

Name: Key

Date:

Wed.
CW+HW

Unit ~ Fractional Exponents & Perfects

Look For 'Perfects'

Evaluate.

1. $8^{1/3} = \sqrt[3]{8} = 2$

2. $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

3. $25^{-1/2} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

4. $49^{3/2} = \sqrt{49}^3 = 7^3 = 343$

5. $32^{-2/5} = \frac{1}{\sqrt[5]{32}^2} = \frac{1}{2^2} = \frac{1}{4}$

6. $(\frac{1}{64})^{2/3} = (\sqrt[3]{\frac{1}{64}})^2 = (\frac{1}{4})^2 = \frac{1}{16}$

7. $(\frac{27}{125})^{-1/3} = \frac{1}{\sqrt[3]{\frac{27}{125}}} = \frac{5}{3}$

8. $(\frac{1}{81})^{-3/4} = 81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

9. $(0.008)^{4/3} = (\frac{8}{1000})^{4/3} = (\sqrt[3]{\frac{8}{1000}})^4 = (\frac{2}{10})^4 = \frac{16}{10,000}$

10. $(9+16)^{1/2} = 25^{1/2} = 5$

11. $(\frac{1}{25})^{-1/2} = 25^{1/2} = 5$

12. $\frac{64^{1/6}}{2^{1/3}} = \frac{(2^6)^{1/6}}{2^{1/3}} = \frac{2^1}{2^{1/3}} = 2^{2/3}$
or $\sqrt[3]{4}$

Rewrite in radical form.

13. $x^{3/2} = \sqrt{x^3}$

14. $x^{2/3} y^{5/6} = \sqrt[3]{x^2} \cdot \sqrt[6]{y^5} = \sqrt[6]{x^4 y^5}$

15. $4x^{1/2} y^{5/2} = 4\sqrt{x} \cdot \sqrt{y^5} = 4\sqrt{xy^5}$

When simplifying radicals, the index tells us:

(root)
the denominator of the exponent

Write each with a rational exponent and simplify if possible.

16. $\sqrt[3]{81}$ 3

17. $\sqrt[3]{x^5}$ $x^{5/3}$

18. $\sqrt[4]{a^6 b^7 c^1}$ $a^{6/4} b^{7/4} c^{1/4} = a^{3/2} b^{7/4} c^{1/4}$

19. $\sqrt[3]{16xy^3}$ $\sqrt[3]{8} \cdot \sqrt[3]{2} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y^3} = 2\sqrt[3]{2x} = 2\sqrt[3]{2x}^{1/3}$
 OR $16^{1/3} x^{1/3} y^{3/3} = 16^{1/3} x^{1/3} y$

20. $\sqrt[5]{10x^{15}y^{12}}$ $10^{1/5} x^{3/5} y^{12/5}$
 $10^{1/5} (5x^3)^{3/5}$
 $(8 \cdot 2)^{1/3} (16x)^{1/3} y$
 $8^{1/3} \cdot 2^{1/3} = 2\sqrt[3]{2x}$

Simplify each and keep answer in radical form, simplify if possible.

21. $10\sqrt[3]{8a^4b}$ $10 \cdot \sqrt[3]{8} \cdot \sqrt[3]{a^3} \cdot \sqrt[3]{a} \cdot \sqrt[3]{b}$
 $10 \cdot 2 \cdot a \cdot \sqrt[3]{ab} = 20a\sqrt[3]{ab}$

22. $\sqrt[3]{125x^9}$ $5x^3 = 5x^3$

23. $\sqrt[6]{27}$ $27^{1/6} = (3^3)^{1/6} = 3^{3/6} = 3^{1/2} = \sqrt{3}$

24. $\sqrt[8]{16y^4}$ $16^{1/8} y^{4/8}$
 $(4^2)^{1/8} y^{1/2}$
 $4^{1/4} y^{1/2}$
 $(2^2)^{1/4} y^{1/2} = \sqrt{2y}$

$\sqrt{x^2} = |x|$
 $\sqrt{x^{10}} = |x^5|$
 $\sqrt[4]{a^6} = \sqrt[4]{a^4} \cdot \sqrt[4]{a^2} = |a| \cdot \sqrt[4]{a^2} = |a| \cdot a^{1/2}$

Exponential Equations Not Requiring Logarithms

Date

Period

Solve each equation.

Power Unknown

1) $5^{3n} = 125$

$$5^{3n} = 5^3$$

$$3n = 3$$

$$\boxed{n = 1}$$

3) $5^{3r} = 5^{-2r}$

$$3r = -2r$$

$$+2r \quad +2r$$

$$\hline 5r = 0$$

$$\boxed{r = 0}$$

5) $2^{-3x} = 2^{x-1}$

$$-3x = x - 1$$

$$-x \quad -x$$

$$\hline -4x = -1$$

$$\hline -4 \quad -4$$

$$\boxed{x = \frac{1}{4}}$$

7) $10^{3x} = 10000$

$$10^{3x} = 10^4$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}}$$

9) $\left(\frac{1}{8}\right)^{3x} \cdot 64^{2x+1} = 4$

$$(8^{-1})^{3x} \cdot (8^2)^{2x+1}$$

$$8^{-3x} \cdot 8^{4x+2}$$

$$8^{x+2} = 4$$

$$(2^3)^{x+2} = 2^2$$

$$3x+6=2$$

$$\boxed{x = -\frac{4}{3}}$$

11) $6^{-3v-2} = 36$

$$6^{-3v-2} = 6^2$$

$$-3v-2 = 2$$

$$-3v = 4$$

$$\boxed{v = -\frac{4}{3}}$$

2) $2^{2n} = 16$

$$2^{2n} = 2^4$$

$$2n = 4$$

$$\boxed{n = 2}$$

4) $3^{-2k} = 81$

$$3^{-2k} = 3^4$$

$$-2k = 4$$

$$\boxed{k = -2}$$

6) $6^{3m} = 36$

$$6^{3m} = 6^2$$

$$\boxed{\frac{3m}{3} = \frac{2}{3}}$$

8) $4^{r+1} = 1$

$$4^{r+1} = 4^0$$

$$r+1 = 0$$

$$\boxed{r = -1}$$

10) $32^{2x} = 8$

$$(2^5)^{2x} = 2^3$$

$$\boxed{\frac{10x}{10} = \frac{3}{10}}$$

12) $243^x = 81$

$$(3^5)^x = 3^4$$

$$\boxed{\frac{5x}{5} = \frac{4}{5}}$$

$$2^{-n+1} = 2^{-2n}$$

$$-n+1 = -2n$$

$$1 = -n \quad \boxed{n = -1}$$

$$15) 3^{2n} = 9$$

$$3^{2n} = 3^2$$

$$2n = 2$$

$$\boxed{n = 1}$$

$$17) \frac{216^{2a}}{36^{-a}} = 216$$

$$\frac{(6^3)^{2a}}{(6^2)^{-a}} = 6^3 \rightarrow \frac{6^{6a}}{6^{-2a}} = 6^3$$

$$\begin{array}{r} 6^{6a} \\ \downarrow \\ 6^{8a} \\ \downarrow \\ 6^{8a} \end{array} \quad \begin{array}{r} 6^3 \\ \downarrow \\ 6^3 \\ \downarrow \\ 6^3 \end{array}$$

$$19) \frac{16}{\left(\frac{1}{16}\right)^x} = \left(\frac{1}{4}\right)^{2-2x}$$

$$\frac{4^2}{4^{-2x}} = (4^{-1})^{2-2x}$$

$$4^{2+2x} = 4^{-2+2x}$$

$$\begin{array}{r} 2+2x = -2+2x \\ +7x \quad +7x \\ \hline 1 = 0 \end{array}$$

No Solution

$$21) 27^{3x} \cdot \left(\frac{1}{9}\right)^{-x} = 243^{-x-3}$$

$$(3^3)^{3x} \cdot (3^{-2})^{-x} = (3^5)^{-x-3}$$

$$\begin{array}{r} 3^{9x} \cdot 3^{2x} = 3^{-5x-15} \\ 11x = -5x-15 \\ +5x \quad +5x \\ \hline 16x = -15 \end{array} \quad \begin{array}{r} 16x = -15 \\ \hline \frac{16x}{16} = \frac{-15}{16} \end{array} \quad \boxed{\frac{-15}{16}}$$

$$23) 125^{3n} \cdot 625^{-n} = 625^{-2n-1}$$

$$(5^3)^{3n} \cdot (5^4)^{-n} = (5^4)^{-2n-1}$$

$$9n + -4n = -8n - 4$$

$$5n = -8n - 4$$

$$\frac{13n}{13} = \frac{-4}{13} \quad \boxed{\frac{-4}{13}}$$

$$14) \left(\frac{1}{16}\right)^{2a} \cdot 16^{-2a-3} = 64^{2a}$$

$$(4^{-2})^{2a} \cdot (4^2)^{-2a-3} = (4^3)^{2a}$$

$$4^{-4a} \cdot 4^{-4a-6} = 4^{6a}$$

$$-4a + -4a - 6 = 6a$$

$$-8a - 6 = 6a$$

$$-6 = 14a$$

$$\frac{-6}{14} = \frac{14a}{14}$$

$$\boxed{a = \frac{-3}{7}}$$

$$16) \left(\frac{1}{6}\right)^n = 36$$

$$(6^{-1})^n = 6^2$$

$$-n = 2$$

$$\boxed{n = -2}$$

$$18) 36^{3n-2} \cdot \frac{1}{216} = 36^{3n-2}$$

$$(6^2)^{3n-2} \cdot \frac{1}{6^3} = (6^2)^{3n-2}$$

$$6^{6n-4} \cdot 6^{-3} = 6^{6n-4}$$

$$6n-4 + -3 = 6n-4$$

can't be!

$$20) 25 \cdot 125^{-3} = 625^{-6n+7}$$

$$5^2 \cdot 125^{-3} = 5^4$$

0 = 3x
NO SOLUTION

$$\begin{array}{r} 2-3x = 4 \\ +2 \quad -2 \\ \hline -3x = 2 \end{array}$$

$$\frac{-3x}{-3} = \frac{2}{-3} \quad \boxed{\frac{-2}{3}}$$

$$22) \left(\frac{1}{6}\right)^{3a} \cdot 36^{-3a} = \frac{1}{36}$$

$$6^{-3a} \cdot (6^2)^{-3a} = 6^{-2}$$

$$\begin{array}{r} -9a = -2 \\ \hline \frac{-9a}{-9} = \frac{-2}{-9} \end{array} \quad \boxed{a = \frac{2}{9}}$$

$$24) \frac{125}{25^{-3m}} = 25^{-2m-2}$$

$$\frac{5^3}{(5^2)^{-3m}} = (5^2)^{-2m-2}$$

$$\begin{array}{r} 3+6m = -4m-4 \\ +4m \quad +4m \\ \hline 3+10m = -4-4 \end{array}$$

$$\frac{10m}{10} = \frac{-7}{10} \quad \boxed{\frac{-7}{10}}$$

Square Root Equations

Tues.

Date

Period

Solve each equation. Remember to check for extraneous solutions.

1) $3 = \sqrt{b-1}$

$9 = b - 1$

$b = 10$

2) $2 = \sqrt{\frac{x}{2}}$

$4 = \frac{x}{2}$

$x = 8$

CW: 0

HW:

3) $\sqrt{-8-2a} = 0$

$-8 - 2a = 0$

$-8 = 2a$
 $\frac{-8}{2} = \frac{2a}{2}$
 $a = -4$

4) $\sqrt{x+4} = 0$

$x + 4 = 0$

$x = -4$

5) $5 = \sqrt{r-3}$

$25 = r - 3$

$r = 28$

6) $\sqrt{2m-6} = \sqrt{3m-14}$

$2m - 6 = 3m - 14$
 $-2m + 14 - 2m + 14$

$8 = m$

7) $\sqrt{8k} = k$

$k^2 = 8k$

$k^2 - 8k = 0$

$k(k-8) = 0$

$k = 0, 8$

8) $\sqrt{9-b} = \sqrt{1-9b}$

$9-b = 1-9b$

$-8+9b - 8+9b$

$8b = -8$

$b = -1$

9) $\sqrt{3-2x} = \sqrt{1-3x}$

$3-2x = 1-3x$

$3+3x - 3+3x$

$x = -2$

10) $\sqrt{3k-11} = \sqrt{5-k}$

$3k-11 = 5-k$

$+k+11 +11+k$

$4k = 16$

$k = 4$

$$(11) (20-r)^{\frac{1}{2}} = r$$

$$\sqrt{20-r} = r$$

$$20-r = r^2$$

$$r^2 + r - 20 = 0$$

$$(r+5)(r-4) = 0$$

$$\boxed{13} \sqrt{56-r} = r^2$$

$$56-r = r^2$$

$$r^2 + r - 56 = 0$$

$$(r+8)(r-7) = 0$$

$$\boxed{15} (18-n)^{\frac{1}{2}} = \left(\frac{n}{8}\right)^{\frac{1}{2}}$$

$$8(18-n) = \frac{n}{8}$$

$$144 - 8n = n$$

$$144 = 9n \quad \boxed{n=16}$$

$$\boxed{17} -3 = (37-3n)^{\frac{1}{2}} - n$$

$$(-3+n) = (37-3n)^{\frac{1}{2}}$$

$$n^2 - 6n + 9 = 37 - 3n$$

$$n^2 - 3n - 28 = 0$$

$$(n-7)(n+4) = 0$$

$$\boxed{19} x = 5 + (3x-11)^{\frac{1}{2}}$$

$$(x-5)^2 = (3x-11)^2$$

$$x^2 - 10x + 25 = 3x - 11$$

$$x^2 - 13x + 36 = 0$$

$$(x-9)(x-4) = 0$$

$$\boxed{9} \quad 4$$

$$12) (6b)^{\frac{1}{2}} = (8-2b)^{\frac{1}{2}}$$

$$6b = 8 - 2b$$

$$8b = 8$$

$$\boxed{b=1}$$

$$\boxed{14} \sqrt{-10+7p} = p$$

$$p^2 - 7p + 10 = 0$$

$$(p-5)(p-2) = 0$$

$$16) \sqrt{2v-7} = v-3$$

$$2v-7 = v^2 - 6v + 9$$

$$0 = v^2 - 8v + 16$$

$$0 = (v-4)(v-4)$$

$$\boxed{4} \quad 4$$

$$\boxed{18} (-3-4x)^{\frac{1}{2}} - (-2-2x)^{\frac{1}{2}} = 1 \quad (\sqrt{-2-2x} + 1)$$

$$\sqrt{-3-4x} = \sqrt{-2-2x} + 1$$

$$-3-4x = -2-2x + 2\sqrt{-2-2x} + 1$$

$$-3-4x = -2-2x + 2\sqrt{-2-2x} + 1$$

$$-2-2x = 2\sqrt{-2-2x}$$

$$\boxed{20} 2 = \sqrt{3b-2} - \sqrt{10-b}$$

$$(2 + \sqrt{10-b}) = \sqrt{3b-2}$$

$$4 + 4\sqrt{10-b} + 10 - b = 3b - 2$$

$$4\sqrt{10-b} = 4b - 16$$

$$\sqrt{10-b} = (b-4)$$

$$10-b = b^2 - 8b + 16$$

$$b^2 - 7b + 6 = 0$$

$$(b-6)(b-1) = 0$$

$$b = \boxed{6}$$

$$(-1-x)^2 = (\sqrt{-2-2x})^2$$

$$1 + 2x + x^2 = -2 - 2x + 2x + x^2$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$\boxed{-3} \quad \boxed{-1}$$

Rational Exponent Equations

Solve each equation.

1) $27 = (x^2)^{\frac{2}{3}}$

$x = \sqrt[3]{27^2}$

$x = 9$

3) $(x^{-\frac{3}{2}})^{\frac{2}{3}} = \frac{1}{729}^{-\frac{2}{3}}$

$x = \sqrt[3]{729^2}$

$x = 9^2$

$x = 81$

5) $(v^{\frac{4}{5}})^{\frac{5}{4}} = 243^{\frac{4}{5}}$

$v = \sqrt[5]{243^4}$

$v = 3^4$

$v = 81$

7) $((n-27)^{\frac{3}{2}})^{\frac{2}{3}} = 64^{\frac{2}{3}}$

$n-27 = \sqrt[3]{64^2}$

$n-27 = 16$

$n = 43$

2) $(m^{\frac{3}{4}})^{\frac{4}{3}} = 27^{\frac{4}{3}}$

$m = \sqrt[3]{27^4}$

$m = 3^4$

$m = 81$

4) $7 = (r^2)^{\frac{1}{2}}$

$49 = r$

6) $(n^{\frac{3}{2}})^{\frac{2}{3}} = 125^{\frac{2}{3}}$

$n = \sqrt[3]{125^2}$

$n = 5^2$

$n = 25$

8) $26 = -1 + (27x)^{\frac{3}{4}}$

$27 = (27x)^{\frac{3}{4}}$

$\sqrt[3]{27^4} = 27x$

$81 = 27x$

$x = 3$

$$9) 3125 = (-1 - 18p)^3$$

$$\sqrt[5]{3125^3} = -1 - 18p$$

$$5^3 = -1 - 18p$$

$$125 = -1 - 18p$$

$$126 = -18p \quad \boxed{p = -7}$$

$$11) 4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$$

$$4b^{-\frac{3}{4}} = \frac{21}{2} - 10 = \frac{1}{2}$$

$$b^{-\frac{3}{4}} = \frac{1}{8}$$

$$b = \left(\frac{1}{8}\right)^{-\frac{4}{3}} = 16 \quad \boxed{16}$$

$$13) -54 = 10 - (m - 10)^{\frac{3}{2}}$$

$$-64 = -(m - 10)^{\frac{3}{2}}$$

$$64 = (m - 10)^{\frac{3}{2}}$$

$$\sqrt[3]{64^2} = m - 10$$

$$+10 \quad \boxed{m = 26}$$

$$15) 9 - 5\sqrt[3]{2m} = 29$$

$$5\sqrt[3]{2m} = 20$$

$$\sqrt[3]{2m} = 4$$

$$2m = 64$$

$$\boxed{m = 32}$$

$$17) -648 = -3(65 - n)^{\frac{3}{2}} + 2$$

$$-648 = -3(65 - n)^{\frac{3}{2}} + 2$$

$$-650 = -3(65 - n)^{\frac{3}{2}}$$

$$216 = (65 - n)^{\frac{3}{2}}$$

$$\sqrt[3]{216} = (65 - n)^{\frac{1}{2}}$$

$$6 = \sqrt{65 - n}$$

$$36 = 65 - n$$

$$-29 = -n \quad \boxed{n = 29}$$

$$10) 5 = 3 + 4a^{-\frac{1}{6}}$$

$$2 = 4a^{-\frac{1}{6}}$$

$$\frac{2}{4} = \frac{4}{4} a^{-\frac{1}{6}}$$

$$\left(\frac{1}{2}\right)^{-6} = (a^{-\frac{1}{6}})^{-6}$$

$$2^6 = \boxed{64}$$

$$12) -x^{\frac{3}{2}} = -27$$

$$x^{\frac{3}{2}} = 27^{\frac{2}{3}}$$

$$x = \sqrt[3]{27^2} = 3^2 = \boxed{9}$$

$$14) -5126 = -6 - 5(3x + 22)^{\frac{5}{3}}$$

$$-5120 = -5(3x + 22)^{\frac{5}{3}}$$

$$1024 = (3x + 22)^{\frac{5}{3}}$$

$$\sqrt[5]{1024^3} = 3x + 22$$

$$4^3 \rightarrow 64 = 3x + 22$$

$$16) 3646 = 1 + 5(4r + 17)^{\frac{3}{2}}$$

$$3645 = 5(4r + 17)^{\frac{3}{2}}$$

$$729 = (4r + 17)^{\frac{3}{2}}$$

$$9^2 = 4r + 17 \rightarrow 81 = 4r + 17$$

$$64 = 4r$$

$$\boxed{r = 16}$$

$$18) -3 + (8 - 2x)^{\frac{5}{4}} = 29$$

$$(8 - 2x)^{\frac{5}{4}} = 32^{\frac{4}{5}}$$

$$8 - 2x = \sqrt[5]{32^4}$$

$$8 - 2x = 16$$

$$-8 = -2x$$

$$\boxed{x = 4}$$

Double Radical Eqns

$$\textcircled{1} (\sqrt{2x-1}) = (\sqrt{2x+15} - 2)^2$$
$$2x-1 = (\sqrt{2x+15} - 2)(\sqrt{2x+15} - 2)$$

$$= 2x+15 - 4\sqrt{2x+15} + 4$$

$$2x-1 = 2x+19 - 4\sqrt{2x+15}$$

$$\cancel{2x} - 19 \quad \cancel{2x} - 19$$

$$\hline -20 = -4\sqrt{2x+15}$$

$$\frac{-20}{-4} = \frac{-4}{-4}\sqrt{2x+15}$$
$$5 = (\sqrt{2x+15})^2$$

$$25 = 2x+15$$

$$10 = 2x$$

$$\boxed{x=5}$$

$$\text{Check: } \sqrt{2 \cdot 5 - 1} \stackrel{?}{=} \sqrt{2 \cdot 5 + 15} - 2$$
$$3 = 5 - 2$$
$$\checkmark$$

$$\textcircled{2} (\sqrt{4x-10}) = (\sqrt{2x-13} + 1)^2$$
$$4x-10 = (\sqrt{2x-13} + 1)(\sqrt{2x-13} + 1)$$
$$= 2x-13 + 2\sqrt{2x-13} + 1$$

$$4x-10 = 2x-12 + 2\sqrt{2x-13}$$

$$\cancel{2x} + 12 \quad \cancel{2x} + 12$$

$$\hline 2x+2 = 2\sqrt{2x-13}$$

$$\frac{2}{2} \quad \frac{2}{2}$$
$$(x+1) = \sqrt{2x-13}$$

$$x^2 + 2x + 1 = 2x - 13$$

$$\cancel{-2x} + 13 \quad \cancel{-2x} + 13$$

$$\hline x^2 + 14 = 0$$

$$\sqrt{x^2} = \sqrt{-14}$$

No Solution

③

$$\sqrt{x-5} + \sqrt{x+6} = 11$$

$$(\sqrt{x-5})^2 = (11 - \sqrt{x+6})^2$$

$$x-5 = (11 - \sqrt{x+6})(11 + \sqrt{x+6})$$

$$= 21 - 22\sqrt{x+6} + x+6$$

$$x-5 = 27 + x - 22\sqrt{x+6}$$

$$-x-17 = 27-x$$

$$-132 = -22\sqrt{x+6}$$

$$-22 = -22$$

$$6^2 = (\sqrt{x+6})^2$$

$$36 = x+6$$

$$x=30$$

$$\text{OK: } 5 + 6 = 11$$



Mon.

$\sqrt[4]{x \cdot x \cdot x \cdot x}$
x

Simplifying Radicals

$$\textcircled{1} \sqrt[4]{80x^4y^5} = 2 \cdot \sqrt[4]{5} \cdot x \cdot y \cdot \sqrt[4]{y} = \boxed{2xy\sqrt[4]{5y}}$$

$$\sqrt[4]{16} \cdot \sqrt[4]{5} \quad \sqrt[4]{\underbrace{y \cdot y \cdot y \cdot y}_{y^4} \cdot y}$$

$$\textcircled{2} \sqrt[3]{18x^5} = 3 \sqrt[3]{2 \cdot x^2} \sqrt{x} = \boxed{3x^2\sqrt{2x}}$$

$$\textcircled{3} \sqrt[4]{a^4b^4} = |ab|$$

$$\textcircled{4} \sqrt[3]{-24y^6} = -2y^2 \cdot \sqrt[3]{3}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{3} \quad \sqrt[3]{\underbrace{y \cdot y \cdot y}_{y^3} \cdot \underbrace{y \cdot y \cdot y}_{y^3}}$$

-2

$$\textcircled{5} \sqrt{\sqrt{2x}} = ((2x)^{1/2})^{1/2} = (2x)^{1/4} = \sqrt[4]{2x}$$

$$\textcircled{6} \sqrt{\sqrt[3]{3x^2}} = [(3x^2)^{1/3}]^{1/2} = (3^{1/3} x^{2/3})^{1/2} = 3^{1/6} x^{1/3}$$

