

Key

How To Find The Equation of the Perpendicular Bisector of a line segment:

- 1) Find the equation of the perpendicular bisector of the segment with endpoints A (1, -3) and B (-3, 5)

Step 1: Find the midpoint of the segment.

$$M(AB) = \left(\frac{1+3}{2}, \frac{-3+5}{2} \right) = \left(\frac{-2}{2}, \frac{2}{2} \right) = (-1, 1)$$

Step 2: Find the slope of the segment.

$$m(AB) = \frac{5+3}{-3-1} = \frac{8}{-4} = -2$$

Step 3: Find the slope of the perpendicular line.

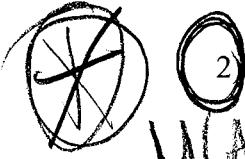
$$m = \frac{1}{2}$$

Step 4: Use the point-slope formula to find the equation of the line.

$$\begin{aligned} y - 1 &= \frac{1}{2}(x + 1) \\ y - 1 &= \frac{1}{2}x + \frac{1}{2} \\ +1 &\quad +\frac{1}{2} \end{aligned}$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

- 2) Find the equation of the perpendicular bisector of the segment with endpoints A (8, 0) and B (0, 4)

 $M(AB) = (4, 2)$

$$m(AB) = \frac{4-0}{-8} = -\frac{1}{2} \rightarrow 2$$

$$y - 2 = 2(x - 4)$$

$$\begin{aligned} y - 2 &= 2x - 8 \\ +2 &\quad +2 \end{aligned}$$

$$y = 2x - 6$$

Recall: concurrent @ circumcenter

Altitude:

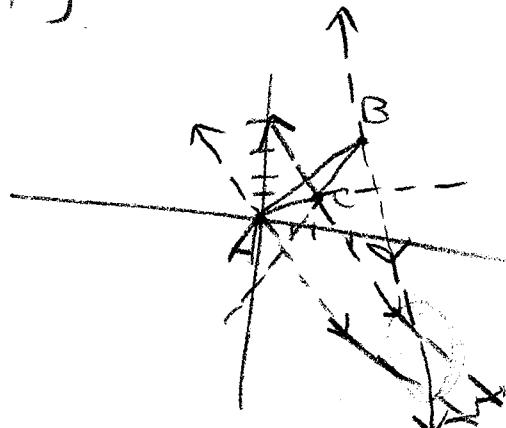
1 line from vertex to opp. side.

~~A(0,0) B(-4,-1) C(2,1)~~

Altitude from A: (to BC)

$$m(BC) = \frac{-1 - 1}{2 - (-4)} = \frac{-2}{6} = -\frac{1}{3}$$

$$\begin{aligned} y - 0 &= -\frac{1}{3}(x - 0) \\ y &= -\frac{1}{3}x \end{aligned}$$



Altitude from B: (to AC)

$$m(AC) = \frac{1 - 0}{2 - 0} = \frac{1}{2} \rightarrow -2$$

$$y - 4 = -2(x - 1)$$

$$\begin{aligned} y - 4 &= -2x + 2 \\ y &= -2x + 6 \end{aligned}$$

$$\begin{array}{rcl} -\frac{1}{3}x &=& -2x + 10 \\ +2x & & +2x \\ \hline \frac{5}{3}x &=& 10 \\ x &=& \frac{10}{\frac{5}{3}} = 6 \end{array}$$

$$y = -\frac{1}{3}(6) = -2$$

Altitude from C: (to AB)

$$m(AB) = \frac{4 - 0}{-4 - 0} = -\frac{3}{4}$$

$$y - 1 = -\frac{3}{4}(x - 1)$$

$$\begin{aligned} y - 1 &= -\frac{3}{4}x + \frac{3}{4} \\ y &= -\frac{3}{4}x + \frac{10}{4} \end{aligned}$$

Find orthocenter:

pt. in common/
system of eqs

$$\star (6, -2) \star$$

$$\begin{aligned} -2 &= -\frac{3}{4}(6) + \frac{10}{4} \\ -2 &= -\frac{18}{4} + \frac{10}{4} \\ -2 &= -\frac{8}{4} \end{aligned}$$

Name _____
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CW

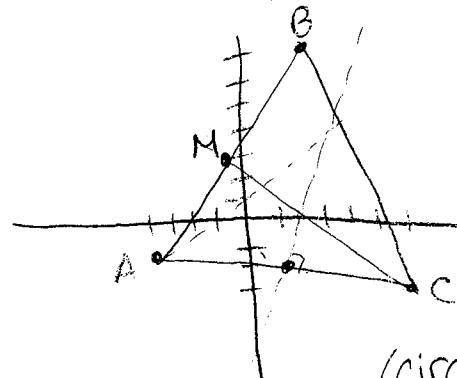
Writing the Equations of the Special Segments of a Triangle

Do Now: For each term write out the definition and identify the two necessary pieces of information needed to determine the equation of each term.

a) Median of a Triangle

b) Altitude of a Triangle

c) Perpendicular Bisector of a Triangle

side of a

(circumcenter)

Practice Problems:

1) The vertices of $\triangle ABC$ are $A(-4, -1)$, $B(2, 7)$, and $C(6, -3)$:

- Write an equation of the line that contains the perpendicular bisector of \overline{AC} .
- Write an equation of the line that contains the median from C to \overline{AB} .
- Write an equation of the line that contains the altitude from A to \overline{BC} .
- Prove that the altitude determined in part c, when extended, passes through the point $(16, 7)$

$$b) m(\overline{AB}) = \left(-\frac{-4+2}{2}, -\frac{-1+7}{2} \right) = (1, 3)$$

$$a) m(\overline{AC}) = \frac{-1+3}{-4-6} = \frac{2}{-10} = \frac{1}{5}$$

$$m(\overline{CM}) = \frac{-3-3}{6+1} = -\frac{6}{7}$$

$$m \perp = 5$$

$$y-3 = -\frac{6}{7}(x+1)$$

$$y-3 = -\frac{6}{7}x - \frac{6}{7} + 3$$

$$\boxed{y = -\frac{6}{7}x + \frac{15}{7}}$$

$$M(\overline{AC}) = \left(-\frac{-4+6}{2}, -\frac{-1+3}{2} \right) = (1, -2)$$

$$y+2 = 5(x-1)$$

$$y+2 = 5x - 5$$

$$\boxed{y = 5x - 7}$$

$$d) 7 = \frac{2}{5}(16) + \frac{3}{5}$$

$$\frac{32}{5} + \frac{3}{5} = \frac{35}{5} = 7 \checkmark$$

$$c) m(\overline{BC}) = \frac{-3-7}{6-2} = \frac{-10}{4} = -\frac{5}{2} \quad m \perp: \frac{2}{5}$$

$$A(-4, -1)$$

$$y+1 = \frac{2}{5}(x+4)$$

$$y+1 = \frac{2}{5}x + \frac{8}{5}$$

$$\boxed{y = \frac{2}{5}x + \frac{3}{5}}$$

(only need 2)

Find alt. from P + Q

- 2) Find the coordinates of the point of intersection of the altitudes of $\triangle PQR$, if the vertices are $P(-2, -2)$, $Q(0, -1)$, and $R(3, -2)$.

alt. from Q: $x=0$

$$\text{alt. from } P: m(QR) = \frac{-2+2}{3-0} = \frac{0}{3} = 0$$

$$m \perp: \frac{1}{2}$$

$$y+2 = \frac{1}{2}(x+2)$$

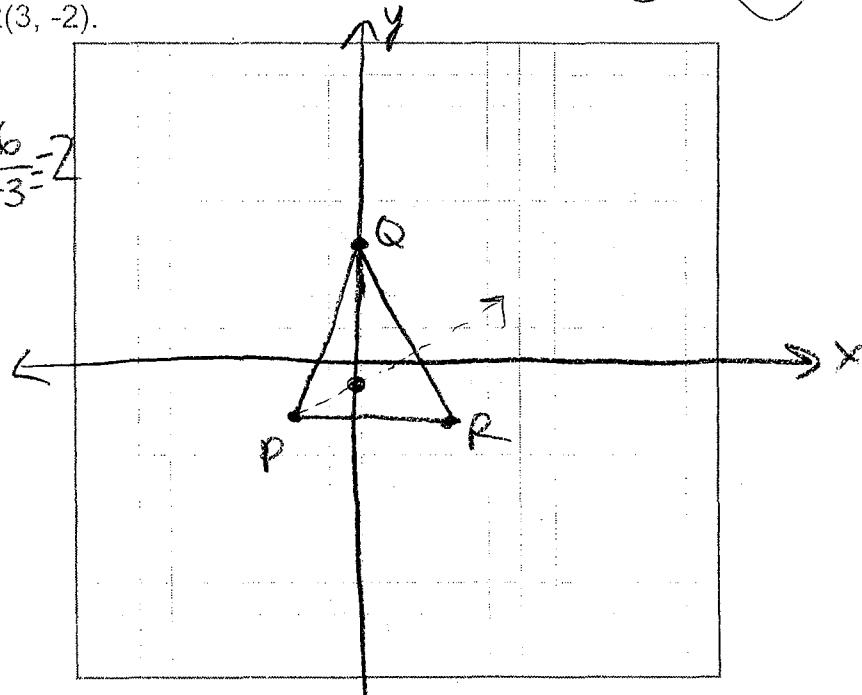
$$y+2 = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}(0) - 1$$

$$y = -1$$

$\boxed{(0, -1)}$ orthocenter



- 3) The vertices of $\triangle MEG$ are $M(1, 2)$, $E(7, 0)$, and $G(1, -2)$.

a. Write the equations of the medians of $\triangle MEG$. $(\text{all } 3)$

b. Find the coordinates of the point of intersection of the medians.

a) med. from E: $y=0$

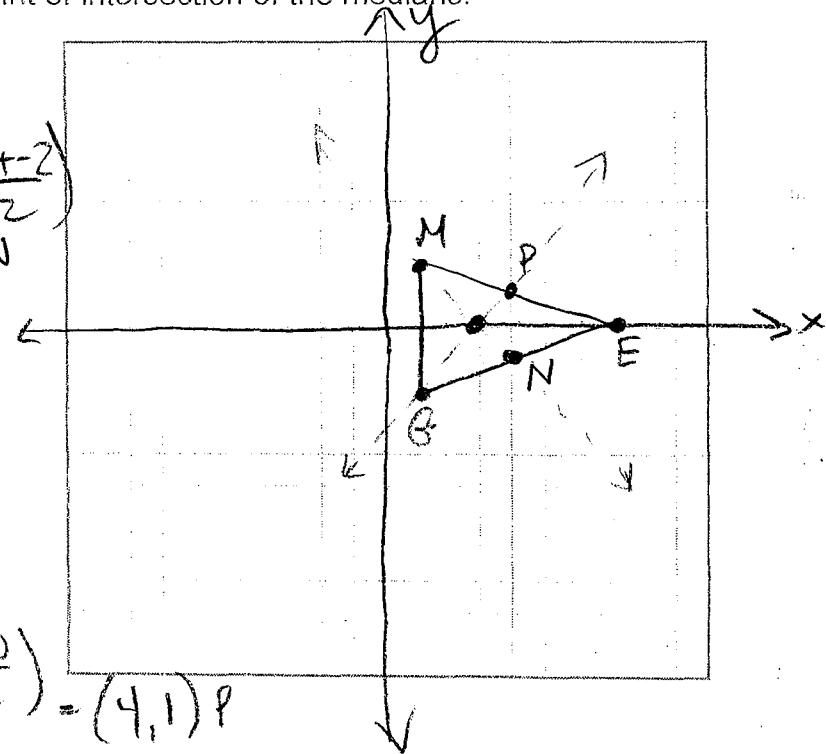
$$\text{med. from } M: M(GE) = \left(\frac{1+7}{2}, \frac{0+0}{2}\right) = (4, -1)$$

$$m(MN) = \frac{2+1}{1-4} = \frac{3}{-3} = -1$$

$$y+1 = -1(x-4)$$

$$y+1 = -x + 4$$

$$\boxed{y = -x + 3}$$



$$\text{med. from } G: M(ME) = \left(\frac{1+7}{2}, \frac{2+0}{2}\right) = (4, 1)$$

$$m(GP) = \frac{-2-1}{1-4} = \frac{-3}{-3} = 1$$

$$y+2 = 1(x-1)$$

$$y+2 = x-1$$

$$\begin{aligned} -y+3 &= x-3 \\ +x &+x \\ \hline 0 &= 2x \end{aligned}$$

$$y = 3 - 3 = 0$$

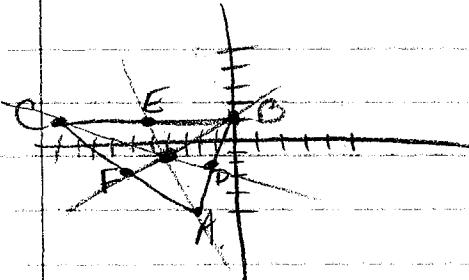
$$\boxed{(3, 0)}$$

cont'd



Find centroid:

$$A(-2, 5) \quad B(0, 1) \quad C(10, 1)$$



$$M(AB) = (-1, -2) = D$$

$$M(BC) = (-5, 1) = E$$

$$M(CA) = (-6, -2) = F$$

Centroid:

$$\begin{aligned} -2x - 3 &= \frac{1}{3}x + 1 \\ -\frac{7}{3}x &= \frac{4}{3} + 9 \end{aligned}$$

$$\begin{aligned} -\frac{7}{2}x &= 10^2 \left(\frac{2}{3}\right) \\ x &= -4 \end{aligned}$$

$$\begin{aligned} y &= -2(-4) - 9 = 8 - 9 = -1 \\ (x, y) &= (-4, -1) \end{aligned}$$

Median from A to E:

$$m(AE) = \frac{-5 - 1}{-2 + 5} = \frac{-6}{3} = -2$$

$$y + 5 = -2(x + 2)$$

$$y + 5 = -2x - 4$$

$$y = -2x - 9$$

CK:

$$-1 = -\frac{1}{3}(-4) - \frac{1}{3}$$

$$-1 = \frac{4}{3} - \frac{1}{3}$$

$$-1 = -\frac{3}{3}$$

$$-1 = -1 \checkmark$$

Median from B to F:

$$m(BF) = 1 + 7 - \frac{3}{6} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - 0)$$

$$y - 1 = \frac{1}{2}x \quad \boxed{y = \frac{1}{2}x + 1}$$

Median from C to D:

$$m(CD) = \frac{1 + 7}{-10 + 1} = \frac{8}{-9} = -\frac{1}{3}$$

$$y - 1 = -\frac{1}{3}(x + 10)$$

$$y - 1 = \frac{1}{3}x - \frac{10}{3}$$

$$y - 1 = \frac{1}{3}x - \frac{7}{3}$$

