.

16. $V = 1726 \text{ in.}^3$



17. $V = 734 \text{ m}^3$



In Exercises 18–23, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

18. $x^4 = 256$
19. $3x^3 = 375$
20. $(x - 6)^2 = 40$
21. $(x + 7)^3 = 1000$
22. $x^5 = -112$
23. $9x^4 = 54$
24. When the average price of an item increases from p_1 to p_2 over a period of n years, the price p_2 is given by $p_2 = p_1(r + 1)^n$, where r is the annual rate of inflation (in decimal form). Find the annual rate of inflation when the price of a loaf of bread was \$1.19 in 1970 and \$3.29 in 2010.

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5.1 Practice B

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = 343$
2. $n = 6, a = -64$
3. $n = 5, a = -243$

In Exercises 4–9, evaluate the expression without using a calculator.

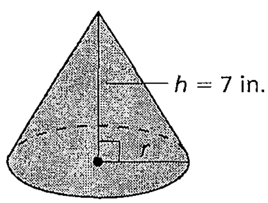
4. $36^{3/2}$
5. $16^{3/4}$
6. $(-32)^{2/5}$
7. $(-125)^{5/3}$
8. $256^{-5/4}$
9. $27^{-4/3}$

In Exercises 10–15, evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

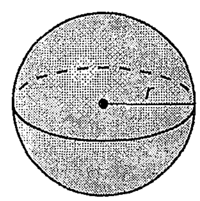
10. $28^{-1/5}$
11. $150^{2/5}$
12. $40{,}351^{6/7}$
13. $750^{-2/5}$
14. $\left(\sqrt[5]{223}\right)^3$
15. $\left(\sqrt[7]{-34}\right)^5$

In Exercises 16 and 17, find the radius of the figure with the given volume.

16. $V = 425 \text{ in.}^3$



17. $V = 1458 \text{ m}^3$



In Exercises 18–23, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

18. $6x^4 = 60$
19. $x^5 = -233$
20. $x^4 + 19 = 100$
21. $x^3 + 17 = 57$
22. $\frac{1}{5}x^4 = 125$
23. $\frac{1}{7}x^3 = -49$

24. Kepler's third law states that the relationship between the mean distance d (in astronomical units) of a planet from the Sun and the time t (in years) it takes the planet to orbit the Sun can be given by $d^3 = t^2$.

    a. It takes Venus 0.61 year to orbit the Sun. Find the mean distance of Venus from the Sun (in astronomical units).

    b. The mean distance of Jupiter from the Sun is 5.24 astronomical units. How many years does it take Jupiter to orbit the Sun?

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5.2 Practice A

In Exercises 1–6, use the properties of rational exponents to simplify the expression.

1. $(7^2)^{1/4}$

2. $(14^3)^{1/2}$

3. $\dfrac{5^{1/5}}{5}$

4. $\dfrac{10}{10^{1/4}}$

5. $\left(\dfrac{6^5}{9^5}\right)^{-1/5}$

6. $\left(7^{-3/4} \bullet 7^{1/4}\right)^{-1}$

In Exercises 7–12, use the properties of radicals to simplify the expression.

7. $\sqrt{3} \bullet \sqrt{75}$

8. $\sqrt[3]{81} \bullet \sqrt[3]{9}$

9. $\sqrt[4]{12} \bullet \sqrt[4]{8}$

10. $\sqrt[4]{9} \bullet \sqrt[4]{9}$

11. $\dfrac{\sqrt[5]{128}}{\sqrt[5]{4}}$

12. $\dfrac{\sqrt{5}}{\sqrt{80}}$

In Exercises 13–18, write the expression in simplest form.

13. $\sqrt[4]{208}$

14. $\dfrac{\sqrt[3]{9}}{\sqrt[3]{4}}$

15. $\sqrt{\dfrac{5}{27}}$

16. $\dfrac{1}{2 + \sqrt{3}}$

17. $\dfrac{6}{4 - \sqrt{5}}$

18. $\dfrac{8}{\sqrt{2} + \sqrt{5}}$

In Exercises 19–24, simplify the expression.

19. $8\sqrt[4]{2} + 5\sqrt[4]{2}$

20. $7\sqrt[5]{13} - 17\sqrt[5]{13}$

21. $4\left(9^{1/4}\right) + 7\left(9^{1/4}\right)$

22. $4\sqrt{18} - 15\sqrt{2}$

23. $8\sqrt{7} + 12\sqrt{63}$

24. $\sqrt[4]{405} + 2\sqrt[4]{5}$

25. The volume of a cube is 80 cubic centimeters.

a. Use exponents to solve the formula for the volume V of a cube with side length s, $V = s^3$, for s.

b. Substitute the expression for s from part (a) into the formula for the surface area of a cube, $S = 6s^2$.

c. Substitute the volume of the given cube into the formula found in part (b) to find the surface area, S. Simplify, if possible.

Name ____ Date __________

5.2 Practice B

In Exercises 1–6, use the properties of tional exponents to simplify the expression.

1. $\dfrac{2^{2/5}}{2}$

2. $\left(\dfrac{\ }{1}\ \right)^{-1/6}$

3. $\left(11^{3/2} \bullet 11^{-5/2}\right)^{-1/3}$

4. $\left(9^{-3/5} \bullet 9^{1/5}\right)^{-1}$

5. $\dfrac{3^{\ } \bullet 27^{3/4}}{9^{3/4}}$

6. $\dfrac{25^{5/9} \bullet 25^{7/9}}{5^{4/3}}$

In Exercises 7–12, use the properties c adicals to simplify the expression.

7. $\sqrt[3]{25} \bullet \sqrt[3]{625}$

8. $\sqrt[5]{\ } \bullet \sqrt[5]{81}$

9. $\dfrac{\sqrt[4]{176}}{\sqrt[4]{11}}$

10. $\dfrac{\sqrt{7}}{\sqrt{700}}$

11. $\dfrac{\sqrt[3]{\ } \bullet \sqrt[3]{50}}{\sqrt[3]{2}}$

12. $\dfrac{\sqrt[4]{4} \bullet \sqrt[4]{12}}{\sqrt[8]{3} \bullet \sqrt[8]{3}}$

In Exercises 13–18, write the expressi in simplest form.

13. $\dfrac{\sqrt[3]{4}}{\sqrt[3]{9}}$

14. $\sqrt[3]{\dfrac{\ }{5}}$

15. $\sqrt[4]{\dfrac{2401}{4}}$

16. $\dfrac{7}{5 - \sqrt{3}}$

17. $\dfrac{6}{\ + \sqrt{7}}$

18. $\dfrac{\sqrt{2}}{\sqrt{15} - \sqrt{3}}$

In Exercises 19–24, simplify the expre ion.

19. $10\left(25^{2/3}\right) - 6\left(25^{2/3}\right)$

20. $2\ \sqrt{54} - 11\sqrt{6}$

21. $13\sqrt[3]{3} - \sqrt[3]{375}$

22. $\sqrt[5]{486} + 10\sqrt[5]{2}$

23. $4\ \ 8^{1/4}) - 3\left(3^{1/4}\right)$

24. $\left(7^{1/3}\right) + 4\left(189^{1/3}\right)$

25. The volume of a right circular cyli ler is $V = 9\pi r^2$, where r is the radius.

    a. Use radicals to solve $V = 9\pi$ for r. Simplify, if possible.

    b. Substitute the expression for r om part (a) into the formula for the surface area of a right cylinder, $S = 1\ \ r + \pi r^2$.

    c. Use the answer to part (b) to f the surface area of a right cylinder when the volume is 108 cubic meter

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5.3 Practice A

In Exercises 1–6, graph the function. Identify the domain and range of the function.

1. $g(x) = \sqrt{x} + 4$
2. $h(x) = \sqrt{x} - 2$
3. $f(x) = -\sqrt[3]{4x}$
4. $h(x) = \sqrt[3]{-2x}$
5. $f(x) = \frac{1}{3}\sqrt{x - 2}$
6. $g(x) = \frac{1}{4}\sqrt{x + 5}$

In Exercises 7–12, describe the transformation of *f* represented by *g*. Then graph each function.

7. $f(x) = \sqrt{x}; g(x) = \sqrt{x - 1} + 4$
8. $f(x) = \sqrt{x}; g(x) = 3\sqrt{x + 2}$
9. $f(x) = \sqrt[3]{x}; g(x) = -2\sqrt[3]{x}$
10. $f(x) = \sqrt[3]{x}; g(x) = \sqrt[3]{x - 1} + 3$
11. $f(x) = x^{1/2}; g(x) = 3(-x)^{1/2}$
12. $f(x) = x^{1/3}; g(x) = -\frac{1}{3}x^{1/3}$

In Exercises 13–15, use a graphing calculator to graph the function. Then identify the domain and range of the function.

13. $f(x) = \sqrt{x^2 - x}$
14. $g(x) = \sqrt[3]{x^2 - x}$
15. $h(x) = \sqrt[3]{2x^2 + 3x}$

In Exercises 16 and 17, write a rule for *g* described by the transformations of the graph of *f*.

16. Let g be a vertical shrink by a factor of $\frac{1}{3}$, followed by a translation 3 units right of the graph of $f(x) = \sqrt{x + 5}$.

17. Let g be a reflection in the x-axis, followed by a translation 2 units down of the graph of $f(x) = 5\sqrt{x} + 3$.

In Exercises 18 and 19, use a graphing calculator to graph the equation of the parabola. Identify the vertex and the direction that the parabola opens.

18. $\frac{1}{2}y^2 = x$
19. $-3y^2 = x + 6$

In Exercises 20 and 21, use a graphing calculator to graph the equation of the circle. Identify the radius and the intercepts.

20. $x^2 + y^2 = 16$
21. $25 - y^2 = x^2$

Name____ Date__________

5.3 Practice B

In Exercises 1–6, graph the function. Identify the domain and range of the function.

1. $g(x) = -\sqrt{x} + 2$
2. $f(x) = \sqrt[3]{-4x}$
3. $f(x) = \frac{1}{4}\sqrt{x + 5}$
4. $h(x) = (5x)^{1/2} - 2$
5. $g(x) = -2(x - 3)^{1/3}$
6. $h(x) = -\sqrt[5]{x}$

In Exercises 7–12, describe the transformation of *f* represented by *g*. Then graph each function.

7. $f(x) = \sqrt{x}; g(x) =$ [illegible] $\sqrt{x - 2}$
8. $f(x) = \sqrt[3]{x}; g(x) = \sqrt[3]{x - 5} - 1$
9. $f(x) = x^{1/4}; g(x) =$ [illegible] $x)^{1/4}$
10. $f(x) = x^{1/3}; g(x) = \frac{1}{2}x^{1/3} - 3$
11. $f(x) = \sqrt[4]{x}; g(x) =$ [illegible] $\sqrt{x - 1} + 3$
12. $f(x) = \sqrt[5]{x}; g(x) = \sqrt[5]{-243x} - 2$

In Exercises 13–15, use a graphing calculator to graph the function. Then identify the domain and range of the function.

13. $g(x) = \sqrt[3]{2x^2 - 3x}$
14. $f(x) = \sqrt{\frac{1}{3}x^2 - x + 2}$
15. $h(x) = \sqrt[3]{3x^2 - 6x + 2}$

In Exercises 16 and 17, write a rule for *g* described by the transformations of the graph of *f*.

16. Let g be a horizontal stretch by a factor of 2, followed by a translation 2 units up of the graph of $f(x) = \sqrt{3x}$.

17. Let g be a translation [illegible] unit up and 4 units left, followed by a reflection in the y-axis of the graph of $f(x) = \sqrt{-x} - \frac{1}{2}$.

In Exercises 18 and 19, use a graphing calculator to graph the equation of the parabola. Identify the vertex and the direction that the parabola opens.

18. $3y^2 + 5 = x$
19. $x - 3 = -\frac{1}{2}y^2$

In Exercises 20 and 21, use a graphing calculator to graph the equation of the circle. Identify the radius and the intercepts.

20. $x^2 + y^2 = 81$
21. $-y^2 = x^2 - 49$

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5.4 Practice A

In Exercises 1–6, solve the equation. Check your solution.

1. $\sqrt{3x - 2} = 5$ **2.** $\sqrt{6x + 1} = 9$ **3.** $\sqrt[3]{x + 10} = 4$

4. $\sqrt[3]{x} - 8 = -2$ **5.** $-3\sqrt{16x} + 14 = -10$ **6.** $6\sqrt[3]{25x} - 16 = 14$

7. Biologists have discovered that the shoulder height h (in centimeters) of a male Asian elephant can be modeled by $h = 62.5\sqrt[3]{t} + 75.8$, where t is the age (in years) of the elephant. Determine the age of an elephant with a shoulder height of 300 centimeters.

In Exercises 8–13, solve the equation. Check your solution(s).

8. $x - 8 = \sqrt{4x}$ **9.** $\sqrt{2x - 14} = x - 7$

10. $\sqrt{x + 22} = x + 2$ **11.** $\sqrt[3]{8x^3 + 27} = 2x + 3$

12. $\sqrt[4]{2 - 9x^2} = 3x$ **13.** $\sqrt{3x - 5} = \sqrt{x + 9}$

In Exercises 14–16, solve the equation. Check your solution(s).

14. $2x^{2/3} = 18$ **15.** $x^{3/4} + 10 = 0$ **16.** $(x + 12)^{1/2} = x$

17. Describe and correct the error in solving the equation.

X
$$\begin{aligned} \sqrt[3]{2x + 1} &= 8 \\ 2x + 1 &= 2 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

In Exercises 18–20, solve the inequality.

18. $3\sqrt{x} - 4 \geq 5$ **19.** $\sqrt{x - 3} \leq 7$ **20.** $5\sqrt{x - 1} > 10$

21. The length ℓ (in inches) of a standard nail can be modeled by $\ell = 54d^{3/2}$, where d is the diameter (in inches) of the nail.

a. What is the diameter of a standard nail that is 2 inches long?

b. What is the diameter of a standard nail that is 4 inches long?

c. The nail in part (b) is twice as long as the nail in part (a). Is the diameter twice as long? Explain.

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5.4 Practice B

In Exercises 1–6, solve the equation. Cl :k your solution.

1. $\sqrt[3]{x-14} = -2$

2. $-5 \quad 16x + 17 = -8$

3. $\frac{1}{4}\sqrt[3]{2x} + 8 = 6$

4. $\sqrt{3x} - \frac{3}{4} = 0$

5. $3\sqrt[5]{\quad} + 9 = 15$

6. $\sqrt[4]{8x} - 16 = -12$

In Exercises 7–12, solve the equation. C :ck your solution(s).

7. $\sqrt{10x + 24} = x + 12$

8. $x + 3 = \sqrt{\frac{22}{3}x + 9}$

9. $\sqrt[4]{2 - 25x^2} = 5x$

10. $\sqrt{4x - 4} - \sqrt{x + 8} = 0$

11. $\sqrt[3]{4x - 1} = \sqrt[3]{6x + 5}$

12. $\sqrt{4x - 10} = \sqrt{2x - 13} + 1$

In Exercises 13–15, solve the equation. heck your solution(s).

13. $3x^{2/3} - 30 = 18$

14. $(6. + 3)^{1/2} - 3x = 0$

15. $(2x^2 + 8)^{1/4} = x$

In Exercises 16–18, solve the inequalit

16. $4\sqrt{x} + 3 \le 23$

17. $\sqrt{\quad + 10} \ge 6$

18. $-3\sqrt{x + 2} < 15$

19. "Hang time" is the time you are sus :nded in the air during a jump. Your hang time t in seconds is given by the fu :ion $t = 0.5\sqrt{h}$, where h is the height (in feet) of the jump. A kite sailor h a hang time of 2.5 seconds. Find the height of the kite sailor's jump.

In Exercises 20–23, solve the nonlinea :ystem. Justify your answer with a graph.

20. $y^2 = x + 2$
    $y = x + 2$

21. $y^2 = -x + 7$
    $y = x - 1$

22. $x^2 + y^2 = 9$
    $y = x - 3$

23. $x^2 + y^2 = 16$
    $y = x + 4$

24. The speed s (in miles per hour) of : :ar can be given by $s = \sqrt{30fd}$, where f is the coefficient of friction and d is t : stopping distance (in feet). The coefficient of friction for a snowy road is 0.3C :ou are driving 20 miles per hour and approaching an intersection. How : away from the intersection must you begin to brake?

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5.5 Practice A

In Exercises 1 and 2, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x.

1. $f(x) = -3\sqrt[4]{x};\ g(x) = 15\sqrt[4]{x};\ x = 81$

2. $f(x) = 9x + 2x^2;\ g(x) = x^2 - 3x + 7;\ x = 1$

In Exercises 3–5, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x.

3. $f(x) = x^2;\ g(x) = 2\sqrt{x};\ x = 9$

4. $f(x) = 10x^3;\ g(x) = 4x^{5/3};\ x = 8$

5. $f(x) = 4x^{2/3};\ g(x) = 2x^{1/3};\ x = -27$

In Exercises 6 and 7, use a graphing calculator to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

6. $f(x) = 5x^3;\ g(x) = 20x^{1/4}$

7. $f(x) = 4x^{2/3};\ g(x) = 16x^{4/3}$

8. Describe and correct the error in stating the domain.

✗ $f(x) = 4x^{1/2} + 2$ and $g(x) = -4x^{1/2}$ The domain of $(f + g)(x)$ is all real numbers.

9. The growth of mold in Specimen A can be modeled by $A(t) = \frac{5}{6}t^{2/3}$. The growth of mold in Specimen B can be modeled by $B(t) = \frac{1}{3}t^{2/3}$.

   a. Find $(A - B)(t)$.

   b. Explain what the function $(A - B)(t)$ represents.

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5.5 Practice B

In Exercises 1 and 2, find $(f$ $g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f +$ and $f - g$ for the given value of x.

1. $f(x) = \sqrt[3]{4x}; g(x) =$ $9\sqrt[3]{4x}; x = -2$

2. $f(x) = 3x - 5x^2 - x$ $g(x) = 6x^2 - 4x; x = -1$

In Exercises 3–5, find $(fg)($ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ fc the given value of x.

3. $f(x) = 3x^3; g(x) =$ $^2; x = -8$

4. $f(x) = 3x^2; g(x) =$ $^{/4}; x = 16$

5. $f(x) = 10x^{5/6}; g(x) =$ $2x^{1/3}; x = 64$

In Exercises 6 and 7, use a raphing calculator to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $= 5$. Round your answers to two decimal places.

6. $f(x) = -3x^{1/3}; g(x)$ $4x^{1/2}$

7. $f(x) = 6x^{3/4}; g(x) = 3x^{1/2}$

8. Describe and correct t error in stating the domain.

> ✗ $f(x) = 4x^{7/3}$ nd $g(x) = 2x^{2/3}$
>
> The domain c $\left(\frac{f}{g}\right)(x)$ is all real numbers.

9. The table shows the o puts of the two functions f and g. Use the table to evaluate $(f + g)(5)$, $(f - g)$, $(fg)(3)$, and $\left(\frac{f}{g}\right)(2)$.

x	0	1	2	3	4	5
f(x)	18	13	8	3	−2	−7
g(x)	64	32	1(	8	4	2

Name____ Date __________

5.6 Practice A

In Exercises 1–3, solve $y = f(x)$ for x. Then find the input(s) when the output is -3.

1. $f(x) = 2x + 3$
2. $f(x) = \frac{1}{3}x - 2$
3. $f(x) = 8x^3$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

4. $f(x) = 4x$
5. $f(x) = 4x - 1$
6. $f(x) = \frac{1}{2}x - 5$

7. Find the inverse of the function $f(x) = \frac{1}{5}x - 2$ by switching the roles of x and y and solving for y. Then find the inverse of the function f by using inverse operations in the reverse order. Which method do you prefer? Explain.

8. Determine whether each pair of functions f and g are inverses. Explain your reasoning.

a.

x	−2	−1	0	1	2
$f(x)$	−3	3	9	15	21

x	−3	3	0	15	21
$g(x)$	−2	−1	0	1	2

b.

x	1	2	3	4	5
$f(x)$	9	7	5	3	1

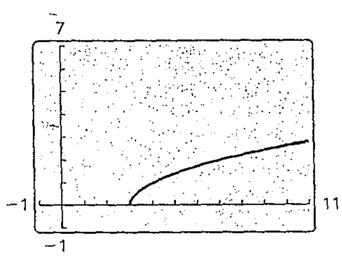
x	9	7	5	3	1
$g(x)$	1	2	3	4	5

In Exercises 9–11, find the inverse of the function. Then graph the function and its inverse.

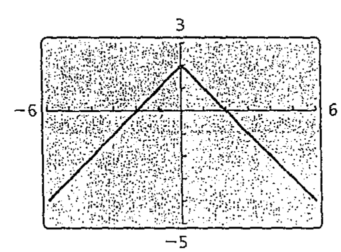
9. $f(x) = 9x^2, x \geq 0$
10. $f(x) = 16x^2, x \leq 0$
11. $f(x) = (x + 2)^3$

In Exercises 12 and 13, use the graph to determine whether the inverse of f is a function. Explain your reasoning.

12.



13.



Name ____ Date __________

5.6 Practice B

In Exercises 1–3, solve $y = f(x)$ for x. Then find the input(s) when the output is -3.

1. $f(x) = -\frac{4}{3}x + 2$
2. $f(x) = 25x^4$
3. $f(x) = (x - 3)^2 - 4$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

4. $f(x) = -3x + 4$
5. $f(x) = -\frac{1}{3}x + 1$
6. $f(x) = \frac{2}{5}x - \frac{1}{5}$

7. Describe and correct the error in finding the inverse function.

$$\begin{aligned} f(x) &= 3x - 8 \\ y &= 3x - 8 \\ x &= 3y - 8 \\ g(x) &= 3x - 8 \end{aligned}$$

In Exercises 8–10, find the inverse function. Then graph the function and its inverse.

8. $f(x) = -9x^2, x \le 0$
9. $f(x) = (x - 1)^3$
10. $f(x) = x^6, x \le 0$

11. Find the inverse of the function $f(x) = 8x^3$ by switching the roles of x and y and solving for y. Then find the inverse of the function f by using inverse operations in the reverse order. Which method do you prefer? Explain.

In Exercises 12–15, determine whether the functions are inverses.

12. $f(x) = 6x + 1; g(x) = 6x - 1$
13. $f(x) = \frac{\sqrt[3]{x - 6}}{2}; g(x) = 8x^3 + 6$
14. $f(x) = \frac{5 - x}{2}; g(x) = 5 - 2x$
15. $f(x) = 4x^2 + 3; g(x) = -\frac{x - 3}{4}$

16. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius.

  a. Find the inverse function. Describe what it represents.

  b. Find the radius of a sphere with a volume of 146 cubic meters.

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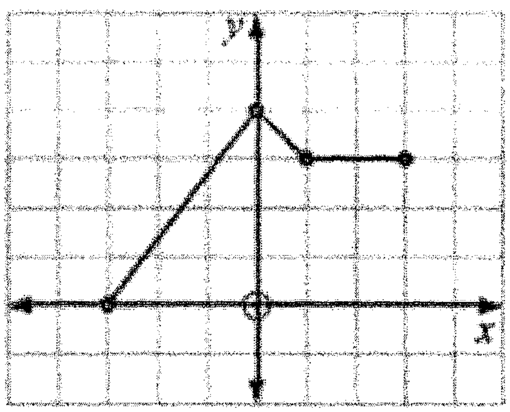
Common Core Algebra 2 Algebra of Functions

Given the following functions:

x	f(x)
1	4
2	1
3	5
4	2
5	3

y=g(x)



$$h(x) = \sqrt[3]{4x}$$

Evaluate:

(f+g)(1)

3g(1)-f(4)

(h·g)(-2)

$\left(\frac{h}{f}\right)(2)$

h(g(0))

g(h(0))

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Inverse of a Function

EXPLORATION 1 Graphing Functions and Their Inverses

Work with a partner. Each pair of functions are *inverses* of each other. Use a graphing calculator to graph f and g in the same viewing window. What do you notice about the graphs?

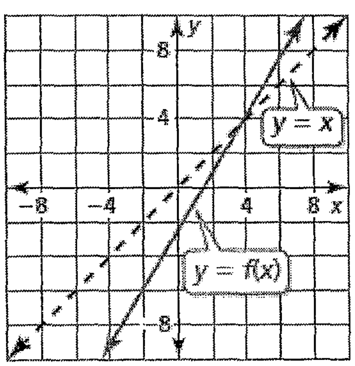
$$f(x) = 4x + 3$$
$$g(x) = \frac{x - 3}{4}$$

$$f(x) = \sqrt{x - 3}$$
$$g(x) = x^2 + 3, x \geq 0$$

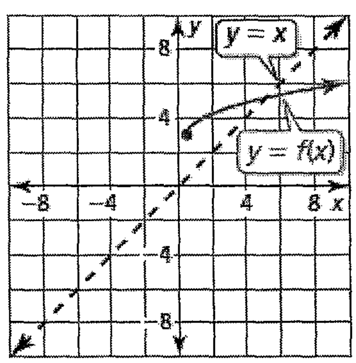
EXPLORATION 2 Sketching Graphs of Inverse Functions

Work with a partner. Use the graph of f to sketch the graph of g, the inverse function of f, on the same set of coordinate axes. Explain your reasoning.

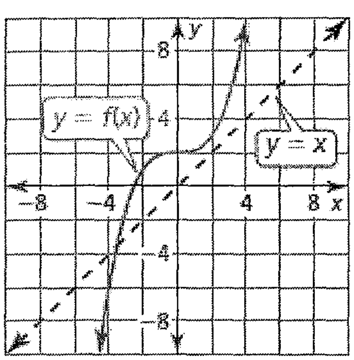
a.



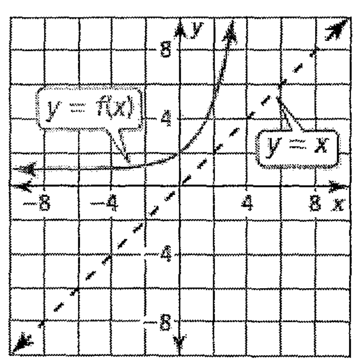
b.



c.



d.



EXAMPLE 1 Writing a Formula for the Input of a Function

Let $f(x) = 2x + 3$.

a. Solve $y = f(x)$ for x.

b. Find the input when the output is -7.

EXAMPLE 2 Finding the Inverse of a Linear Function

Find the inverse of $f(x) = 3x - 1$.

EXAMPLE 3 Finding the Inverse of a Quadratic Function

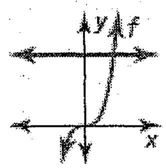
Find the inverse of $f(x) = x^2, x \geq 0$. Then graph the function and its inverse.

Core Concept

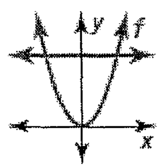
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function



Inverse is not a function



EXAMPLE 4 Finding the Inverse of a Cubic Function

Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

EXAMPLE 5 Finding the Inverse of a Radical Function

Consider the function $f(x) = 2\sqrt{x - 3}$. Determine whether the inverse of f is a function. Then find the inverse.

EXAMPLE 6 Verifying Functions Are Inverses

Verify that $f(x) = 3x - 1$ and $g(x) = \frac{x+1}{3}$ are inverse functions.

Solving Real-Life Problems

In many real-life problems, formulas contain meaningful variables, such as the radius r in the formula for the surface area of a sphere, $S = 4\pi r^2$. In this situation, switching the variables to find the inverse would create confusion by switching the meanings of S and r. So, when finding the inverse, solve for r without switching the variables.

EXAMPLE 7 Solving a Multi-Step Problem

Find the inverse of the function that represents the surface area of a sphere, $S = 4\pi r^2$. Then find the radius of a sphere that has a surface area of 100π square feet.