Ops. of Fcns.

Name________________________________ Date__________

5.5 Practice B

In Exercises 1 and 2, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x.

1. $f(x) = \sqrt[3]{4x}$; $g(x) = -9\sqrt[3]{4x}$; $x = -2$

2. $f(x) = 3x - 5x^2 - x^3$; $g(x) = 6x^2 - 4x$; $x = -1$

In Exercises 3–5, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x.

3. $f(x) = 3x^3$; $g(x) = \sqrt[3]{x^2}$; $x = -8$

4. $f(x) = 3x^2$; $g(x) = 5x^{1/4}$; $x = 16$

5. $f(x) = 10x^{5/6}$; $g(x) = 2x^{1/3}$; $x = 64$

In Exercises 6 and 7, use a graphing calculator to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

6. $f(x) = -3x^{1/3}$; $g(x) = 4x^{1/2}$

7. $f(x) = 6x^{3/4}$; $g(x) = 3x^{1/2}$

8. Describe and correct the error in stating the domain.

> ✗ $f(x) = 4x^{7/3}$ and $g(x) = 2x^{2/3}$
>
> The domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers.

9. The table shows the outputs of the two functions f and g. Use the table to evaluate $(f + g)(5)$, $(f - g)(0)$, $(fg)(3)$, and $\left(\frac{f}{g}\right)(2)$.

x	0	1	2	3	4	5
f(x)	18	13	8	3	−2	−7
g(x)	64	32	16	8	4	2

① $(f+g)(x) =$
$\sqrt[3]{4x} + -9\sqrt[3]{4x}$
$= \boxed{-8\sqrt[3]{4x}}$
$-8\sqrt[3]{4(-2)}$
$-8\sqrt[3]{-8}$
$-8(-2) = 16$
D: $(-\infty, \infty)$

4) $(f \cdot g)(x)$
$3x^2 \cdot 5x^{1/4}$
$\boxed{15x^{9/4}}$
$15\sqrt[4]{x^9}$
even root
D: $[0, \infty)$
$5\sqrt[4]{16^9}$
15.512
7680

5 $\left(\frac{f}{g}\right)(x)$
$= \frac{10x^{5/6}}{2x^{1/3}}$ $\frac{1}{3} = \frac{2}{6}$
can't be 0
$= 5x^{3/6}$
$= \boxed{5x^{1/2}}$ or
$5\sqrt{x}$
D: $(0, \infty)$
$5\sqrt{64}$
$5 \cdot 8$
40

$y = -y - 3$
$0 = -3$

Name Key $x^3-y^3=(x-y)(x^2+xy+y^2)$ Date $(x^3+y^3)(x^3-y^3)$

Inverse of a Function

EXPLORATION 1 Graphing Functions and Their Inverses

Work with a partner. Each pair of functions are *inverses* of each other. Use a graphing calculator to graph f and g in the same viewing window. What do you notice about the graphs?

$f(x) = 4x + 3$

$g(x) = \dfrac{x-3}{4}$

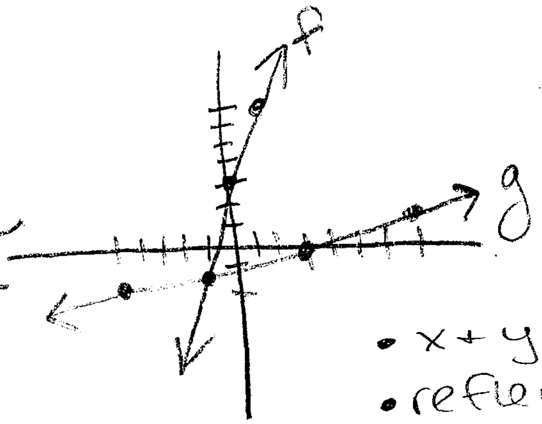
$f(x) = \sqrt{x-3}$

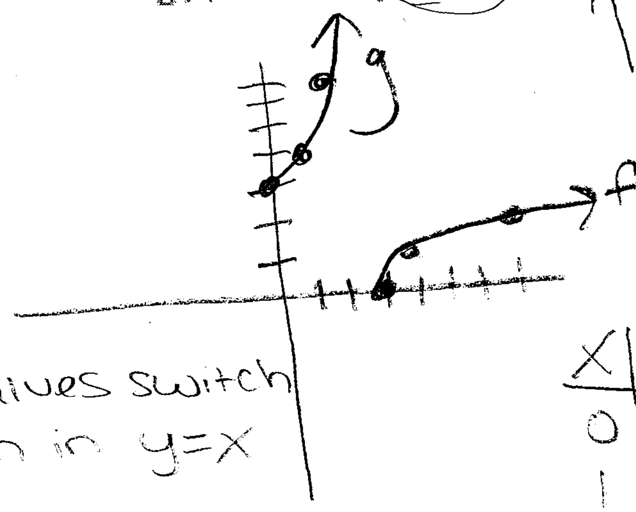
$g(x) = x^2 + 3, x \geq 0$

x	y
3	0
4	1
7	2

x	y
-2	-5
-1	-1
0	3
1	7
2	11

x	y
-5	-2
-1	-1
3	0
7	1
11	2





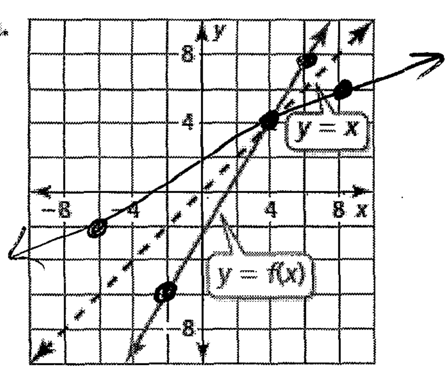
- x+y values switch
- reflection in y=x

x	y
0	3
1	4
2	7

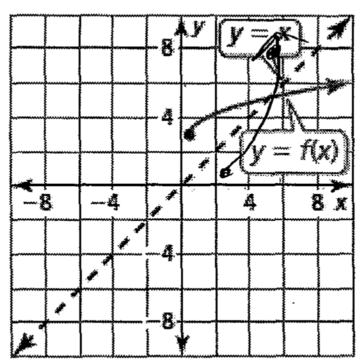
EXPLORATION 2 Sketching Graphs of Inverse Functions

Work with a partner. Use the graph of f to sketch the graph of g, the inverse function of f, on the same set of coordinate axes. Explain your reasoning.

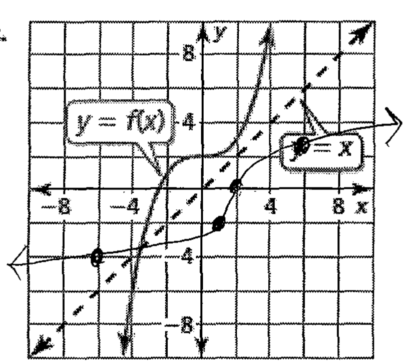
a.



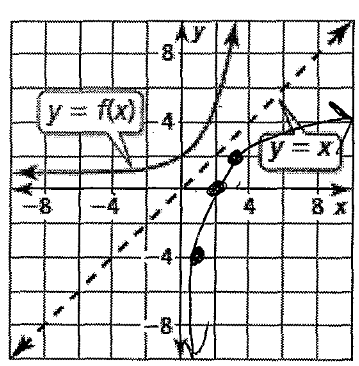
b.



c.



d.



EXAMPLE 1 **Writing a Formula for the Input of a Function**

Let $f(x) = 2x + 3$.

a. Solve $y = f(x)$ for x.

b. Find the input when the output is -7.

$x = 2y + 3$

$\frac{x-3}{2} = \frac{2y}{2}$

a) $y = 2x + 3$

$\frac{y-3}{2} = \frac{2x}{2}$

b) Find x when $y = -7$

$x = \frac{-7-3}{2} = -5$ $\therefore (-5, -7)$

EXAMPLE 2 **Finding the Inverse of a Linear Function**

Find the inverse of $f(x) = 3x - 1$.

$y = 3x - 1$

$x = 3y - 1$

$\frac{x+1}{3} = \frac{3y}{3}$

$f^{-1}(x) = \frac{x+1}{3} = \frac{1}{3}x + \frac{1}{3}$

OR reverse operations

"x" input is being mult. by 3, then sub.

so, ... add 1 to "x", then div. by 3

EXAMPLE 3 **Finding the Inverse of a Quadratic Function**

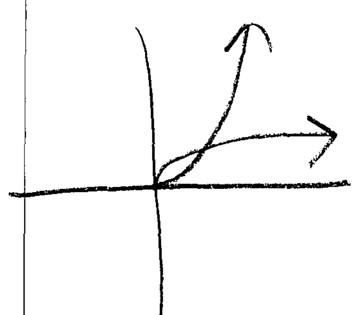
Find the inverse of $f(x) = x^2, x \geq 0$. Then graph the function and its inverse.

Why is domain restricted?

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$y = \pm\sqrt{x}$

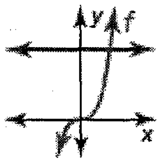


Core Concept

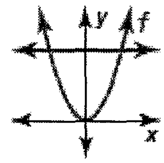
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function



Inverse is not a function



Steps:

① set $f(x) = y$

② switch x & y

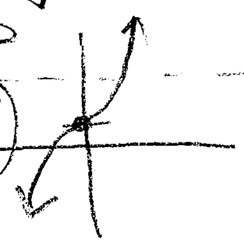
③ Solve for y

EXAMPLE 4 Finding the Inverse of a Cubic Function

Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

yes ✓

passes HLT



$y = 2x^3 + 1$

$x = 2y^3 + 1$

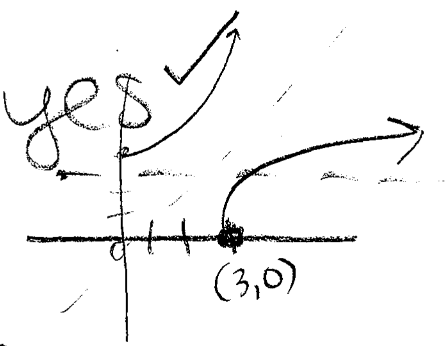
$\frac{x-1}{2} = \frac{2y^3}{2}$

$\sqrt[3]{y^3} = \sqrt[3]{\frac{x-1}{2}}$

EXAMPLE 5 Finding the Inverse of a Radical Function

Consider the function $f(x) = 2\sqrt{x-3}$. Determine whether the inverse of f is a function. Then find the inverse.

yes ✓



$y = 2\sqrt{x-3}$

$\frac{x}{2} = \frac{2\sqrt{y-3}}{2}$

$\left(\frac{x}{2}\right)^2 = \left(\sqrt{y-3}\right)^2$

$\frac{x^2}{4} = y - 3$
$+3 \quad +3$

$y = \frac{x^2}{4} + 3, \; x \geq 0$

Round-trip Thm:

$f(g(x))=x$

$g(f(x))=x$ ✓

EXAMPLE 6 **Verifying Functions Are Inverses**

Verify that $f(x) = 3x - 1$ and $g(x) = \dfrac{x+1}{3}$ are inverse functions.

$f(g(x)) = f\left(\frac{x+1}{3}\right) = \cancel{3}\left(\frac{x+1}{\cancel{3}}\right) - 1$

$= x + \cancel{1} - \cancel{1}$

$= x$ ✓

$g(f(x)) = g(3x-1) = \dfrac{3x + \cancel{1} + \cancel{-1}}{3} = \dfrac{\cancel{3}x}{\cancel{3}}$

$= x$ ✓

CK: find $f(g(3))$ and $g(f(3))$

$g(3) = \frac{4}{3}$

$f\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right) - 1 = 3$

$f(3) = 8$

$g(8) = \frac{8+1}{3} = 3$

Solving Real-Life Problems

In many real-life problems, formulas contain meaningful variables, such as the radius r in the formula for the surface area S of a sphere, $S = 4\pi r^2$. In this situation, switching the variables to find the inverse would create confusion by switching the meanings of S and r. So, when finding the inverse, solve for r without switching the variables.

* **EXAMPLE 7** **Solving a Multi-Step Problem**

Find the inverse of the function that represents the surface area of a sphere, $S = 4\pi r^2$. Then find the radius of a sphere that has a surface area of 100π square feet. $y = 4\pi x^2$

① ***

$$\boxed{\begin{aligned} \frac{S}{4\pi} &= \frac{4\pi r^2}{4\pi} \\ \sqrt{r^2} &= \sqrt{\frac{S}{4\pi}} \\ r &= \pm\sqrt{\frac{S}{4\pi}} \end{aligned}}$$

$\dfrac{x}{(4\pi)} = \dfrac{4\pi y^2}{(4\pi)}$

$\sqrt{y^2} = \sqrt{\dfrac{x}{(4\pi)}}$

$y = \pm\sqrt{\dfrac{x}{4\pi}}$

$r = \sqrt{\dfrac{S}{4\pi}}$

② $r = \sqrt{\dfrac{100\pi}{4\pi}} = \sqrt{25}$

$r = 5$

Name Key Inverses Date ________

5.6 Practice A

$y = \frac{1}{3}x - 2$

$3(y+2) = \frac{1}{3}x(3)$

$\boxed{x = 3y + 6}$ (−3)

$x = -9 + 6 = -3$

$\boxed{x = -3}$

In Exercises 1–3, solve $y = f(x)$ for x. Then find the input(s) when the output is -3.

1. $f(x) = 2x + 3$ 2. $f(x) = \frac{1}{3}x - 2$ 3. $f(x) = 8x^3$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

$x = \frac{1}{2}y - 5$

$2(x+5) = \frac{1}{2}y(2)$

$\boxed{2x + 10 = y}$

4. $f(x) = 4x$ 5. $f(x) = 4x - 1$ 6. $f(x) = \frac{1}{2}x - 5$

$x = \frac{1}{5}y - 2$

$x + 2 = \frac{1}{5}y(5)$

$\boxed{y = 5x + 10}$

7. Find the inverse of the function $f(x) = \frac{1}{5}x - 2$ by switching the roles of x and y and solving for y. Then find the inverse of the function f by using inverse operations in the reverse order. Which method do you prefer? Explain.

$y = \frac{1}{5}x - 2$

$+2 \quad +2$

$5(y+2) = \frac{1}{5}x(5)$

$\boxed{5y + 10 = x}$

8. Determine whether each pair of functions f and g are inverses. Explain your reasoning.

a.

x	−2	−1	0	1	2
f(x)	−3	3	9	15	21

x	−3	3	0	15	21
g(x)	−2	−1	0	1	2

NO.

b.

x	1	2	3	4	5
f(x)	9	7	5	3	1

x	9	7	5	3	1
g(x)	1	2	3	4	5

Yes.

In Exercises 9–11, find the inverse of the function. Then graph the function and its inverse.

$\frac{x}{9} = \frac{9y^2}{9}$

$y = \pm\frac{\sqrt{x}}{3}$

$\sqrt[3]{x} = \sqrt[3]{(y+2)^3}$

$\sqrt[3]{x} = y + 2$

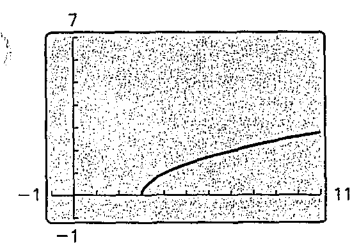
$-2 \quad -2$

$\boxed{y = \sqrt[3]{x} - 2}$

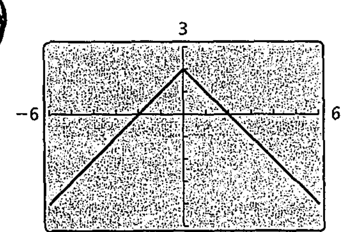
9. $f(x) = 9x^2, x \geq 0$ 10. $f(x) = 16x^2, x \leq 0$ 11. $f(x) = (x + 2)^3$

In Exercises 12 and 13, use the graph to determine whether the inverse of f is a function. Explain your reasoning.

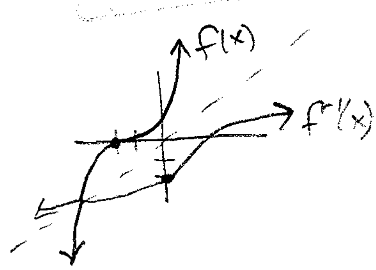
12.



13.



No, fails HLT.



(12) $f(g(x))$

$f(6x-1) = 6(6x-1)+1$

$36x-1+1$

$36x \cdot 5 \neq x$ No.

Name ______________________ Date _________

5.6 Practice B

In Exercises 1–3, solve $y = f(x)$ for x. Then find the input(s) when the output is -3.

1. $f(x) = -\frac{4}{3}x + 2$
2. $f(x) = 25x^4$
3. $f(x) = (x-3)^2 - 4$

In Exercises 4–6, find the inverse of the function. Then graph the function and its inverse.

$x = -3y + 4$

$x - 4 = -3y$

$\frac{x-4}{-3} = \frac{-3y}{-3}$

$y = \frac{x-4}{-3}$

4. $f(x) = -3x + 4$
5. $f(x) = -\frac{1}{3}x + 1$
6. $f(x) = \frac{2}{5}x - \frac{1}{5}$

7. Describe and correct the error in finding the inverse function.

$f(x) = 3x - 8$
$y = 3x - 8$
$x = 3y - 8$
$g(x) = 3x - 8$

$\sqrt[6]{x} = \sqrt[6]{y^6}$

$y = -\sqrt[6]{x}$

$f(g(x))$

$f(-\sqrt[6]{x})^6 = x$

$g(f(x))$

$-\sqrt[6]{x^6}$

In Exercises 8–10, find the inverse function. Then graph the function and its inverse.

$\sqrt[3]{x} = \sqrt[3]{(y-1)^3}$

8. $f(x) = -9x^2, x \le 0$
9. $f(x) = (x-1)^3$
10. $f(x) = x^6, x \le 0$

$y = \sqrt[3]{x} + 1$

11. Find the inverse of the function $f(x) = 8x^3$ by switching the roles of x and y and solving for y. Then find the inverse of the function f by using inverse operations in the reverse order. Which method do you prefer? Explain.

In Exercises 12–15, determine whether the functions are inverses.

12. $f(x) = 6x + 1$; $g(x) = 6x - 1$ No.
13. $f(x) = \frac{\sqrt[3]{x-6}}{2}$; $g(x) = 8x^3 + 6$
14. $f(x) = \frac{5-x}{2}$; $g(x) = 5 - 2x$
15. $f(x) = 4x^2 + 3$; $g(x) = -\frac{x-3}{4}$

14) $f(g(x))$

$f(5-2x) = \frac{5-(5-2x)}{2}$

$= \frac{5-5+2x}{2} = x$

$g(f(x)) =$

$g\left(\frac{5-x}{2}\right) =$

$5 - 2\left(\frac{5-x}{2}\right)$

$5 - 5 + x = x$ ✓

$f(g(x)) =$

$f(8x^3+6) = \frac{\sqrt[3]{8x^3+6-6}}{2}$

$\frac{\sqrt[3]{8x^3}}{2} = x$

$g(f(x)) =$

$g\left(\frac{\sqrt[3]{x-6}}{2}\right) = 8\left(\frac{\sqrt[3]{x-6}}{2}\right)^3 + 6$

$= 8\left(\frac{x-6}{8}\right) + 6$

$= x - 6 + 6$

$= x$ ✓

16. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius.

   a. Find the inverse function. Describe what it represents.

   b. Find the radius of a sphere with a volume of 146 cubic meters.

a) $\left(\frac{3}{4}\right)V = \frac{4}{3}\pi r^3\left(\frac{3}{4}\right)$

$\frac{3}{4}V = \frac{\pi r^3}{\pi}$

$r^3 = \frac{3}{4}V \cdot \frac{1}{\pi}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{3V}{4\pi}}$

$\boxed{r = \sqrt[3]{\frac{3V}{4\pi}}}$

b) $r = \sqrt[3]{\frac{3 \cdot 146}{4\pi}}$

$\boxed{r = 3.2665\text{ m}}$

Solving Radical Eq'ns

give to 1^{st} per Friday

1. isolate radical on one side of eq'n (if necessary)
2. raise each side of eq'n to the same exponent to eliminate the radical
3. Solve the resulting eq'n
4. Check.

EX: ① $2\sqrt{x+1} = 4$

$\frac{2\sqrt{x+1}}{2} = \frac{4}{2}$

$\left(\sqrt{x+1}\right)^2 = 2^2$

$x+1 = 4$

$\boxed{x=3}$ ✓

② $\sqrt[3]{2x-9} - 1 = 2$

$\left(\sqrt[3]{2x-9}\right)^3 = 3^3$

$2x-9 = 27$

$2x = 36$

$\boxed{x=18}$ ✓

③ $(x+1)^2 = \left(\sqrt{7x+15}\right)^2$

$x^2 + 2x + 1 = 7x + 15$

$x^2 - \cancel{2}\,5x - 14 = 0$

$(x-7)(x+2) = 0$

✓ $\boxed{x=7}$ $\quad$ $x = \cancel{-2}$ extraneous

④ $(2x)^{3/4} + 2 = 10$

$\left[(2x)^{3/4}\right]^{4/3} = 8^{4/3}$

$2x = \sqrt[3]{8}^{\,4}$

$2x = 2^4$

$2x = 16$

$\boxed{x=8}$ ✓

OR

$\left(\sqrt[4]{(2x)^3}\right)^4 = 8^4$

$\sqrt[3]{(2x)^3} = \sqrt[3]{8^4}$

4096

$2x = 16$

$\boxed{x=8}$

Last 20 points Quest

Jan 17 #37

#8, 11, 16

Name____ Date__________

5.4 Practic A

In Exercises 1–6, solve the uation. Check your solution.

1. $\sqrt{3x-2} = 5$ 9
2. $\sqrt{6x+1} = 9$ $\frac{40}{3}$
3. $\sqrt[3]{x+10} = 4$ 54
4. $\sqrt[3]{x} - 8 = -2$ 216
5. $-3\sqrt{16x} + 14 = -10$ 4
6. $6\sqrt[3]{25x} - 16 = 14$ 5

7. Biologists have discov d that the shoulder height h (in centimeters) of a male Asian elephant can be deled by $h = 62.5\sqrt[3]{t} + 75.8$, where t is the age (in years) of the elepha . Determine the age of an elephant with a shoulder height of 300 centimet .

In Exercises 8–13, solve th equation. Check your solution(s).

8. $x - 8 = \sqrt{4x}$ 16
9. $\sqrt{2x-14} = x - 7$ 7, 9
10. $\sqrt{x+22} = x + 2$
11. $\sqrt[3]{8x^3+27} = 2x + 3$ $0, -\frac{3}{2}$
12. $\sqrt[4]{2-9x^2} = 3x$
13. $\sqrt{3x-5} = \sqrt{x+9}$ 7

In Exercises 14–16, solve t equation. Check your solution(s).

14. $2x^{2/3} = 18$

$(x^{2/3})^{3/2} = 9^{3/2}$, $x = 27$

15. $x^{3/4} + 10 = 0$

16. $[(x+12)^{1/2}]^2 = x^2$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

4 ✓ ~~−3~~

17. Describe and correct t error in solving the equation.

✗ $\sqrt[3]{2x+1} = $
$2x + 1 = $
$2x = $
$x = $

In Exercises 18–20, solve inequality.

18. $3\sqrt{x} - 4 \geq 5$ $x \geq 9$
19. $\sqrt{x-3} \leq 7$ $3 \leq x \leq 52$
20. $5\sqrt{x-1} > 10$ $x > 5$

$x \geq 2$

$\frac{5\sqrt{x-2}}{5} \leq \frac{1(}{5}$

$\sqrt{x-2}^2 \leq 2^2$

$x - 2 \leq 4$

$\boxed{2 \leq x \leq 6}$

21. The length ℓ (in inch) of a standard nail can be modeled by $\ell = 54d^{3/2}$, where d is the diameter (in i hes) of the nail.

a. What is the diame r of a standard nail that is 2 inches long?

b. What is the diame r of a standard nail that is 4 inches long?

c. The nail in part (b s twice as long as the nail in part (a). Is the diameter twice as long? Ex ain.

Base Unknown

Kuta Software - Infinite Algebra 2 Name Key

Square Root Equations

Tues.

Date________________ Period____

Solve each equation. Remember to check for extraneous solutions.

CW: O

HW: □

1) $3^2 = \sqrt{b-1}^{\,2}$

$9 = b - 1$

$\boxed{b = 10}$

2) $2^2 = \sqrt{\frac{x}{2}}^{\,2}$

$(2)\,4 = \frac{x}{2}\,(2)$

$\boxed{x = 8}$

3) $\sqrt{-8-2a}^{\,2} = 0^2$

$-8 - 2a = 0$

$\frac{+8}{-2} = \frac{-2a}{-2}$ $\boxed{a = -4}$

4) $\sqrt{x+4}^{\,2} = 0^2$

$x + 4 = 0$

$\boxed{x = -4}$

5) $5^2 = \sqrt{r-3}^{\,2}$

$25 = r - 3$

$\boxed{r = 28}$

6) $\sqrt{2m-6}^{\,2} = \sqrt{3m-14}^{\,2}$

$2m - 6 = 3m - 14$

$-2m + 14 \quad -2m + 14$

$\boxed{8 = m}$

7) $\sqrt{8k}^{\,2} = k^2$

$k^2 = 8k$

$k^2 - 8k = 0$

$k(k-8) = 0$

$\boxed{k = 0, 8}$

8) $\sqrt{9-b}^{\,2} = \sqrt{1-9b}^{\,2}$

$9 - b = 1 - 9b$

$-9 + 9b \quad -9 + 9b$

$8b = -8$

$\boxed{b = -1}$

9) $\sqrt{3-2x}^{\,2} = \sqrt{1-3x}^{\,2}$

$3 - 2x = 1 - 3x$

$-3 + 3x \quad -3 + 3x$

$\boxed{x = -2}$

10) $\sqrt{3k-11}^{\,2} = \sqrt{5-k}^{\,2}$

$3k - 11 = 5 - k$

$+k + 11 \quad +11 + k$

$4k = 16$

$\boxed{k = 4}$

11) $(20-r)^{\frac{1}{2}} = r$

$\sqrt{20-r}^{\,2} = r^2$

$20 - r = r^2$

$r^2 + r - 20 = 0$

$(r+5)(r-4) = 0$

$\cancel{-5}$ | 4

12) $(6b)^{\frac{1}{2}} = (8-2b)^{\frac{1}{2}}$

$6b = 8 - 2b$

$8b = 8$

$\boxed{b = 1}$

13) $\sqrt{56-r} = r$

$56 - r = r^2$

$r^2 + r - 56 = 0$

$(r+8)(r-7) = 0$

$\cancel{-8}$ | 7

14) $\sqrt{-10+7p} = p$

$p^2 - 7p + 10 = 0$

$(p-5)(p-2) = 0$

5 | 2

15) $(18-n)^{\frac{1}{2}} = \left(\frac{n}{8}\right)^{\frac{1}{2}}$

$8(18-n) = \frac{n}{8}(8)$

$144 - 8n = n$

$144 = 9n$ $\boxed{n = 16}$

16) $\sqrt{2v-7} = v - 3$

$2v - 7 = v^2 - 6v + 9$

$0 = v^2 - 8v + 16$

$0 = (v-4)(v-4)$

4 | 4

1 - 0

3 - 2

17) $-3 = (37-3n)^{\frac{1}{2}} - n$

$(-3+n)^2 = (37-3n)^{\frac{1}{2} \cdot 2}$

$n^2 - 6n + 9 = 37 - 3n$

$n^2 - 3n - 28 = 0$

$(n-7)(n+4) = 0$

7 | $\cancel{-4}$

don't do here

18) $(-3-4x)^{\frac{1}{2}} - (-2-2x)^{\frac{1}{2}} = 1$

$(\sqrt{-2-2x} + 1)$

$\left(\sqrt{-3-4x} = \sqrt{-2-2x} + 1\right)^2$

$-3 - 4x = -2 - 2x + 2\sqrt{-2-2x}$ $+1$

$-3 - 4x = -2x + 2\sqrt{-2-2x} + 1$

$+1 \; +2x \quad +2x \quad +1$

$-2 - 2x = 2\sqrt{-2-2x}$

$(-1-x)$

$(-1-x)^2 = (\sqrt{-2-2x})^2$

$1 + 2x + x^2 = -2-2x$

$+2+2x \quad +2+2x$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

-3 | -1

19) $x = 5 + (3x-11)^{\frac{1}{2}}$

$-5 \quad -5$

$(x-5)^2 = (3x-11)^{\frac{1}{2} \cdot 2}$

$x^2 - 10x + 25 = 3x - 11$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

9 | $\cancel{4}$

20) $2 = \sqrt{3b-2} - \sqrt{10-b}$

$(2 + \sqrt{10-b})^2 = \sqrt{3b-2}^{\,2}$

$4 + 4\sqrt{10-b} + 10 - b = 3b - 2$

$-10 + b \quad -4 - 10 + b$

$\frac{4\sqrt{10-b}}{4} = \frac{4b - 16}{4}$

$\sqrt{10-b}^{\,2} = (b-4)^2$

$10 - b = b^2 - 8b + 16$

$b^2 - 7b + 6 = 0$

$(b-6)(b-1) = 0$

$b = 6, \cancel{1}$

4 - 2

1 - 3

[Base Unknown]

Kuta Software - Infinite Algebra 2 Thurs. Name Key

Rational Exponent Equations

Date________________ Period____

Solve each equation.

1) $27^{\frac{2}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}$

$x = \sqrt[3]{27}^{\,2}$

$\boxed{x = 9}$

2) $\left(m^{\frac{3}{4}}\right)^{\frac{4}{3}} = 27^{\frac{4}{3}}$

$m = \sqrt[3]{27}^{\,4}$

$m = 3^4$

$\boxed{m = 81}$

3) $\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = \dfrac{1}{729}^{-\frac{2}{3}}$

$x = \sqrt[3]{729}^{\,2}$

$x = 9^2$

$\boxed{x = 81}$

4) $7^2 = \left(r^{\frac{1}{2}}\right)^2$

$\boxed{49 = r}$

5) $\left(v^{\frac{5}{4}}\right)^{\frac{4}{5}} = 243^{\frac{4}{5}}$

$v = \sqrt[5]{243}^{\,4}$

$v = 3^4$

$\boxed{v = 81}$

6) $\left(n^{\frac{3}{2}}\right)^{\frac{2}{3}} = 125^{\frac{2}{3}}$

$n = \sqrt[3]{125}^{\,2}$

$n = 5^2$

$\boxed{n = 25}$

7) $\left((n-27)^{\frac{3}{2}}\right)^{\frac{2}{3}} = 64^{\frac{2}{3}}$

$n - 27 = \sqrt[3]{64}^{\,2}$

$n - 27 = 16$

$\boxed{n = 43}$

8) $26 = -1 + (27x)^{\frac{3}{4}}$

$27^{\frac{4}{3}} = \left((27x)^{\frac{3}{4}}\right)^{\frac{4}{3}}$

$\sqrt[3]{27}^{\,4} = 27x$

$81 = 27x$

$\boxed{x = 3}$

9) $3125 = (-1-18p)^{\frac{5}{3}}$

$\sqrt[5]{3125^3} = -1-18p$

$5^3 = -1-18p$

$125 = -1-18p$

$126 = -18p$

$\boxed{p=-7}$

10) $5 = 3 + 4a^{-\frac{1}{6}}$

$\frac{2}{4} = \frac{4a^{-1/6}}{4}$

$\left(\frac{1}{2}\right)^{-6} = \left(a^{-1/6}\right)^{-6}$

$2^6 = \boxed{64}$

11) $4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$ $-\frac{20}{2}$

$\frac{4b^{-3/4}}{4} = \frac{\frac{1}{2}}{4}$

$\left(b^{-3/4} = \frac{1}{8}\right)^{-4/3}$

$b = \sqrt[3]{8}^{\,4}$

$2^4 = \boxed{16}$

12) $\frac{-x^{\frac{3}{2}}}{-1} = \frac{-27}{-1}$

$\left(x^{3/2}\right)^{2/3} = 27^{2/3}$

$x = \sqrt[3]{27}^{\,2} = 3^2 = \boxed{9}$

13) $-54 = 10 - (m-10)^{\frac{3}{2}}$

$-10 \quad -10$

$-64 = -(m-10)^{3/2}$

$64^{2/3} = \left((m-10)^{3/2}\right)^{2/3}$

$\sqrt[3]{64}^{\,2} = m - 10$

$+10$

$\boxed{m=26}$

14) $-5126 = -6 - 5(3x+22)^{\frac{5}{3}}$

$+6 \quad +6$

$-5120 = -5(3x+22)^{5/3}$

$1024^{3/5} = \left[(3x+22)^{5/3}\right]^{3/5}$

$\sqrt[5]{1024^3} = 3x+22$

$4^3 \rightarrow 64 = 3x+22$

$-22 \quad -22$

$42 = 3x$

$\boxed{x=14}$

15) $9 + 5\sqrt[3]{2m} = 29$

$5\sqrt[3]{2m} = 20$

$\sqrt[3]{2m} = 4^3$

$2m = 64$

$\boxed{m=32}$

16) $3646 = 1 + 5(4r+17)^{\frac{3}{2}}$

$3645 = 5(4r+17)^{3/2}$

$729^{2/3} = \left[(4r+17)^{3/2}\right]^{2/3}$

$9^2 = 4r+17 \longrightarrow 81 = 4r+17$

$64 = 4r$

$\boxed{r=16}$

17) $-646 = -3(65-n)^{\frac{3}{2}} + 2$

$\frac{-648}{-3} = \frac{-3(65-n)^{3/2}}{-3}$

$216^{2/3} = \left[(65-n)^{3/2}\right]^{2/3}$

$\left(\sqrt[3]{216}\right)^2 = 65-n$

$36 = 65-n$

$-65 \quad -65$

$-29 = -n$

$\boxed{n=29}$

18) $-3 + (8-2x)^{\frac{5}{4}} = 29$

$\left[(8-2x)^{5/4}\right]^{4/5} = 32^{4/5}$

$8-2x = \sqrt[5]{32}^{\,4}$

$8-2x = 16$

$-8 \quad -8$

$-2x = 8$

$\boxed{x=-4}$

Review:

① $\sqrt{9x^2} \cdot x^{\frac{4}{3}} + \sqrt[3]{27x^5}$

$3x^{3/3} \cdot x^{4/3}$

$3x^{7/3} + 3x^{5/3}$

② $x^2 = \sqrt{2x+8}^{\,2}$

$x^2 = 2x+8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ | $\cancel{x = -2}$

③ $x = 5 + \sqrt{3x-11}$

$(x-5)^2 = \sqrt{3x-11}^{\,2}$

$x^2 - 10x + 25 = 3x - 11$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

$x = 9$ | $\cancel{x = 4}$

2-PUZZLES

④ $\frac{2x^{4/5}}{2} = \frac{32}{2}$

$(x^{4/5})^{5/4} = 16^{5/4}$

$x = \sqrt[4]{16}^{\,5}$

$\boxed{x = 32}$

⑤ $\left[(n+10)^{-2/3}\right]^{-3/2} = \frac{1}{16}^{-3/2}$

$n + 10 = 16^{3/2}$

$n + 10 = \sqrt{16}^{\,3}$

$n + 10 = 64$

$\boxed{n = 54}$

⑥ $f^{-1}(x)$ of $f(x) = \frac{1}{2}x + 5$

$x = \frac{1}{2}y + 5$

$2(x-5) = \frac{1}{2}y \cdot 2$

$\boxed{f^{-1}(x) = 2x - 10}$

(7) $f^{-1}(x)$ of $f(x) = \sqrt[3]{x+4}$ → Is inv. a fcn?

↓

yes, passes H

$x^3 = \sqrt[3]{y+4}^{\,3}$

$x^3 = y+4$

$\boxed{y = x^3 - 4} = f^{-1}(x)$

Prove: $f(g(x))$

$f(x^3-4)$

$= \sqrt[3]{x^3-4+4}$

$= \sqrt[3]{x^3} = x$ ✓

$g(f(x))$

$g(\sqrt[3]{x+4})$

$= (\sqrt[3]{x+4})^3 - 4$

$= x+4-4$

$= x$ ✓

(8) $f(x) = 4x^{1/2}$

$g(x) = \frac{5}{2}x^{5/2}$

$(f \cdot g)(x) = 4x^{1/2} \cdot \frac{5}{2} x^{5/2}$

$= 10x^{6/2} = 10x^3$ D: $(-\infty, \infty)$

↓

What if it was

$10x^{3/2}$

↓

D: $[0, \infty)$

$(f \cdot g)(-2) = -80$