Name Key Date________________

Ch. 2 Quadratic Functions

1. Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for g.

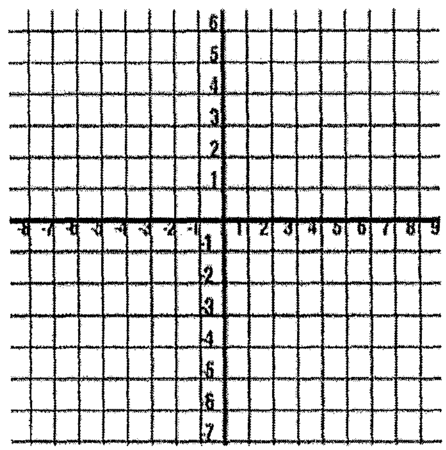
$(x-3)^2 - 5(x-3) + 2$

$x^2 - 6x + 9 - 5x + 15 + 2$

$x^2 - 11x + 26$

$(-x)^2 - 11(-x) + 26$

$\boxed{g(x) = x^2 + 11x + 26}$



Quadratic Functions

3 ways to write the equation:

- $f(x)=ax^2 +bx +c$ **Standard Form**
- $f(x)=a(x-p)(x-q)$ **Intercept Form**
- $f(x)=a(x-h)^2 +k$ **Vertex Form**

Investigating the graphs of quadratics in the form $y=a(x-h)^2+k$

Without a calculator, describe each transformation of the following on the parent graph $y = x^2$.

$y=(x-5)^2+3$ 5 right, 3 up

$y=6(x-5)^2+3$ vert stretch by 6

* $y=-6(x-5)^2+3$ reflect over x-axis
BUT happened 1st *

The Benefits of Vertex Form $y=a(x-h)^2+k$

$$f(x) = a(x-h)^2 + k$$

a indicates a reflection in the *x*-axis and/or a vertical stretch or shrink

h indicates a horizontal translation.

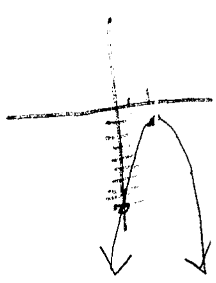
k indicates a vertical translation.

Example 5

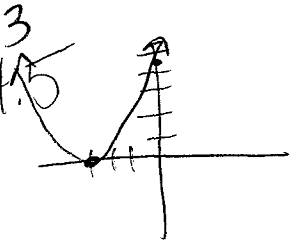
Use the vertex, axis of symmetry and *y*-intercept to sketch the graph of:

a $y = -2(x - \ )^2 - 1$ b $y = \frac{1}{2}(x+3)^2$

V: (2, -1)
A: X=2
y-int: $-2(-2)^2-1$
$-2(4)-1$
-9



V: (-3, 0)
A: X=-3
y-int: 4.5



The Benefits of Intercept Form

- $f(x)=a(x-p)(x-q)$

p, q: x-ints, roots, zeros

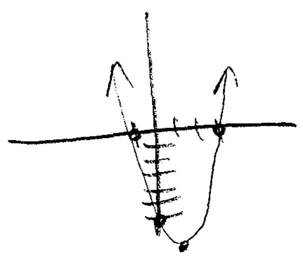
Example 4

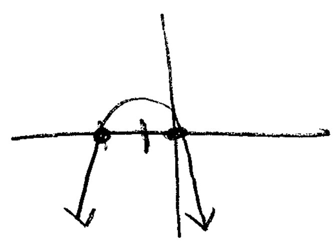
Using axis intercepts only, sketch the graphs of:

a $y = 2(x+1)(x-3)$ b $y = -2x(x+2)$

x-ints: $-1, 3$
y-int: $2(1)(-3) = -6$

x-ints: $0, -2$
y-int: 0



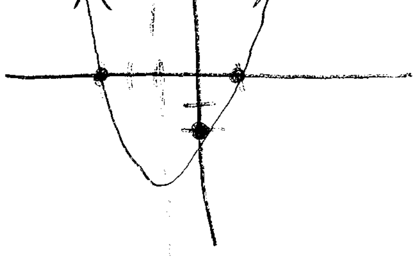


Example 6

Sketch the parabola which has x-intercepts -3 and 1, and y-intercept -2. Find the equation of the axis of symmetry.

x-ints: $-3, 1$
y-int: -2

Axis: $x = -1$



The Benefits of S andard Form • $f(x)=ax^2+bx+c$

For the following quadratics find:

i the equation of the xis of symmetry **ii** the coordinates of the vertex
iii the axes intercepts, f they exist. **iv** Hence, sketch the graph.

$$y = -x^2 + 3x - 2$$

i) $X = \frac{-3}{2(-1)} = \frac{3}{2}$

ii) V: $\left(\frac{3}{2}, \frac{1}{4}\right)$

$-\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) - 2$

$-\frac{9}{4} + \frac{9}{2} - 2$

$-\frac{9}{4} + \frac{18}{4} - \frac{8}{4} = \frac{1}{4}$

iii) x-ints:

$(0 = -x^2 + 3x - 2)(-1)$

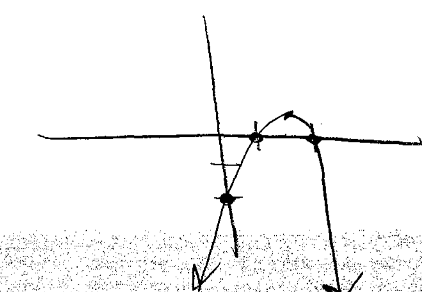
$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x = 2 \quad x = 1$

y-int: -2

iv)



Example 31

Find, in the form $y = a$ $+ bx + c$, the equation of the quadratic whose graph cuts the x-axis at 4 and − and passes through the point $(2, -20)$.

x-ints: 4, −3

$y = a(x-4)(x+3)$

$y = a(x^2 - x - 12)$

$-20 = a(2^2 - 2 - 12)$

$-20 = a(-10)$

$a = 2$

$y = 2(x^2 - x - 12)$

$\boxed{y = 2x^2 - 2x - 24}$

DETERMINING THE QUADRATIC FROM A GRAPH

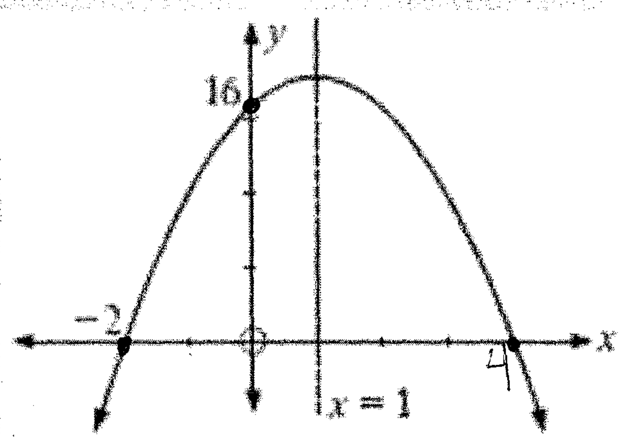
If we are given sufficient information on or about a graph we can determine the quadratic function in whatever form required.

Let's Review!

What are the forms a quadratic may be written in? List the characteristics from a parabola would best fit each form.

Example 30

Find the equation of the quadratic with graph:



$y = a(x+2)(x-4)$

$16 = a(2)(-4)$

$16 = -8a$

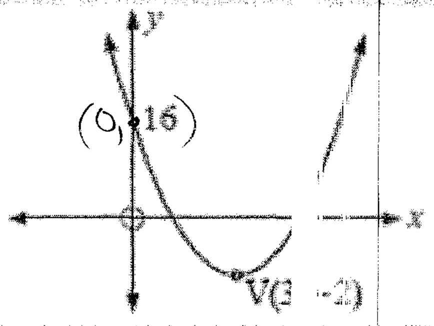
$a = -2$

$y = -2(x+2)(x-4)$

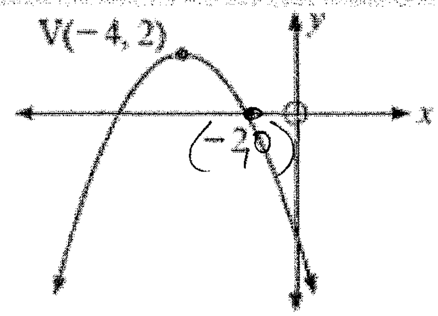
Example 3.2

Find the equation of the quadratic given its graph is:

a



b



$y = a(x-3)^2 - 2$

$16 = a(-3)^2 - 2$

$16 = 9a - 2$

$18 = 9a$

$a = 2$ $\quad \boxed{y = 2(x-3)^2 - 2}$

$y = a(x+4)^2 + 2$

$0 = a(-2+4)^2 + 2$

$0 = 4a + 2$

$-2 = 4a$

$a = -\frac{1}{2}$

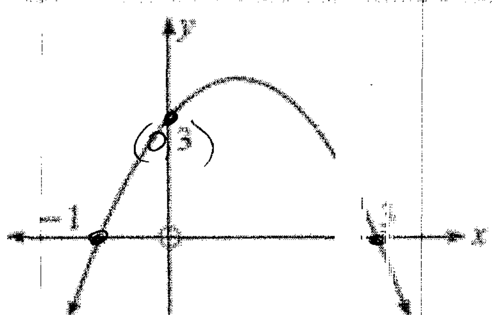
$\boxed{y = -\frac{1}{2}(x+4)^2 + 2}$

Example 3.3

Find the equation of the quadratic with graph:

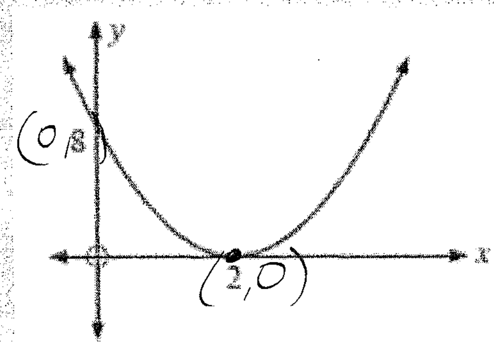
stopped here w/ 5th per

a



8th per.

b



$y = a(x+1)(x-3)$

$3 = a(1)(-3)$

$3 = -3a$

$a = -1$

$\boxed{y = -1(x+1)(x-3)}$

$y = a(x-2)^2 + 0$

$8 = a(0-2)^2$

$8 = 4a$

$a = 2$

$\boxed{y = 2(x-2)^2}$

Name Key Date

Writing Equations to Model Data

When data have equally-spaced inputs, you can analyze patterns in the differences of the outputs to determine what type of function can be used to model the data. Linear data have constant *first differences*. Quadratic data have constant *second differences*. The first and second differences of $f(x) = x^2$ are shown below.

Equally-spaced x-values

x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

first differences: −5 −3 −1 1 3 5

second differences: 2 2 2 2 2

Ex. 1

Time, t	Height, h
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

NASA can create a weightless environment by flying a plane in parabolic paths. The table shows heights h (in feet) of a plane t seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

SOLUTION

Step 1 The input values are equally spaced. So, analyze the differences in the outputs to determine what type of function you can use to model the data.

$h(10)$	$h(15)$	$h(20)$	$h(25)$	$h(30)$	$h(35)$	$h(40)$
26,900	29,025	30,600	31,625	32,100	32,025	31,400

First differences: 2125 1575 1025 475 −75 −625

Second differences: −550 −550 −550 −550 −550

Because the second differences are constant, you can model the data with a quadratic function.

Step 2 Write a quadratic function of the form $h(t) = at^2 + bt + c$ that models the data. Use any three points (t, h) from the table to write a system of equations.

Use (10, 26,900): $100a + 10b + c = 26{,}900$ Equation 1
Use (20, 30,600): $400a + 20b + c = 30{,}600$ Equation 2
Use (30, 32,100): $900a + 30b + c = 32{,}100$ Equation 3

Use the elimination method to solve the system.

Step 3 Evaluate the function when $t = 20.8$.

Real-life data that show a quadratic relationship usually do not have constant second differences because the data are not *exactly* quadratic. Relationships that are *approximately* quadratic have second differences that are relatively "close" in value. Many technology tools have a *quadratic regression* feature that you can use to find a quadratic function that best models a set of data.

Ex. 2

The table shows fue fficiencies of a vehicle at different speeds. Write a function that models the data. Us he model to approximate the optimal driving speed.

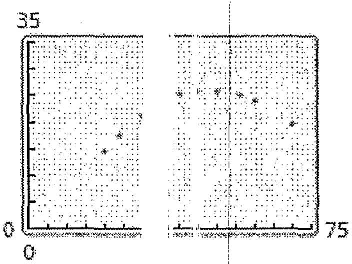
SOLUTION

Because the x-value are not equally spaced, you cannot analyze the differences in the outputs. Use a grapl g calculator to find a function that models the data.

not evenly spaced

Miles per hour, x	Miles per gallon, y
20	14.5
24	17.5
30	21.2
36	23.7
40	25.2
45	25.8
50	25.8
56	25.1
60	24.0
70	19.5

Step 1 Enter the da ir a graphing calculator u g two lists and create a scat r plot. The data show a quac tic relationship.



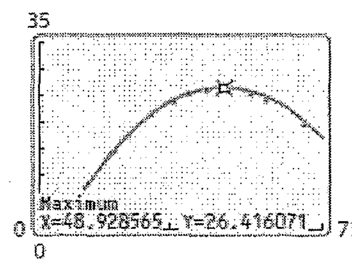
Step 2 Use the *quadratic regression* feature. A quadratic model that represents the data is $y = -0.014x^2 + 1.37x - 7.1$.

QuadReg
y=ax²+bx+c
a=-.014097349
b=1.366218867
c=-7.144052413
R^2=.9992475882

coefficient of determinati

Step 3 Graph the r ression equation with the scatter plot.

In this conte t, the "optimal" driving speed is the speed at hich the mileage per gallon is maximized. sing the *maximum* feature, you can see that e maximum mileage per gallon is about 26.4 r les per gallon when driving about 48.9 miles p r hour.



▶ So, the optima driving speed is about 49 miles per hour.

Practice:

(1) Write an equation c the parabola that passes through the points $(-1, 4)$, $(0, 1)$, and $(2, 7)$.

(2) The table shows the stimated profits y (in dollars) for a concert when the charge is x dollars p r ticket. Write and evaluate a function to determine what the charge per ticke hould be to maximize the profit.

evenly spaced

Ticket price, x	2	5	8	11	14	17
Profit, y	600	6500	8600	8900	7400	4100

$y = -100x^2 + 200x - 1000$

$R^2 = 1$

(3) The table shows the esults of an experiment testing the maximum weights y (in tons) supporte by ice x inches thick. Write a function that models the data. How much weight n be supported by ice that is 22 inches thick?

not evenly spaced

Ice thickness, x	12	14	15	18	20	24	27
Maximum weigl , y	3.4	7.6	10.0	18.3	25.0	40.6	54.3

$y = .0198x^2 - .4978x - 5.0072$

$R^2 = .9999$

EX1: ditto (NASA)

$$-(100a + 10b + c = 26{,}900)$$
$$400a + 20b + c = 30{,}600$$
$$900a + 30b + c = 32{,}100$$

(1)

$$-100a - 10b - \cancel{c} = -26{,}900$$
$$+\ 400a + 20b + \cancel{c} = 30{,}600$$
$$-2(300a + 10b = 3700)$$

(2)

$$-100a - 10b - \cancel{c} = -26{,}900$$
$$+\ 900a + 30b + \cancel{c} = 32{,}100$$
$$800a + 20b = 5200$$

(3)

$$-600a - \cancel{20b} = -7400$$
$$800a + \cancel{20b} = 5200$$
$$200a = -2200$$
$$\boxed{a = -11}$$

(4) Back substitute:

$$800(-11) + 20b = 5200$$
$$-8800 + 20b = 5200$$
$$+8800 \qquad +8800$$
$$20b = 14{,}000$$
$$\boxed{b = 700}$$

(5)

$$100(-11) + 10(700) + c = 26{,}900$$
$$-1100 + 7000 + c = 26{,}900$$
$$5900 + c = 26900$$
$$\boxed{c = 21{,}000}$$

$$\boxed{h(t) = -11t^2 + 700t + 21{,}000}$$

* Can check w/ a Quad Regression OR in a Matrix.

Practice $y = ax^2 + bx + c$

(1) $4 = a(-1)^2 + b(-1) + c \rightarrow a - b + c = 4$

$1 = a(0)^2 + b(0) + c$ $\quad c = 1$

$7 = a(2)^2 + b(2) + c$ $\quad 4a + 2b + c = 7$

Plug in c: $a - b + 1 = 4$

$2(a - b = 3)$

$4a + 2b + 1 = 7$

$4a + \cancel{2b} = 6$

$+\ 2a - \cancel{2b} = 6$

$6a = 12$

$a = 2$

$2 - b = 3$

$-2 \quad -2$

$-b = 1$

$b = -1$

$\boxed{y = 2x^2 - x + 1}$