

Logarithms Review Sheet

1. What are logs?

Know all Log Properties

2. Evaluate each of the following. All must be done without a calculator.

a. $\ln e^{189,004,258}$

$$= 189,004,258$$

b. $\log(\ln(e^{100}))$

$$\log 100 = 2$$

c. $\log\left(\frac{1}{10}\right) + \log(0.0001)$

$$-1 + -4 = -5$$

d. $\log_4\left(\frac{1}{64}\right)$

$$= -3$$

e. $\log_2(\sqrt[3]{2})$

$$\log_2(2^{1/3}) = \frac{1}{3}$$

3. Write as a single logarithm (a.k.a. condense):

a. $8\log x - \frac{1}{2}\log y + 6\log z - 1$

$$\log\left(\frac{x^8 z^6}{\sqrt{y}}\right)$$

b. $4\log_3(x-2) - (\log_3 x + \log_3(x-5))$

$$\log_3\left(\frac{(x-2)^4}{x(x-5)}\right)$$

4. Write as a series of logarithms with integer exponents (a.k.a. expand):

a. $\log\left(\frac{6x^2y^3}{5z}\right) =$

$$\log 6 + 2\log x + 3\log y - (\log 5 + \log z)$$

b. $\ln\left(\frac{\sqrt{5x^4}}{\sqrt[3]{12}}\right) =$

$$\frac{1}{2}\ln 5 + 4\ln x - \frac{1}{3}\ln 12$$

5. Solve each of the following for x. Look for the most efficient method! Round your answers to the nearest hundredth and be sure to check for extraneous solutions:

a. $8 \cdot 2^x = 13$

$$2^x = \frac{13}{8}$$

$$\log_2\left(\frac{13}{8}\right) = \boxed{.7 = x}$$

b. $\log(x+5) - \log(x-2) = \log(6) + \log(2)$

$$\log\left(\frac{x+5}{x-2}\right) = \log(12)$$

$$\frac{x+5}{x-2} = \frac{12}{1}$$

$$12x - 24 = x + 5$$

$$\boxed{x = \frac{29}{11}}$$

c. $\log(2x-3) + \log(x+6) = \log(6x+2)$

$$\log(2x-3)(x+6) = \log(6x+2)$$

$$(2x-5)(x+4) = 0$$

$$2x^2 + 9x - 18 = 6x + 2$$

$$2x^2 + 3x - 20 = 0$$

$$\boxed{x = 2.5, -4}$$

Key

d. $3 \cdot 4^x = 2 \cdot 3^{x-1}$

$$\log 3 + x \cdot \log 4 = \log 2 + (x-1) \log 3$$

$$\log 3 - \log 2 = x \log 3 - \log 3 - x \log 4$$

$$2 \log 3 - \log 2 = x (\log 3 - \log 4)$$

$$\frac{2 \log 3 - \log 2}{\log 3 - \log 4} =$$

$$x \approx -5.2$$

e. $\left(\frac{1}{8}\right)^{x+2} = 4^{1-2x}$

$$x = 8$$

f. $\log(8-x) = 1$

$$10^1 = 8 - x$$

$$x = -2$$

g. $8e^{2x-3} = 24$

$$e^{2x-3} = 3$$

$$x = \frac{3 + \ln 3}{2}$$

6. Find the amount of time it takes for an investment of \$1000 to triple if it is invested into an account bearing 8% interest compounded:

- a. Continuously
- b. Monthly

Use $A = P \left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$

a) $3000 = 1000 e^{.08t}$
 $3 = e^{.08t}$

$$\ln 3 = .08t$$

$$\frac{\ln 3}{.08} = t$$

$$13.7 = t$$

b) $3000 = 1000 \left(1 + \frac{.08}{12}\right)^{12t}$
 $3 = (1.006)^{12t}$

$$\log_{1.006} (3) = 12t$$

$$t = 13.8$$

7. Explain why the domain of $f(x) = \log_b x$ is $x > 0$.

b/c $\log_b x$ is undefined when $x \leq 0$ or neg.
 ex. $\log_b -8$ has no answer.

8. Explain the relationship between $f(x) = e^x$ and $g(x) = \ln x$. How do the graphs compare? How do the domain and range of each compare? What is the value of $(f \circ g)(x)$? Why?

They are inverses (reflections over $y=x$ line)

D+R Switch

$$(f \circ g)(x) = e^{\ln x} = x \quad (\text{They are inverses})$$

9. If $\log x = A$ and $\log y = B$, write $\log \left(\frac{x^5}{\sqrt{y}} \right)$ in terms of A and B .

$$5 \log x - \frac{1}{2} \log y$$

$$\boxed{5A - \frac{1}{2}B}$$

10. Evaluate without a calculator:

a $\log_2 16$

$$\boxed{4}$$

b $\log_{10} \sqrt[5]{100}$

$$\log_{10} 10^{2/5} = \boxed{\frac{2}{5}}$$

c $\log_2 \left(\frac{1}{\sqrt{2}} \right)$

$$\boxed{-\frac{1}{2}}$$