

Solving Systems of Equations

① Solve graphically: $y = x^2 - 2x - 1$
 $y = -2x - 1$

Now, solve algebraically... (by substitution)

$$\begin{array}{r} x^2 - 2x - 1 = -2x - 1 \\ +2x + 1 \quad +2x + 1 \\ \hline x^2 = 0 \end{array}$$

$$x = 0$$

$$y = -2(0) - 1 = -1$$

$$\boxed{\text{Sol: } (0, -1)}$$

② Solve by substitution:

$$x^2 + x - y = -1$$

$$\begin{array}{r} x + y = 4 \\ -x \quad -x \\ \hline \end{array}$$

$$y = (-x + 4)$$

1 eq'n
in terms
of 1
variable

$$x^2 + x - (-x + 4) = -1$$

$$x^2 + x + x - 4 = -1$$

$$\begin{array}{r} x^2 + 2x - 4 = -1 \\ \quad +1 \quad +1 \\ \hline \end{array}$$

$$\begin{array}{r} x^2 + 2x - 3 = 0 \\ \quad A \quad M \end{array}$$

FACTOR

$$(x-1) | (x+3) = 0$$

$$\begin{array}{|l} x=1 \\ x=-3 \end{array}$$

$$\begin{array}{|l} 1+y=4 \\ -3+y=4 \end{array}$$

$$y=3$$

$$y=7$$

Sols: (1, 3), (-3, 7)

③ Solve by elimination:

$$\begin{array}{r} 2x^2 - 5x - y = -2 \\ + \quad x^2 + 2x + \cancel{y} = 0 \\ \hline \end{array}$$

$$\begin{array}{r} 3x^2 - 3x = -2 \\ \quad \quad +2 \quad +2 \\ \hline \end{array}$$

$$3x^2 - 3x + 2 = 0 \quad \leftarrow$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - (4 \cdot 3 \cdot 2)}}{2(3)}$$

$$x = \frac{3 \pm \sqrt{-15}}{6}$$

No Solution

Since this quadratic has no REAL solutions, the system has no real solution / pts. of intersection

④ Solve: $x^2 + y^2 = 10 \rightarrow$ Circle
 $y = -3x + 10 \rightarrow$ line

By substitution:

$$x^2 + (-3x + 10)^2 = 10$$

$$x^2 + 9x^2 - 30x - 30x + 100$$

$$10x^2 - 60x + 100 = 10$$

$$\frac{10x^2 - 60x + 90 = 0}{10}$$

$$x^2 - 6x + 9 = 0$$

$$\sqrt{(x-3)^2} = \sqrt{0}$$

$$x - 3 = 0$$

$$x = 3$$

$$y = -3(3) + 10 = 1$$

$$\boxed{\text{Sol. } (3, 1)}$$

⑤ Solve: $y = x^2 - 6x + 15$

$y = -(x-3)^2 + 6$

PEMDAS

$x^2 - 6x + 15 = -1(x-3)^2 + 6$

$(x-3)(x-3)$

$-1(x^2 - 6x + 9)$

$-x^2 + 6x - 9 + 6$

$x^2 - 6x + 15 = -x^2 + 6x - 3$
 $+x^2 - 6x + 3 \quad +x^2 - 6x + 3$

$2x^2 - 12x + 18 = 0$
 2

$x^2 - 6x + 9 = 0$

$\sqrt{(x-3)^2} = \sqrt{0}$

$x - 3 = 0$

$x = 3$

$y = 3^2 - 6(3) + 15 = 6$

Sol: (3, 6)