Name: ______________________ Period: _____ Date: ____________________

Solving Polynomial Equations Using Synthetic Division

For each of the following, use synthetic division to simplify the polynomial expression. Then use factoring or the quadratic formula to find the other two roots of the expression. Write all answers in **exact** form (that is, no non-terminating decimals; leave your answers in simplest radical form).

1. $x^3 - 5x^2 - 8x + 12 = 0$, given that one of the solutions is $x = 1$.

2. $x^3 + 4x^2 - 6x - 12 = 0$, given that one of the solutions is $x = 2$.

3. $x^3 - 19x + 30 = 0$, given that one of the solutions is $x = -5$.

4. $x^3 - 3x^2 + 7x - 5 = 0$, given that one of the solutions is $x = 1$.

For the following, sketch the function in your calculator to find one of the roots. Then use synthetic division to simplify the polynomial so that you can find the other roots in exact form.

5. $x^3 - x^2 - 14x + 8 = 0$

6. $x^3 + 8x^2 + 5x + 40 = 0$

Could you have factored any of the polynomials above by using the method of grouping?

Name: ____________________ Remainders and Roots Date: ____________

1) Given $p(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$
   a) Find the quotient and remainder when $p(x)$ is divided by $(x - 2)$.

   b) find $p(2)$

2) Given $p(x) = 3x^4 - 5x^2 + 9x + 10$
   a) Find the quotient and remainder when $p(x)$ is divided by $(x + 2)$.

   b) find $p(-2)$

3) If $p(x) = 4x^3 - 5x^2 + 7x - 9$
   a) Find the quotient and remainder when $p(x)$ is divided by $(x - 3)$.

   b) find $p(3)$

Remainder Theorem: If a polynomial $p(x)$ is divided by $(x - c)$, then the remainder is the number $p(c)$.

4) Let $f(x) = x^3 - 4x^2 + 2x + 3$

a) Show that 3 is a root of $f(x)$

b) Show that $(x - 3)$ is a factor $f(x)$.

Factor Theorem: A polynomial $f(x$ has a factor $x - k$ if and only if $f(k) = 0$.

Ex: Determine whether (a) $x - 2$ is a factor $f(x) = x^2 + 2x - 4$ and (b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

Number of Roots: A polynomial of de ree n has at most n distinct roots.

Practice Problems:

1) Given: $f(x) = x^4 + 6x^3 - x^2 - 30x$

a) Determine if 2 is a root of).

b) Determine if -1 is a root o x).

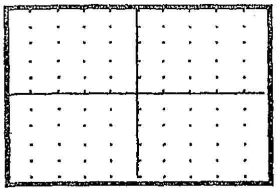
2) Given: $f(x) = x^{10} + x^8$ and $g(x) =$ -1 find the remainder when f(x) is divided by g(x).

3) Given: $f(x) = x^3 - 3x^2 - 4x - 12$ a $h(x) = x + 2$ find the remainder when f(x) is divided by h(x).

Investigating End Behavior

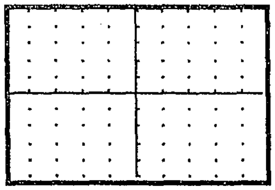
se your calculator to graph each polynomial and complete the information below each graph. You will be ng this information to draw a conclusion about the end behavior of polynomials of degree 'n'.

$f(x)=(x+2)(x-1)$



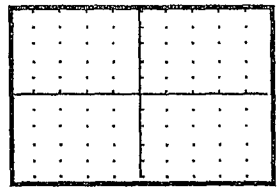
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=(x+3)(x-2)(x+1)(x-4)$



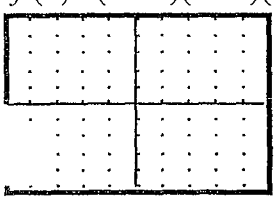
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=-(x+3)(x-1)$



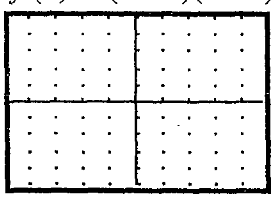
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=(x+2)(x-1)(x-4)(x+5)$



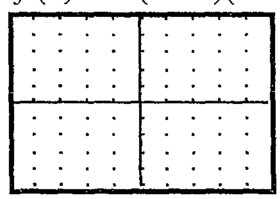
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=(x+2)(x-3)(x-1)$



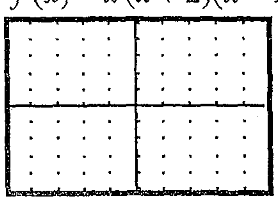
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=-(x+2)(x-1)(x+1)$



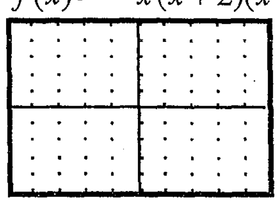
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=x(x+2)(x-1)(x-3)(x+4)$



Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
nd behavior: _________

$f(x)=-x(x+2)(x-1)(x-3)(x+4)$



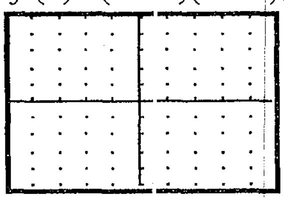
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

How does a negative in front of the polynomial affect the end behavior?

Use your observations from the previous graphs, explain how you would predict the end behavior of a given polynomial of degree *n*.

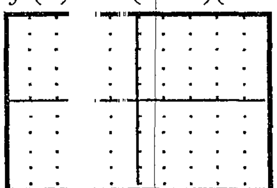
Using your prediction, sketch the following graphs without using a calculator.

$f(x)=(x+4)(x-2)(x+3)(x-4)$



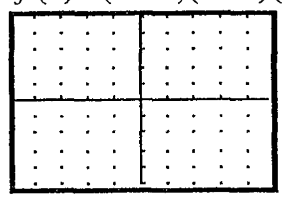
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x) = -(x+3)(x-1)$



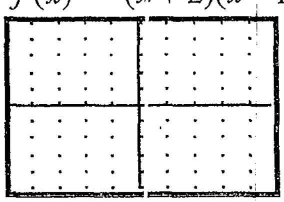
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=(x+2)(x-1)(x+4)$



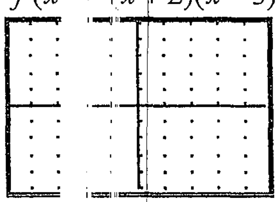
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)=-(x+2)(x-1)(x-4)$



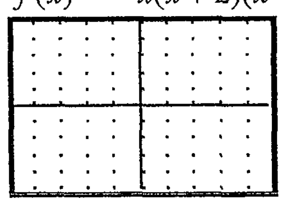
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x) = (x+2)(x-3)(x-1)$



Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

$f(x)= -x(x+2)(x-1)(x+1)$



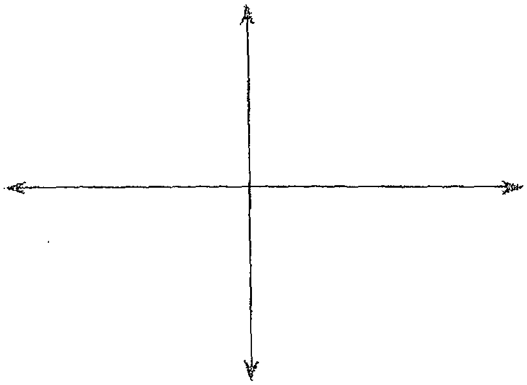
Zeroes: ______________
Degree of polynomial: ___
Leading Coefficient: + / -
End behavior: _________

Compare your answers with others, then make any necessary adjustments to your conjecture at the top of this page.

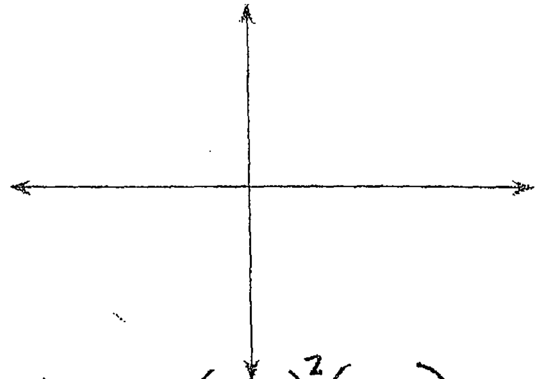
Name:________________________________ "Pass Through" and "Bounce" Points

'e want to examine the role of the exponent on each factor and its effect on the graph of the polynomial. Without using your calculator, make a quick sketch of the graph of each of the following functions. It is not important to have the heights drawn to scale. Draw smooth flowing curves.

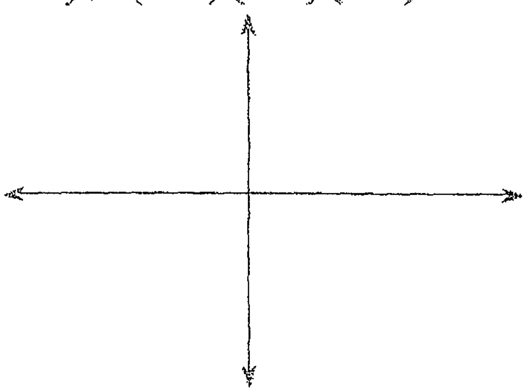
1) $y_1 = (x+2)(x-1)(x-3)$



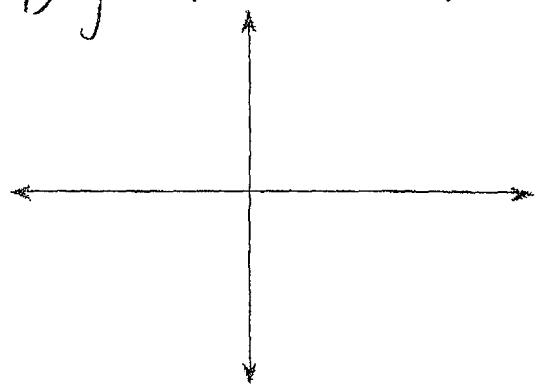
2) $y_2 = (x+2)^2(x-1)(x-3)$



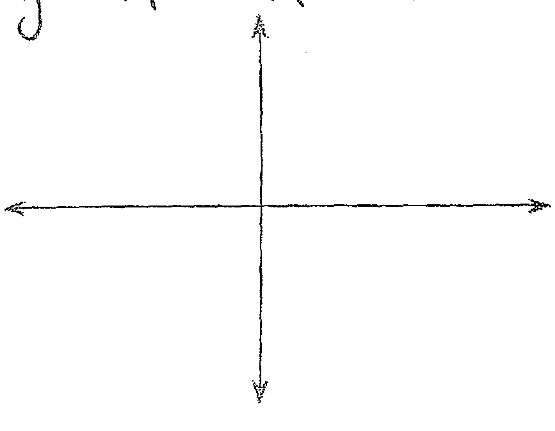
3) $y_3 = (x+2)^2(x-1)^2(x-3)$



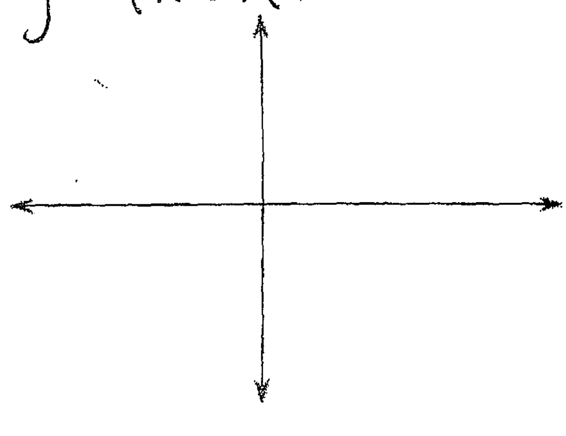
4) $y = -(x+4)^2(x-3)$



5) $y = x^2(x-4)(x+1)^2$



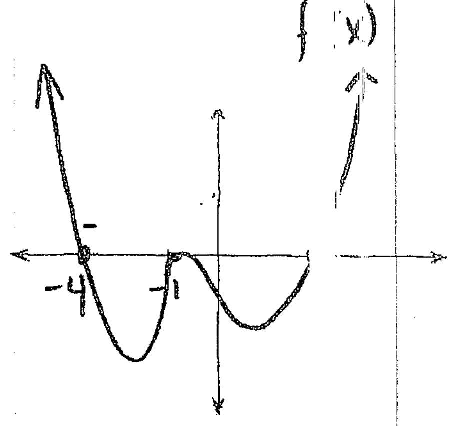
6) $y = -(x-2)^2(x+3)^2$



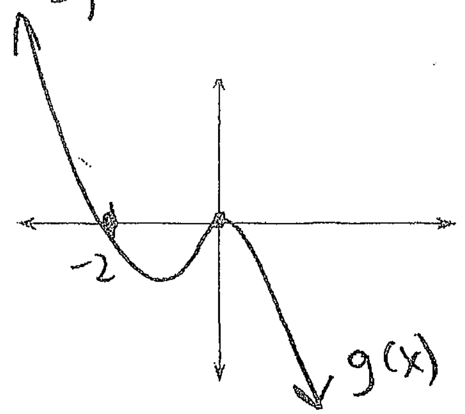
I.

Find a possible equation for each of these functions

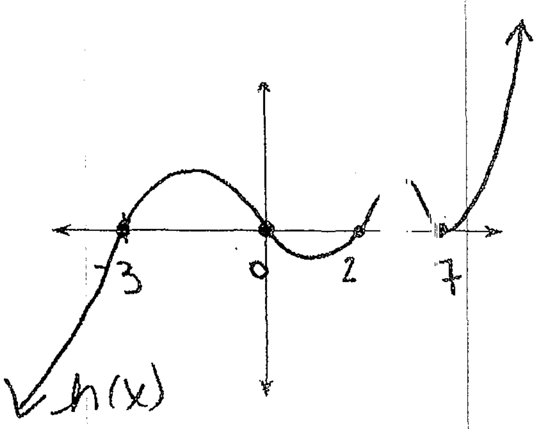
1)



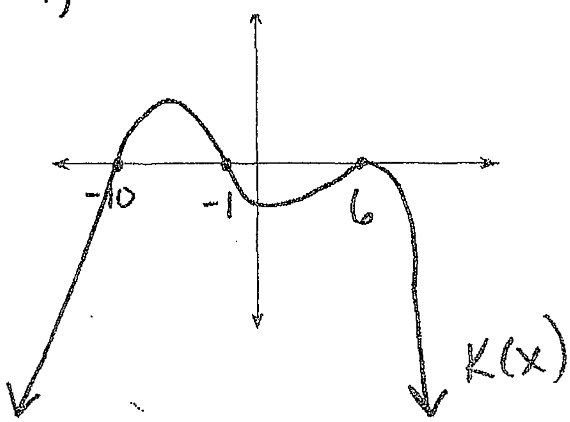
2)



3)

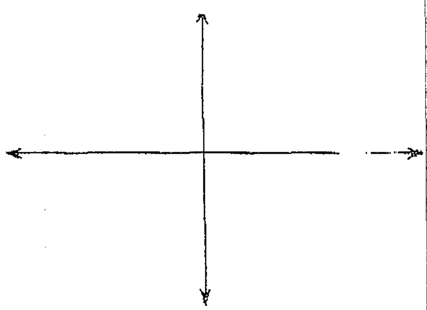


4)

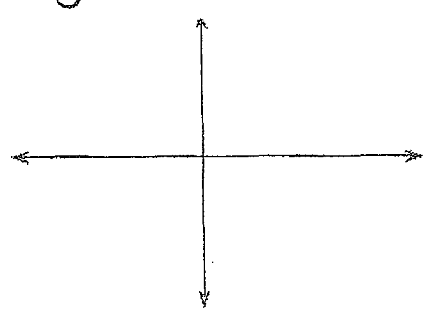


Sticker work: either name the graph or sketch the graph

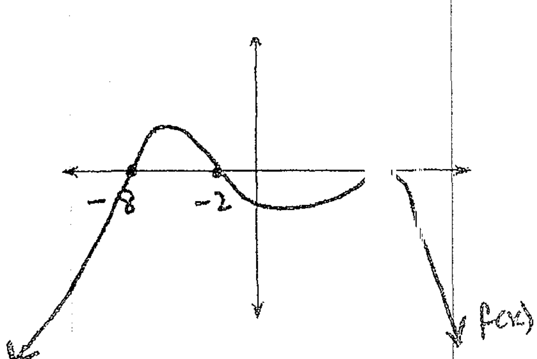
1) $f(x) = (x+2)^2(x-1)(x-5)^3$

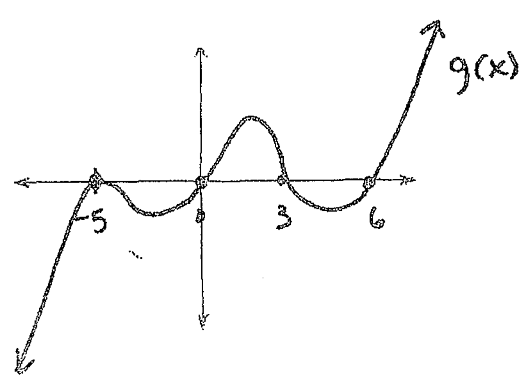


2) $y = -x^2(x-3)(x+2)^4$



1)

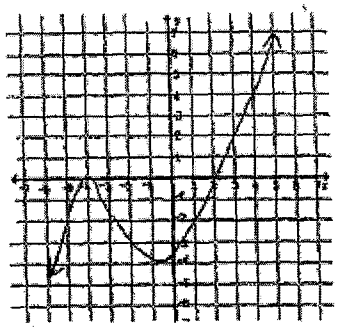


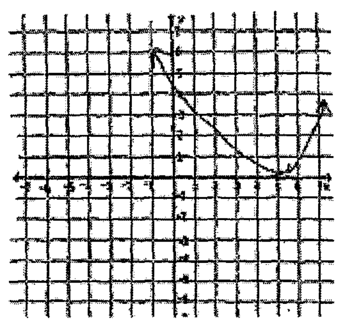


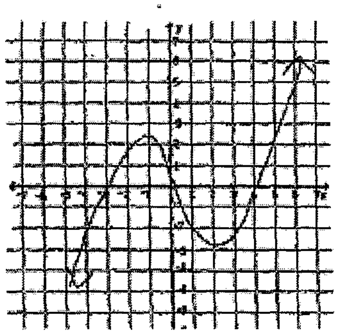
Name: ____________________ Period: _____ Date: ____________________

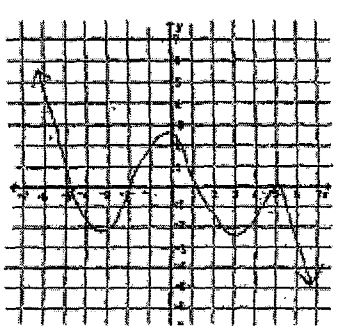
Writing *Possible* Polynomial Functions from Graphs

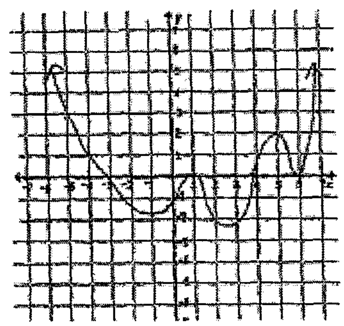
For each of the following, write two *possible* functions that *could* have the given graph. Base your answers on the number of zeros, the multiplicity of the zeros (does the graph bounce off of the x axis or cut through it?), the number of extrema, and the end behavior.

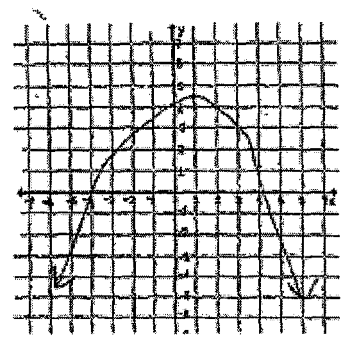
1.


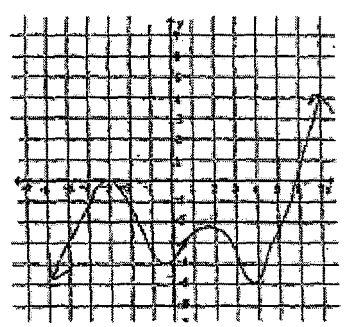
2.


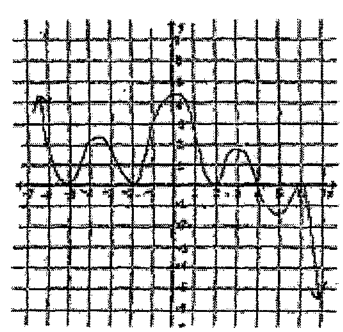
3.


4.


5.


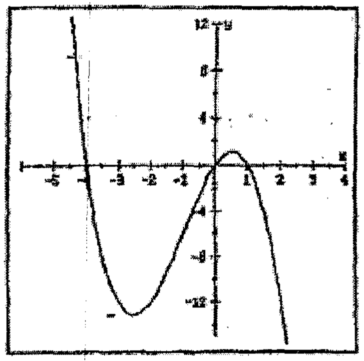
6.


7.


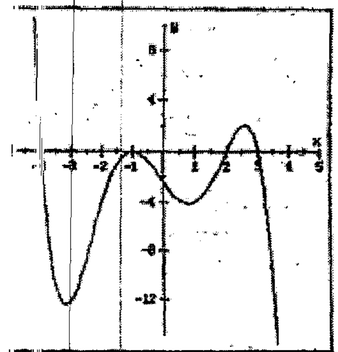
8.


Factoring a Polynomial by Using Its Graph I

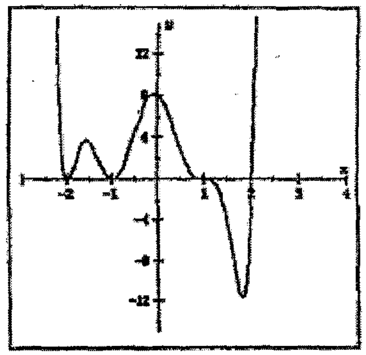
Give a possible factorization of the following polynomials. Do NOT multiply out the factors! Be sure to use your knowledge of the Leading Coefficient Test and Repeated Zeros.



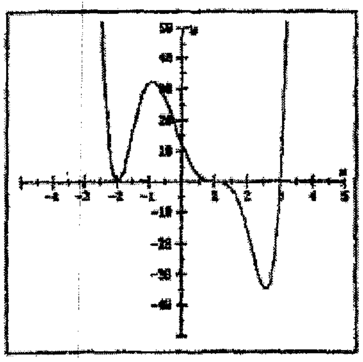
1)



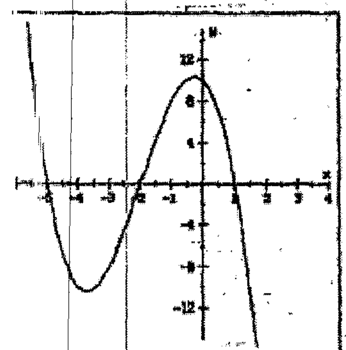
2)



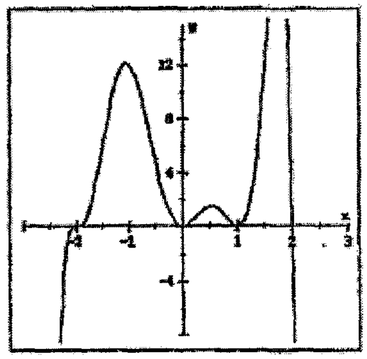
3)



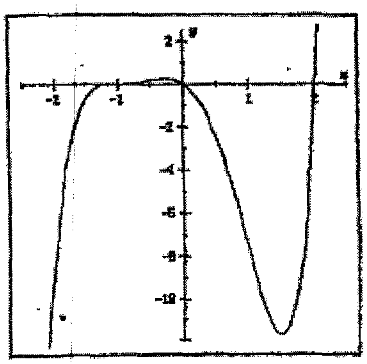
4)



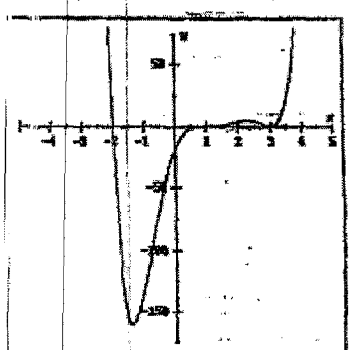
5)



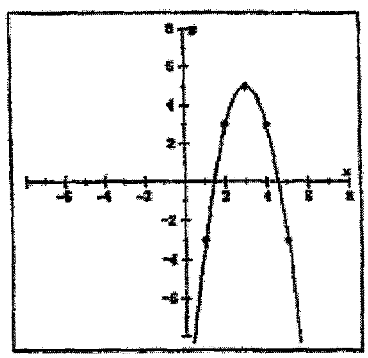
6)



7)



8)



9)

Name: Key Period: _____ Date: ____________________

Solving Polynomial Equations Using Synthetic Division

For each of the following, use synthetic division to simplify the polynomial expression. Then use factoring or the quadratic formula to find the other two roots of the expression. Write all answers in **exact** form (that is, no non-terminating decimals; leave your answers in simplest radical form).

1. $x^3 - 5x^2 - 8x + 12 = 0$, given that one of the solutions is $x = 1$.

1	1	-5	-8	12
	↓	1	-4	-12
	1	-4	-12	0

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6 \quad x = -2$

2. $x^3 + 4x^2 - 6x - 12 = 0$, given that one of the solutions is $x = 2$.

2	1	4	-6	-12
	↓	2	12	12
	1	6	6	0

$x^2 + 6x + 6 = 0$

$x = \frac{-6 \pm \sqrt{36 - (4 \cdot 1 \cdot 6)}}{2} = \frac{-6 \pm \sqrt{12}}{2}$

$x = \frac{-6 \pm 2\sqrt{3}}{2}$

$x = -3 \pm \sqrt{3}$

3. $x^3 - 19x + 30 = 0$, given that one of the solutions is $x = -5$.

$x^3 + 0x^2 - 19x + 30 = 0$

-5	1	0	-19	30
	↓	-5	25	-30
	1	-5	6	0

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2 \quad x = 3$

4. $x^3 - 3x^2 + 7x - 5 = 0$, given that one of the solutions is $x = 1$.

1	1	-3	7	-5
	↓	1	-2	5
	1	-2	5	0

$x^2 - 2x + 5 = 0$

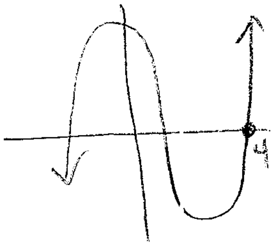
$x = \frac{2 \pm \sqrt{4 - (4 \cdot 1 \cdot 5)}}{2}$

$x = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$

$x = 1 \pm 2i$

For the following, sketch the function in your calculator to find one of the roots. Then use synthetic division to simplify the polynomial so that you can find the other roots in exact form.

5. $x^3 - x^2 - 14x + 8 = 0$ $\boxed{x = 4}$



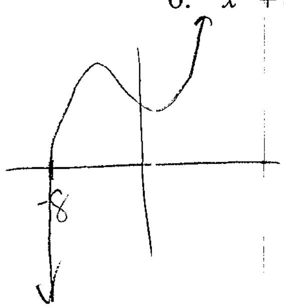
$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 8 \\ & \downarrow & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

$x^2 + 3x - 2 = 0$

$x = \dfrac{-3 \pm \sqrt{9 - (4 \cdot 1 \cdot -2)}}{2}$

$\boxed{x = \dfrac{-3 \pm \sqrt{17}}{2}}$

6. $x^3 + 8x^2 + 5x + 40 = 0$ $\boxed{x = -8}$



$$\begin{array}{r|rrrr} -8 & 1 & 8 & 5 & 40 \\ & \downarrow & -8 & 0 & -40 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$x^2 + 5 = 0$

$-5 \quad -5$

$\sqrt{x^2} = \sqrt{-5}$

$\boxed{x = \pm i\sqrt{5}}$

Could you have factored any of the polynomials above by using the method of grouping? yes #6

$x^3 + 5x + 8x^2 + 40$

$x(x^2 + 5) + 8(x^2 + 5) = 0$

$(x + 8)(x^2 + 5) = 0$

$x = -8 \quad \sqrt{x^2} = \sqrt{-5}$

$x = \pm \sqrt{5}$

Name: Key Remainders and Roots Date: __________

1) Given $p(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$

a) Find the quotient and remainder when $p(x)$ is divided by $(x - 2)$.

2	3	-7	-5	9	10
	↓	6	-2	-14	-10
	3	-1	-7	-5	0

$3x^3 - x^2 - 7x - 5$

$R = 0$

b) find $p(2)$

$p(2) = 3(2)^4 - 7(2)^3 - 5(2)^2 + 9(2) + 10 = 0$

2) Given $p(x) = 3x^4 - 5x^2 + 9x + 10$

a) Find the quotient and remainder when $p(x)$ is divided by $(x + 2)$.

-2	3	0	-5	9	10
	↓	-6	12	-14	10
	3	-6	7	-5	20

$3x^3 - 6x^2 + 7x - 5$ R 20

b) find $p(-2)$

$p(-2) = 3(-2)^4 - 5(-2)^2 + 9(-2) + 10 = 20$

3) If $p(x) = 4x^3 - 5x^2 + 7x - 9$

a) Find the quotient and remainder when $p(x)$ is divided by $(x - 3)$.

3	4	-5	7	-9
	↓	12	21	84
	4	7	28	75

b) find $p(3)$

$P(3) = 4(3)^3 - 5(3)^2 + 7(3) - 9 = 75$

Remainder Theorem: If a polynomial $p(x)$ is divided by $(x - c)$, then the remainder is the number $p(c)$.

4) Let: $f(x) = x^3 - 4x^2 + 2x + 3$

a) Show that 3 is a root of $f(x)$

$f(3) = 3^3 - 4(3)^2 + 2(3) + 3 = 0$ ✓

b) Show that $(x - 3)$ is a factor of $f(x)$.

3) 1 -4 2 3
↓ 3 -3 -3
1 -1 -1 0 ✓

Factor Theorem: A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Ex: Determine whether (a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$ and (b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

a) $f(2) = 2^2 + 2(2) - 4 = 4$
No.

b) $f(-5) = 3(-5)^4 + 15(-5)^3 - (-5)^2 + 25 = 0$
Yes.

Number of Roots: A polynomial of degree n has at most n distinct roots.

Practice Problems:

1) Given: $f(x) = x^4 + 6x^3 - x^2 - 30x$

a) Determine if 2 is a root of $f(x)$. $f(2) = 0$
Yes.

b) Determine if -1 is a root of $f(x)$. $f(-1) = 24$
No.

2) Given: $f(x) = x^{10} + x^8$ and $g(x) = x - 1$ find the remainder when ~~$h(x)$~~ $f(x)$ is ~~a factor of~~ divided by ~~$f(x)$~~ $g(x)$.

$f(1) = 1^{10} + 1^8 = 2$

3) Given: $f(x) = x^3 - 3x^2 - 4x - 12$ and $h(x) = x + 2$ find the remainder when ~~$h(x)$~~ $f(x)$ is ~~a factor of~~ divided by ~~$f(x)$~~ $h(x)$.

$f(-2) = -24$

Name: Key "Pass Through" and "Bounce" Points # 1

("Pass Through" → odd power/mult. ; "Bounce" → even)

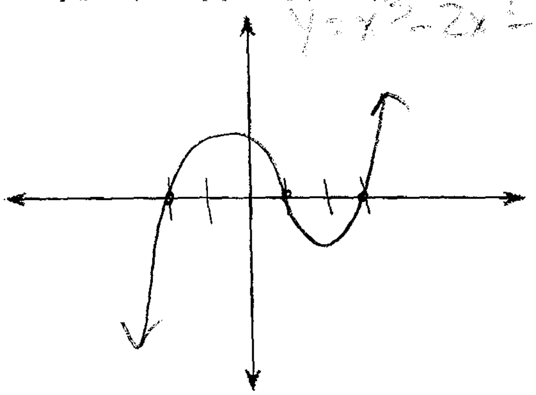
We want to examine the role of the exponent on each factor and its effect on the graph of the polynomial. Without using your calculator, make a quick sketch of the graph of each of the following functions. It is not important to have the heights drawn to scale. Draw smooth flowing curves.

$x^3-3x^2+x^2-3x-2x+6$

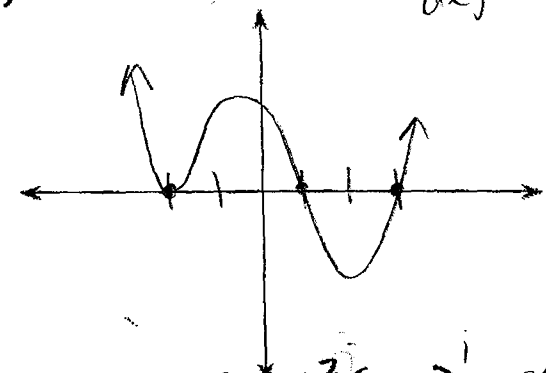
$(x^2+x-2)(x-3)$

1) $y = (x+2)(x-1)(x-3)$

$y = x^3-2x^2-5x+6$

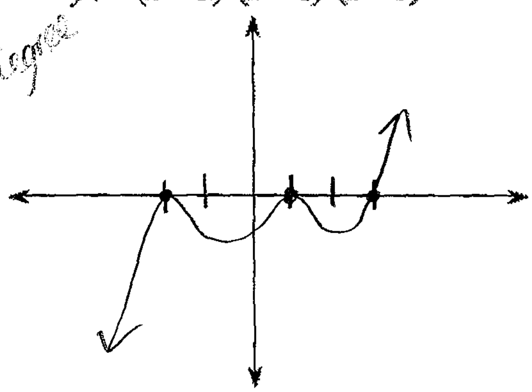


2) $y = (x+2)^2(x-1)(x-3)$ even degree (4)

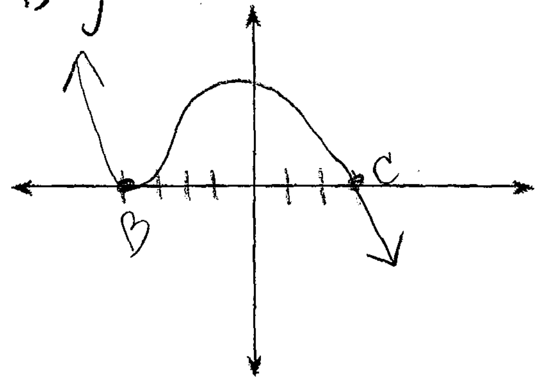


3) $y = (x+2)^2(x-1)^2(x-3)$

5th degree

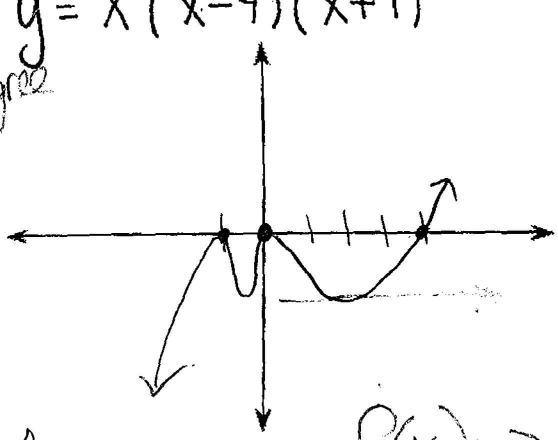


4) $y = -(x+4)^2(x-3)^1$ 3rd degree



5) $y = +x^2(x-4)^1(x+1)^2$

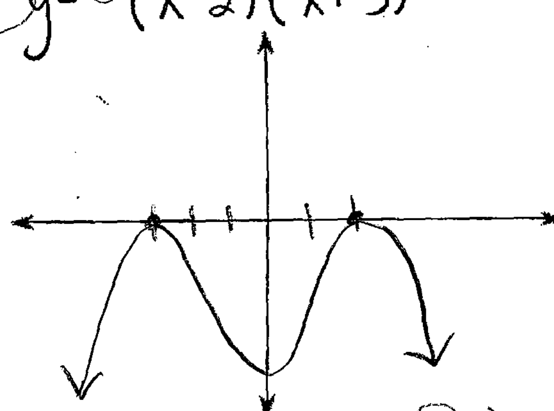
5th degree



As $x \to \infty$, $f(x) \to$ ∞

As $x \to -\infty$, $f(x) \to$ $-\infty$

6) $y = -(x-2)^2(x+3)^2$ 4th degree



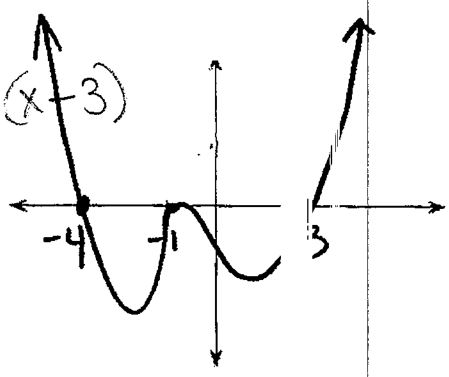
As $x \to \infty$, $f(x) \to$ $-\infty$

As $x \to -\infty$, $f(x) \to$ $-\infty$

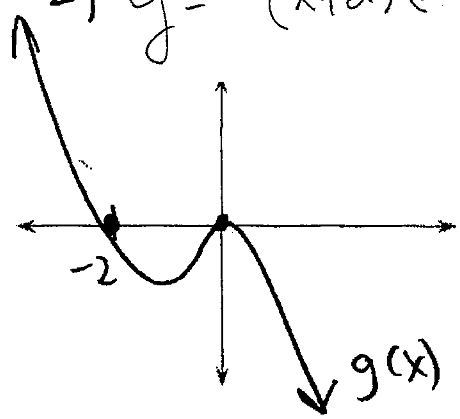
II.

Find a possible equation for each of these functions

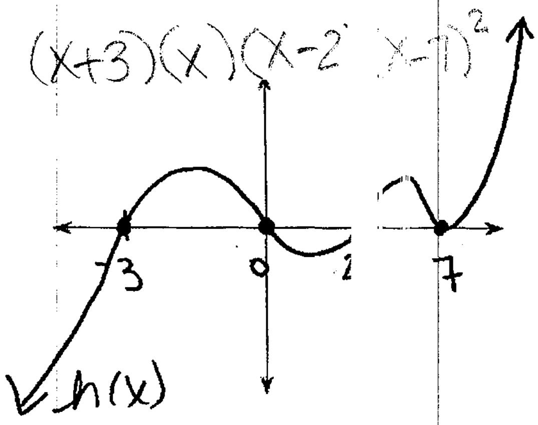
1) $y=(x+4)(x+1)^2(x-3)$



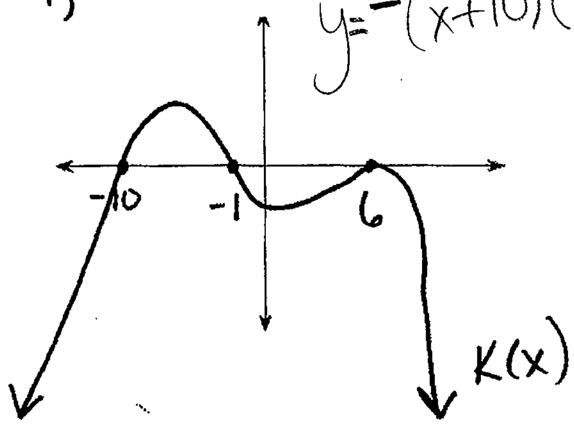
2) $y=-(x+2)(x)^2$



3) $y=(x+3)(x)(x-2)(x-7)^2$



4) $y=-(x+10)(x+1)(x-6)^2$

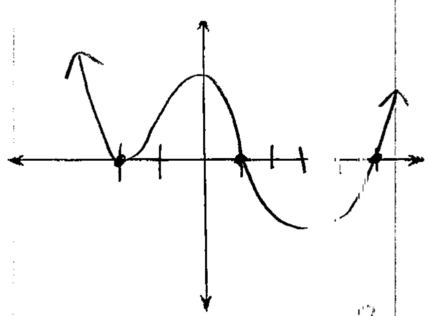


III. Do Now

~~Sticker work:~~ either name the graph or sketch the graph

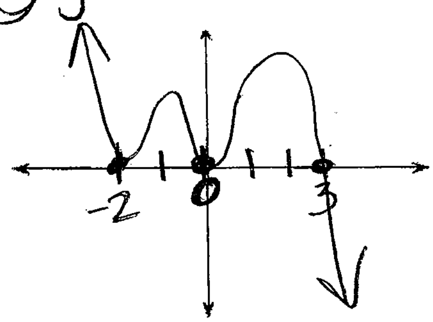
1) $f(x)=(x+2)^2(x-1)^1(x-5)^3$

⊕ 6th degree

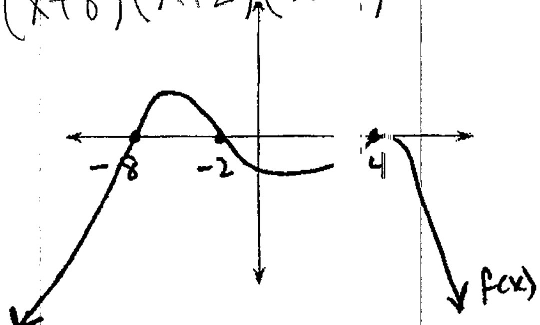


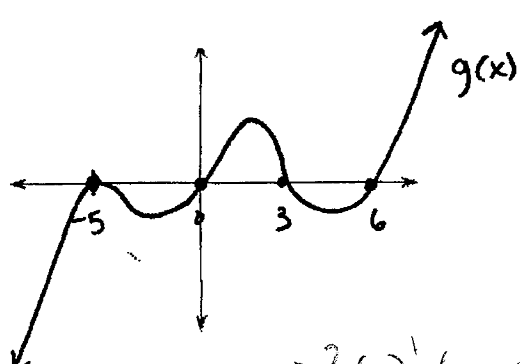
2) $y=-x^2(x-3)^1(x+2)^4$

⊖ 7th degree



3) $y=-(x+8)(x+2)(x-4)^2$





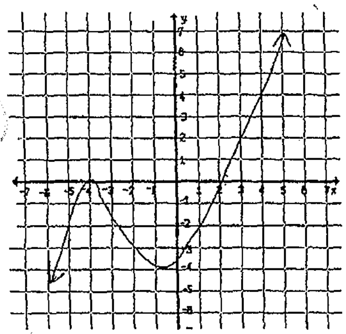
$y=(x+5)^2(x)^1(x-3)^1(x-6)^1$

Need to print

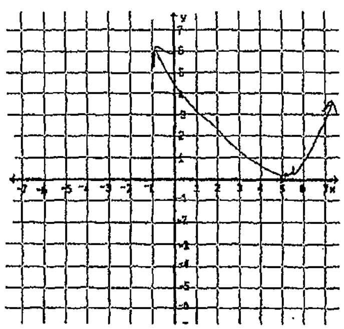
Name: Key Period: ____ Date: ____________ #2

Writing *Possible* Polynomial Functions from Graphs

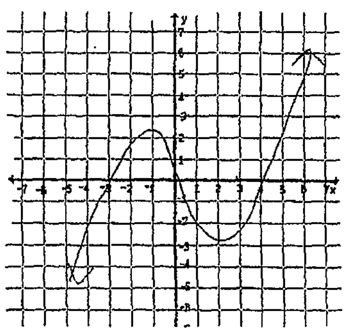
For each of the following, write two *possible* functions that *could* have the given graph. Base your answers on the number of zeros, the multiplicity of the zeros (does the graph bounce off of the x axis or cut through it?), the number of extrema, and the end behavior.

1. 

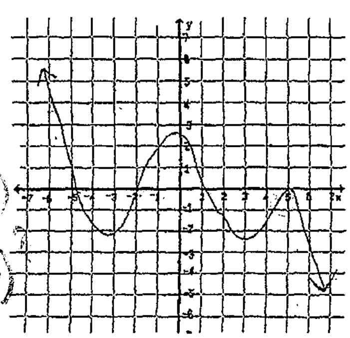
$y=(x+4)^2(x-2)$

2. 

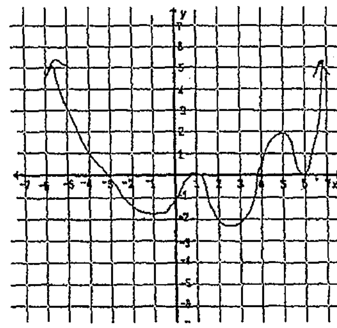
$y=(x-5)^2$

3. 

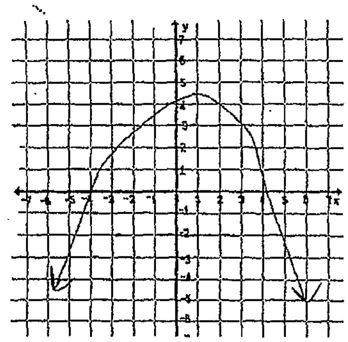
$y=x(x+3)(x-4)$

4. 

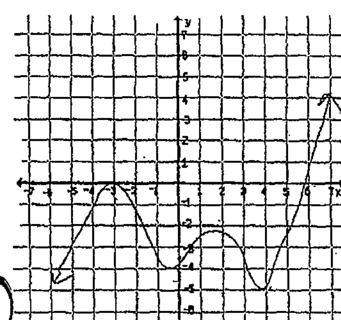
$y=-(x+5)(x+2)(x-1)(x-5)$

5. 

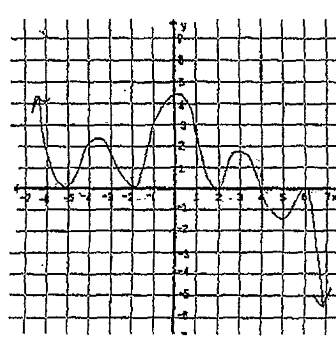
$y=(x+3)(x-1)^2(x-4)(x-6)^2$

6. 

$y=-(x+4)(x-4)$

7. 

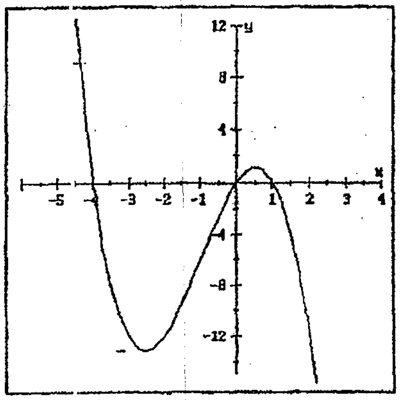
$y=(x+3)^2(x-6)$

8. 

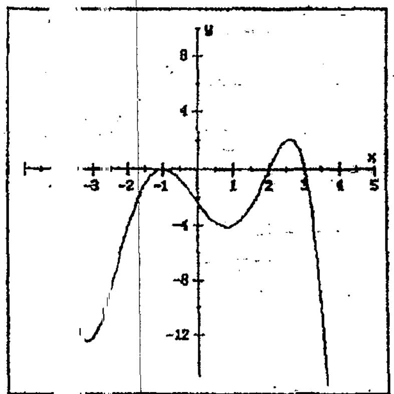
$y=-(x+5)^2(x+2)^2(x-2)^2(x-4)(x-6)^2$

Factoring a olynomial by Using Its Graph I

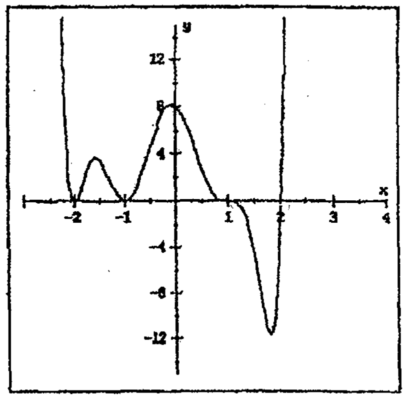
Give a possible factorization of the follov :g polynomials. Do NOT multiply out the factors!
Be sure to use your knowledge of the Lea :ng Coefficient Test and Repeated Zeros.



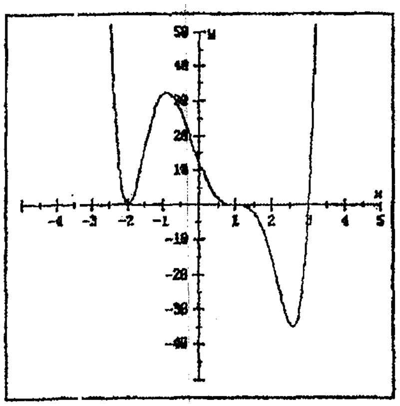
1) $y = x(x+4)(x-1)$



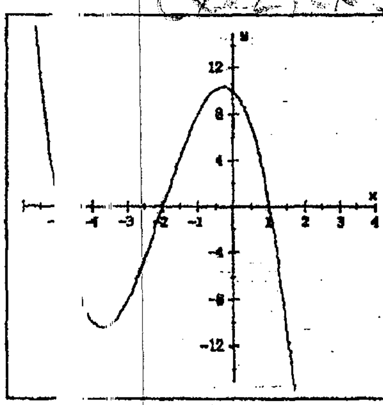
2) $y = -(x+4)(x+1)^2(x-2)(x-3)$



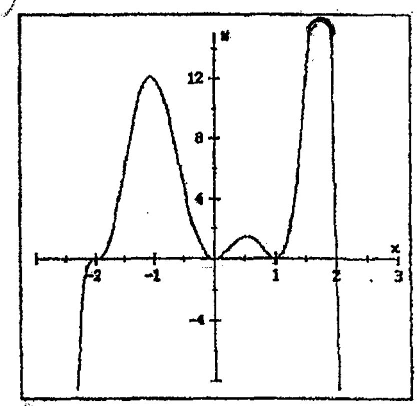
3) $y = (x+2)^2(x+1)^2(x-1)(x-2)$



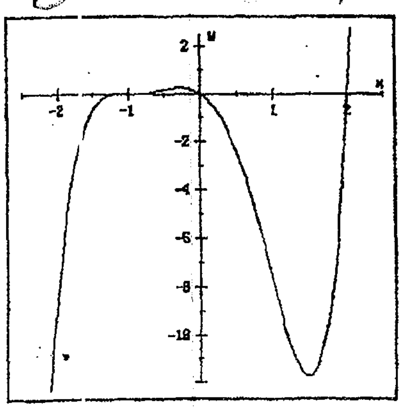
4) $y = (x+2)^2(x-1)(x-3)$



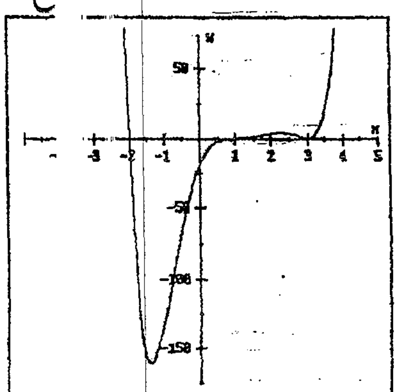
5) $y = -(x+5)(x+2)(x-1)$



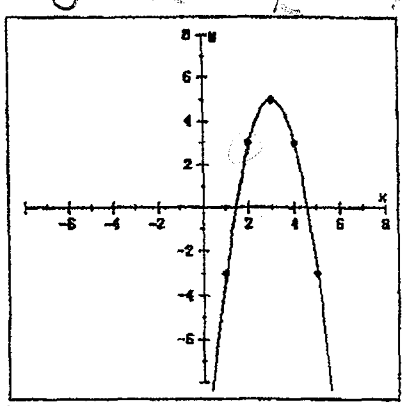
6) $y = x^2(x+2)(x-1)^2(x-2)$



7) $y = x^2(x+1)(x-2)$



8) $y = (x+2)(x-1)(x-3)^3$



$y = (x - \frac{3}{2})(x-$

$y = (2x-3)(2x-$

9) $y = a(x-3)^2+5$

$3 = a(2-3)^2+5$

$3 = 1a+5$

$-2 = a$

$y = -2(x-3)^2+5$

$2x^2-12x+13$

$2(x^2-6x+9)+5$

$2x^2-12x+18+5$