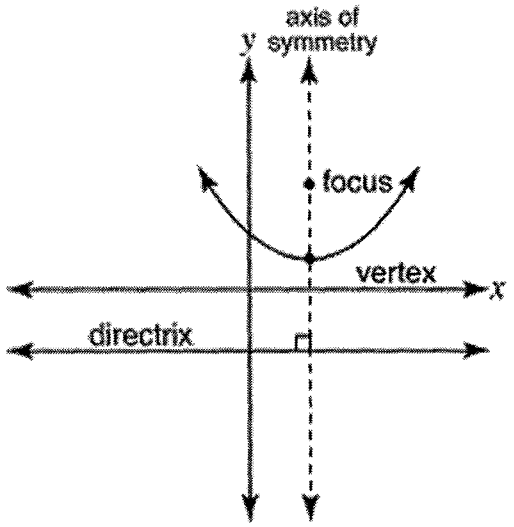
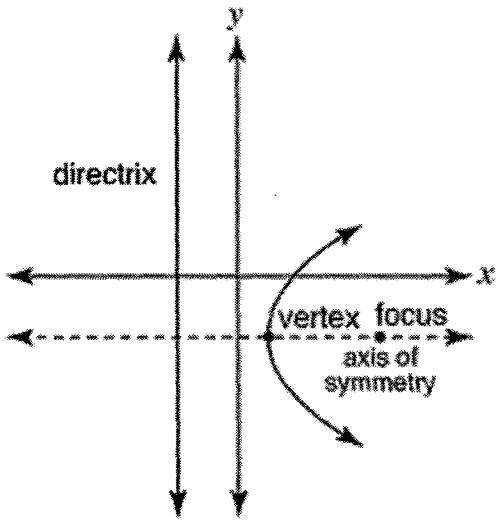
Name_____ Date_____

CC Algebra 2 Period _____

EXPLORING THE FOCUS AND DIRECTRIX





Teacher Demonstration: http://www.youtube.com/watch?v=wtk5q8wGAe0

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1. What is the name of the shape that is formed by all the folds in this activity?

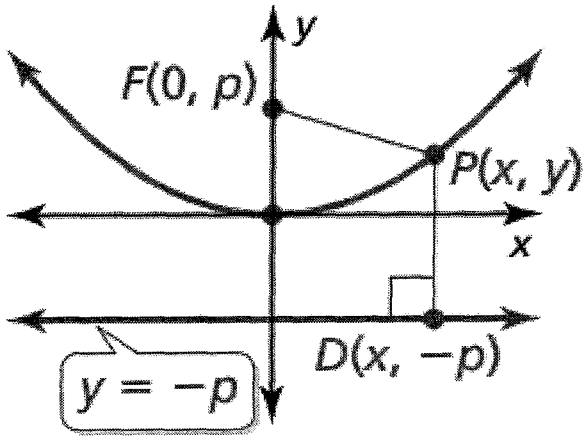
2. Fill in the blank: The vertex is _____ between the _____ and the _____.

3. The directrix is _____ to a parabolas axis of symmetry.

A **parabola** is the set of points in a place that are equidistant from a given point and a given line in a plane. The given point is called the **focus**, and the line is called the **directrix**.

The midpoint on the perpendicular segment from the focus to the directrix is call the **vertex of the parabola**. The line that passes through the vertex and focus is called the **axis of symmetry**.

We can derive the equation of a parabola that opens up or down with vertex $(0, 0)$, focus $(0, p)$, and directrix $y = -p$ using the distance formula.



Practice: Use the distance formula to write an equation of the parabola with a focus at (0,-4) and directrix of y = 4.

Steps for writing the equation of a parabola given the focus and directrix.

Step 1: Graph the directrix and focus.

Step 2: Find the distance in between the directrix and focus, this number is $2p$.

Step 3: Plug in values for (h, k): the vertex of the parabola

Step 4: Solve for y (for parabola opening up/down) or x (for parabolas opening left/right)

* p is the directed distance from vertex to focus

Core Concept

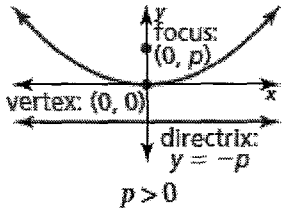
Standard Equations of a Parabola with Vertex at the Origin

Vertical axis of symmetry ($x = 0$)

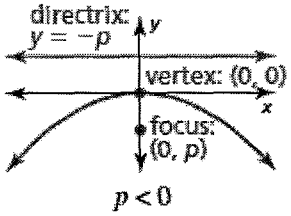
Equation: $y = \frac{1}{4p}x^2$

Focus: $(0, p)$

Directrix: $y = -p$



$p > 0$



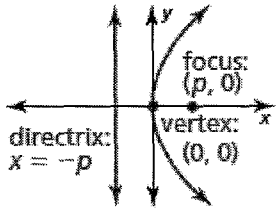
$p < 0$

Horizontal axis of symmetry ($y = 0$)

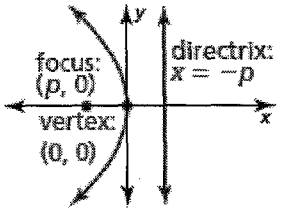
Equation: $x = \frac{1}{4p}y^2$

Focus: $(p, 0)$

Directrix: $x = -p$

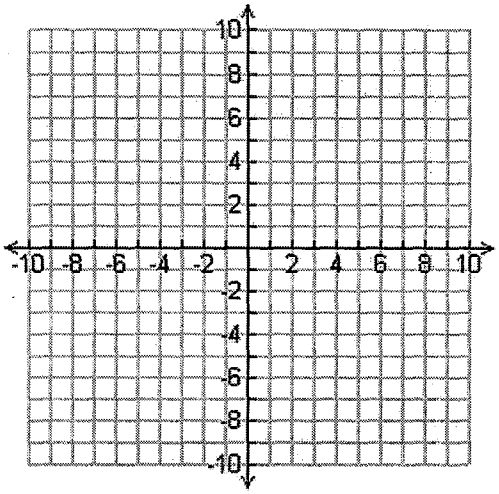


$p > 0$

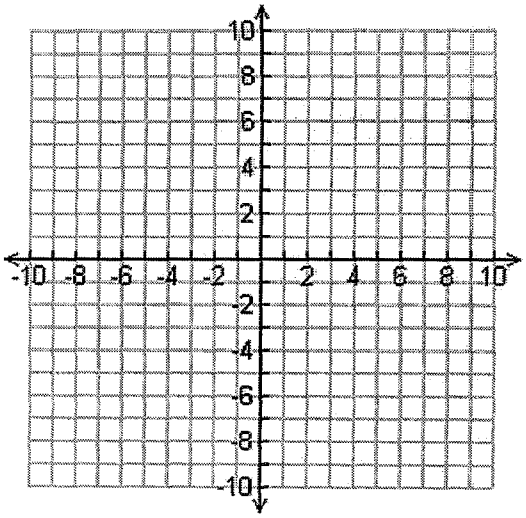


$p < 0$

Example 1: Write the equation of the parabola given the focus $(0, 4)$ and directrix $y = -4$.

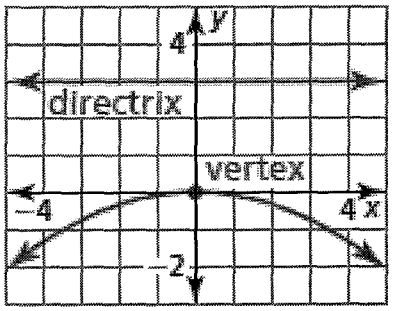


Example 2: Write the equation of the parabola given the vertex $(0, -2)$ and directrix $x = 2$.

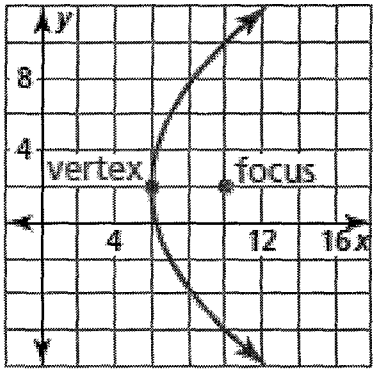


Example 3: Identify the focus, directrix, and axis of symmetry of $-4x = y^2$. Graph the parabola.

Example 4: Write the equation of the parabola shown.



Example 5: Write the equation of the parabola shown.



Core Concept

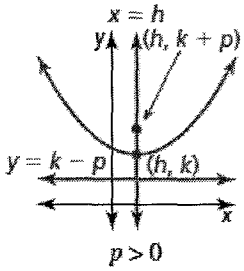
Standard Equation of a Parabola with Vertex at (h, k)

Vertical axis of symmetry ($x = h$)

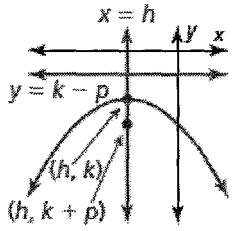
Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$



$p > 0$



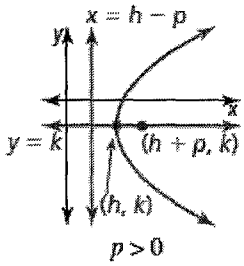
$p < 0$

Horizontal axis of symmetry ($y = k$)

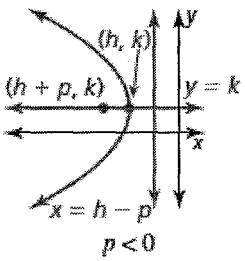
Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



$p > 0$



$p < 0$

Solving Real-Life Problems

Parabolic reflectors have cross sections that are parabolas. Incoming sound, light, or other energy that arrives at a parabolic reflector parallel to the axis of symmetry is directed to the focus (Diagram 1). Similarly, energy that is emitted from the focus of a parabolic reflector and then strikes the reflector is directed parallel to the axis of symmetry (Diagram 2).

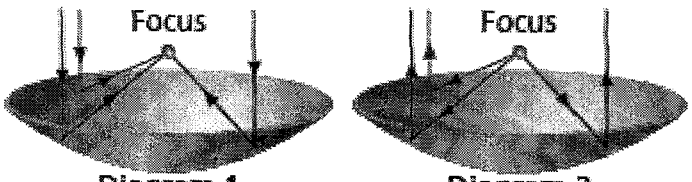
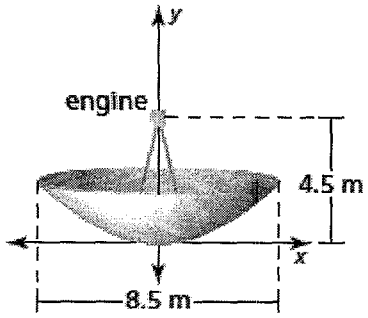


Diagram 1 Diagram 2

Example 6:

An electricity-generating dish uses a parabolic reflector to concentrate sunlight onto a high-frequency engine located at the focus of the reflector. The sunlight heats helium to 650°C to power the engine. Write an equation that represents the cross section of the dish shown with its vertex at (0, 0). What is the depth of the dish?

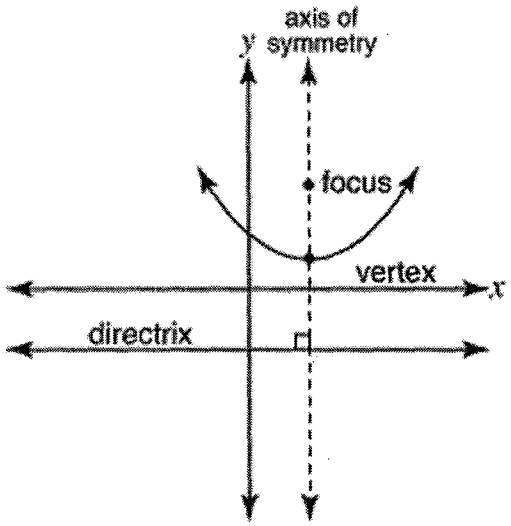


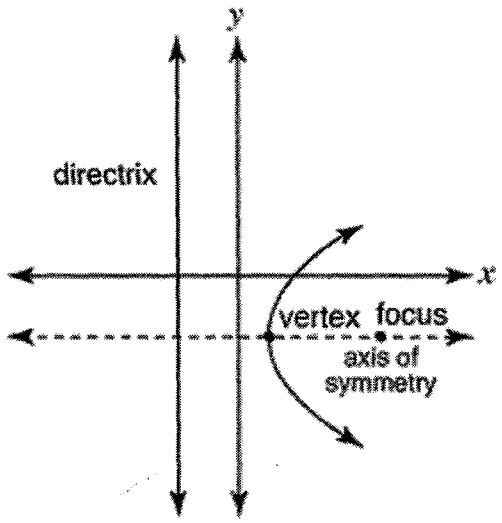
Check this out:
http://www.mathwarehouse.com/quadratic/parabola/focus-and-directrix-of-parabola.php

Name Key
CC Algebra 2

Date Saved as "parabolas"
Period _____

EXPLORING THE FOCUS AND DIRECTRIX





Video 1st

Teacher Demonstration: http://www.youtube.com/watch?v=wtk5q8wGAe0

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1. What is the name of the shape that is formed by all the folds in this activity?

parabola

2. Fill in the blank: The vertex is equidistant between the focus and the directrix.

3. The directrix is perpendicular to a parabolas axis of symmetry.

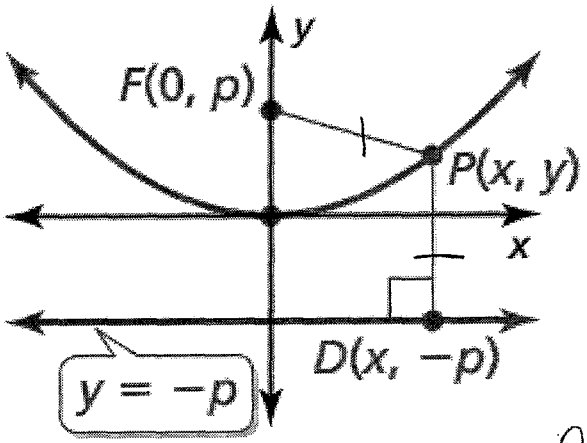
A **parabola** is the set of points in a place that are equidistant from a given point and a given line in a plane. The given point is called the **focus**, and the line is called the **directrix**.

The midpoint on the perpendicular segment from the focus to the directrix is call the **vertex of the parabola**. The line that passes through the vertex and focus is called the **axis of symmetry**.

We can derive the equation of a parabola that opens up or down with vertex $(0, 0)$, focus $(0, p)$, and directrix $y = -p$ using the distance formula.

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$FP = DP$



$\sqrt{(x-0)^2 + (y-p)^2} = \sqrt{(x-x)^2 + (y+p)^2}$

$\left(\sqrt{x^2 + (y-p)^2}\right)^2 = \left(\sqrt{(y+p)^2}\right)^2$

$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$

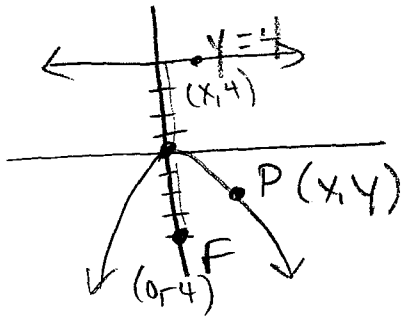
$+2py \qquad\qquad -2py$

$\frac{x^2}{4p} = \frac{4py}{4p}$

$y = \frac{1}{4p}x^2$

basically the same where p is directed dist from V to F

Practice: Use the distance formula to write an equation of the parabola with a focus at (0,-4) and directrix of y = 4.



$\sqrt{(x-x)^2 + (y-4)^2} = \sqrt{(x-0)^2 + (y+4)^2}$

$\left(\sqrt{(y-4)^2}\right)^2 = \left(\sqrt{x^2 + (y+4)^2}\right)^2$

$y^2 - 8y + 16 = x^2 + y^2 + 8y + 16$

$-8y \qquad\qquad -8y$

$\frac{-16y}{-16} = \frac{x^2}{-16}$

$y = \frac{1}{-16}x^2$ → $y = \frac{1}{4p}x^2$

$\therefore p = -4$

Steps for writing the equation of a parabola given the focus and directrix.

Step 1: Graph the directrix and focus.

Step 2: Find the distance in between the directrix and focus, this number is our p.

Step 3: Plug in values for (h, k): the vertex of the parabola

Step 4: Solve for y (for parabola opening up/down) or x (for parabolas opening left/right)

dist. from vertex to foc

Core Concept

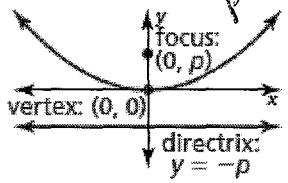
Standard Equations of a Parabola with Vertex at the Origin

Vertical axis of symmetry ($x = 0$)

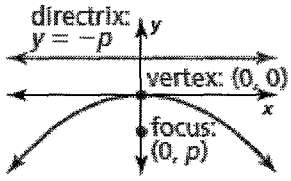
Equation: $y = \frac{1}{4p}x^2$

Focus: $(0, p)$

Directrix: $y = -p$



$p > 0$



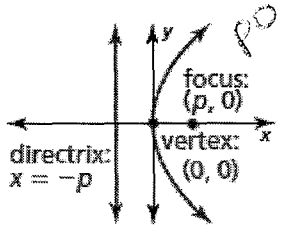
$p < 0$

Horizontal axis of symmetry ($y = 0$)

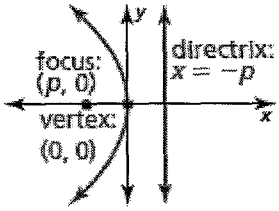
Equation: $x = \frac{1}{4p}y^2$

Focus: $(p, 0)$

Directrix: $x = -p$



$p > 0$



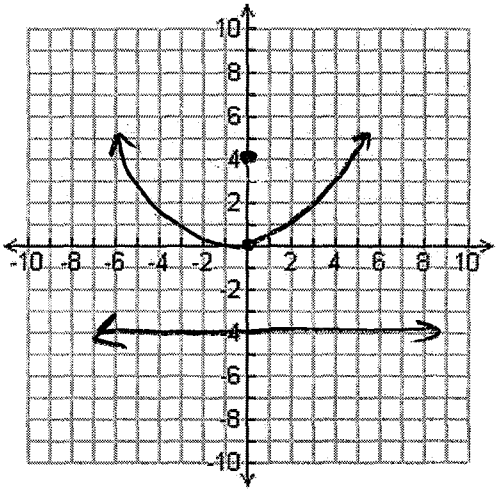
$p < 0$

Example 1: Write the equation of the parabola given the ~~vertex~~ focus $(0, 4)$ and directrix $y =$

$y = \frac{1}{4p}x^2$

$y = \frac{1}{4(4)}x^2$

$y = \frac{1}{16}x^2$



$x = \frac{1}{4p}(y-k)^2 + h$ (not $y = \frac{1}{4p}(x-h)^2+k$)

Example 2: Write the equation of the parabola given the vertex (h, k) $(0, -2)$ and directrix $x = 2$.

$x = \frac{1}{4p}(y+2)^2$

$x = \frac{1}{4(-2)}(y+2)^2$

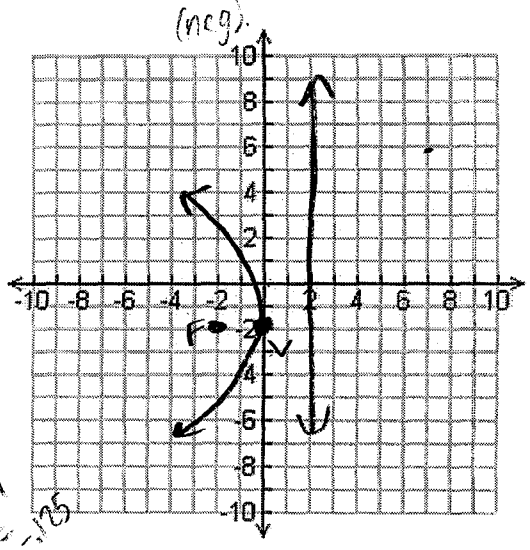
$x = -\frac{1}{8}(y+2)^2$

$x = -\frac{1}{8}(y^2+4y+4)$

Ck: ① $y = 4$ $x = -4.5$

② $y = -2$ $x = 0$

③ $y = -1$ $x = -3.5$



Example 3: Identify the focus, directrix, and axis of symmetry of $-4x = y^2$. Graph the parabola.

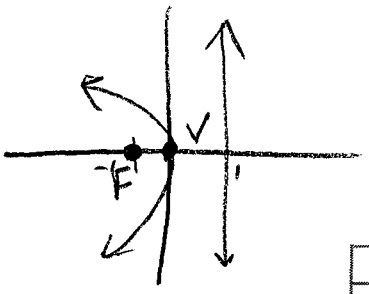
$x = -\frac{1}{4}y^2 \Rightarrow \frac{1}{4p}y^2$ $\quad \frac{-4}{-4} = 4$

$p = -1$ $\quad F: (-1, 0)$

Vertex: (0,0)

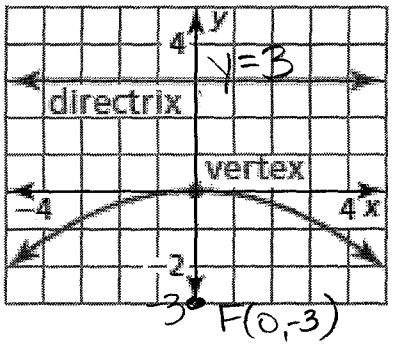
Directrix: $x = 1$

Axis: $y = 0$



OR table of values

Example 4: Write the equation of the parabola shown.



V: (0,0)

$p = -3$

$y = \frac{1}{4p}x^2$

$y = \frac{1}{4(-3)}x^2 \Rightarrow$ $\boxed{y = -\frac{1}{12}x^2}$

Example 5: Write the equation of the parabola shown.

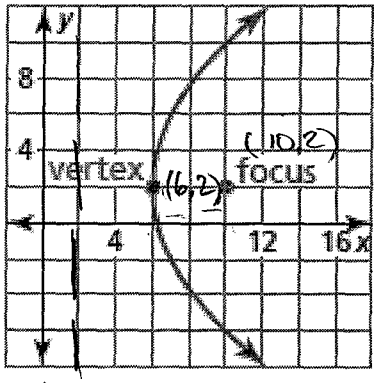
V: (6,2) $\quad F: (10,2)$

(h,k)

$p = 4$

$x = \frac{1}{4p}(y-k)^2 + h$

$\boxed{x = \frac{1}{16}(y-2)^2 + 6}$



Directrix $x = 2$.

Core Concep

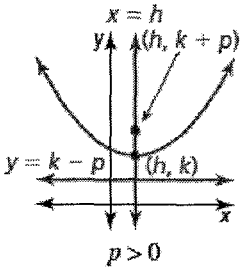
Standard Equation of a Parabola with Vertex at (h, k)

Vertical axis of symmetry ($x = h$)

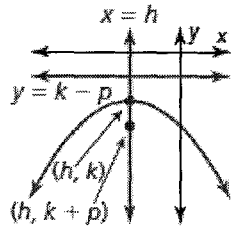
Equation: $y = \frac{1}{4p}(x - \quad)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$



$p > 0$



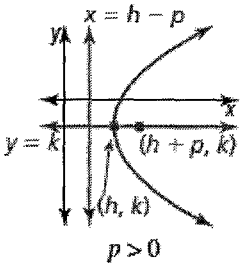
$p < 0$

Horizontal axis of symmetry ($y = k$)

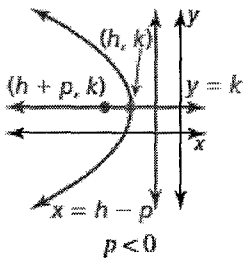
Equation: $x = \frac{1}{4p}(y - \quad)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



$p > 0$



$p < 0$

Solving Real-Life Problems

Parabolic reflectors have cross sections that are parabolas. Incoming sound, light, or other energy that arrives at a parabolic reflector parallel to the axis of symmetry is directed to the focus (Diagram 1). Similarly, energy that is emitted from the focus of a parabolic reflector and then strikes the reflector is directed parallel to the axis of symmetry (Diagram 2).

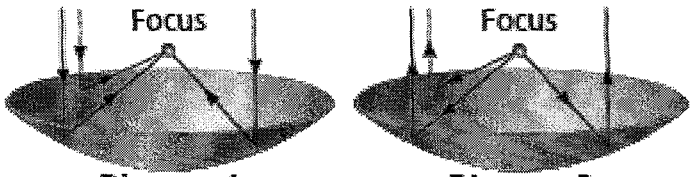


Diagram 1 Diagram 2

Example 6:

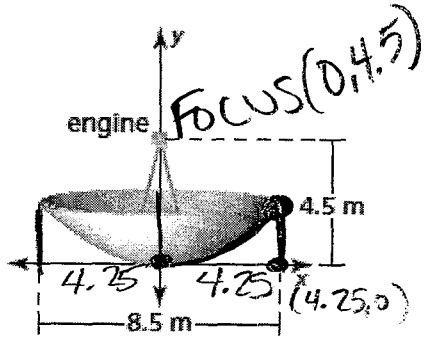
An electricity-generating dish uses a parabolic reflector to concentrate sunlight onto a high-frequency engine located at the focus of the reflector. The sunlight heats helium to 650°C to power the engine. Write an equation that represents the cross section of the dish shown with its vertex at (0, 0). What is the depth of the dish?

$y = \frac{1}{4p}x^2$

$p = 4.5$

$y = \frac{1}{4(4.5)}x^2$

$y = \frac{1}{18}x^2$



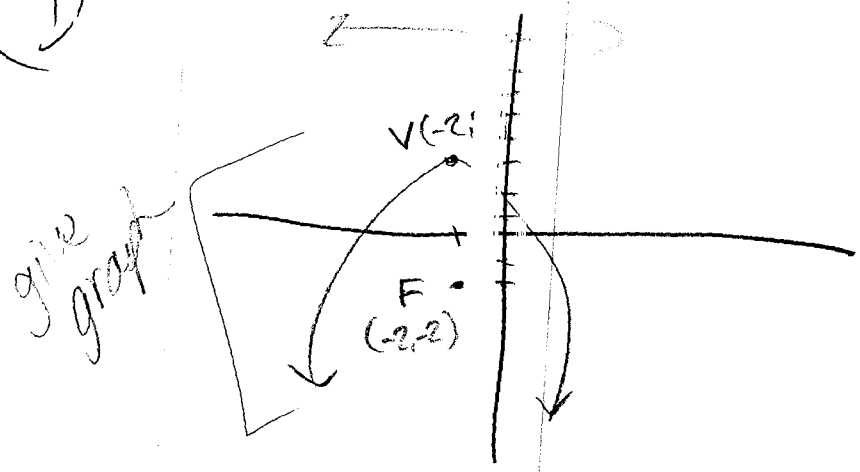
So, plug in 4.25 for x to find y (the height/depth)

$y = \frac{1}{18}(4.25)^2 = 1.003472$

depth ≈ 1m

Check this out:
http://www.mathwarehouse.com/quadratic/parabola/focus-and-directrix-of-parabola.php

(7)



give graph

D: $y \neq 8$.

Axis: $y = -1$

eqn: $y = \frac{1}{4(-5)}(x+2)^2 + 3$

$y = \frac{1}{-20}(x+2)^2 + 3$

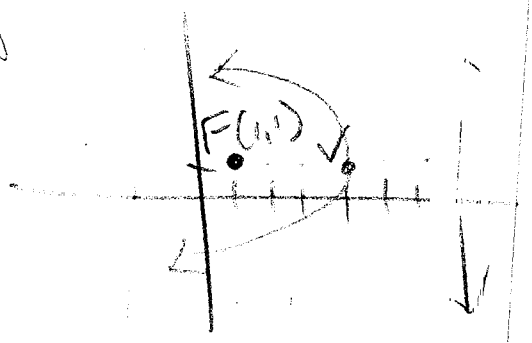
(8) V: (4,1)

D: $x = 7$

give

$x = \frac{1}{4(-3)}(y-1)^2 + 4$

$x = \frac{1}{-12}(y-1)^2 + 4$



axis: $y = 1$