Transformations / Rigid Motions

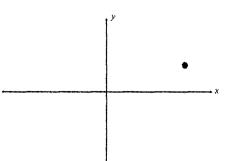
Lesson 1: Reflections on the Coordinate Plane

Opening Exercise

What do you remember about reflections???

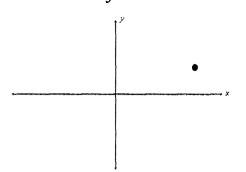
Take the point (4, 2) and reflect it as stated. Plot the new point and state its coordinates.

Reflection in the x-axis



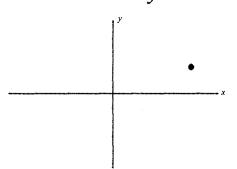
Coordinates of the New Point:

Reflection in the y-axis



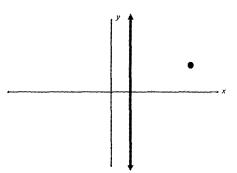
Coordinates of the New Point: _____

Reflection in the line y = x



Coordinates of the New Point: _____

Reflection in the line x = 1



Coordinates of the New Point: _____

Summary of the Rules:

$$r_{x-axis}: (x,y) \rightarrow$$

$$r_{y-axis}: (x,y) \rightarrow$$

$$r_{y=x}$$
: $(x,y) \rightarrow$

Vocabulary

A **transformation** is a chang in the position, shape, or size of a figure.

A **rigid motion** is a transforr stion that changes only the position of the figure (length and angle measures are preserve

An **image** is the result of a trule sformation of a figure (called the pre-image). To identify the image of a point, use prime number ation. The image of point A is A' (read as A prime).

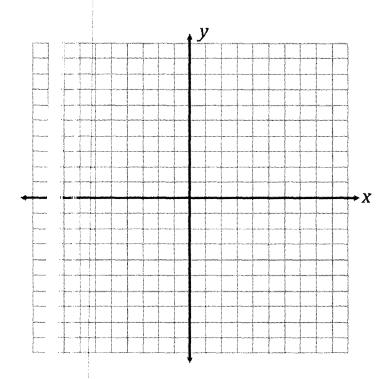
Example 1

Given $\triangle ABC$ with vertices A 5, 1), B (-1, 1) and C (-1, 7).

a. Graph $\triangle ABC$ on the zest provided below.

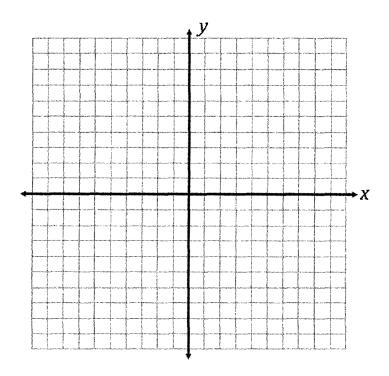
b. On the same set of ax graph $\Delta A'B'C'$, the image of ΔABC reflected over the x-axis.

c. On the same set of ax graph ΔA " B" C", the image of ΔABC reflected over the y-axis.



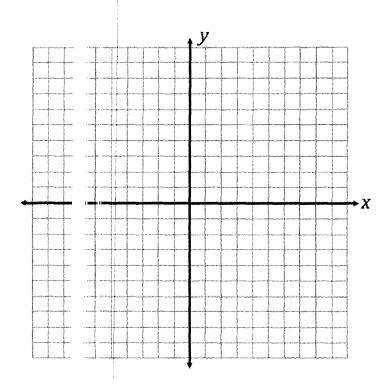
Given $\triangle ABC$ with vertices A (2, 3), B (0, 6) and C (2, 6).

- a. Graph $\triangle ABC$ on the axes provided below.
- b. Graph and state the coordinates of $\Delta A'B'C'$, the image of ΔABC reflected over the line y = x.
- c. Graph and state the coordinates of $\Delta A"B"C"$, the image of $\Delta A'B'C'$ reflected over the line y=-2.



 ΔDOG has vertices D (-1, 1), (-2, 5) and G (-5, 2) and $\Delta D'O'G'$ has vertices D' (3, -3), O' (7, -4) and G' (4, -7).

- a. Graph and label ΔDO and $\Delta D'O'G'$
- *b*. Graph the line y = x -
- c. What is the relationshal between the line of reflection and the segments connecting the corresponding policy? (Think back to Unit 1)



1. Using the point (-5, 3), find its image after the following reflections (the use of the grid is optional).

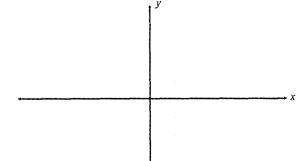




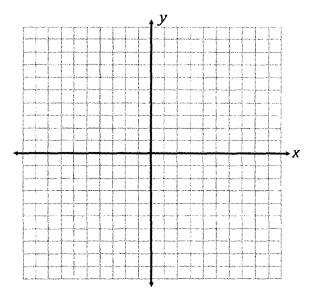
$$c. r_{y=y}$$

$$d. \qquad r_{x=-1}$$

$$e$$
. $r_{y=2}$



- 2. Given $\triangle ABC$ with vertices A (2, 1), B (3, 4) and C (-4, 5).
 - a. Graph $\triangle ABC$ on the axes provided.
 - b. Graph and state the coordinates of $\Delta A'B'C'$, the image of ΔABC reflected over the x-axis.
 - c. Graph and state the coordinates of $\Delta A''B''C''$, the image of $\Delta A'B'C'$ reflected over the line y = x.



Lesson 2: Reflections o the Coordinate Plane

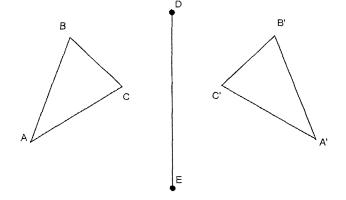
Opening Exercise

You will need a compass and cotraightedge

As shown in the diagram to t = r ght, $\triangle ABC$ is reflected across DE and map: In to $\triangle A'B'C'$.

- a. Use your straightedge \Rightarrow draw in segments AA', BB' a $\mid CC' \mid$.
- b. Use your compass to a sasure the distances from the properties of DE and from the image point to DE.

 What do you notice all ut these distances?

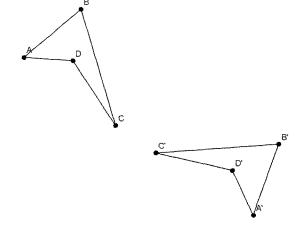


c. What is the relationslant between segment *DE* and each of the segments that were drawn in *part a*?

You will need a compass and a straightedge

We now know that the line of reflection is the *perpendicular bisector* of the segments connecting the pre-image to the image point. We are going to use this, along with our knowledge of constructions, to construct the line of reflection.

- *a.* Connect any point to its image point.
- b. Draw the perpendicular bisector of this segment.



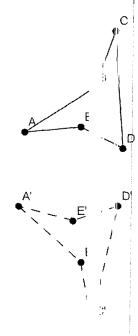
This is the line of reflection! Each point and its image point are equidistant from this line!!! Selecting a second pair of points and constructing its perpendicular bisector can verify this.

Exercises

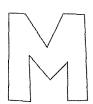
You will need a compass and cotraightedge

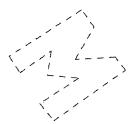
Construct the line of reflectio for each image and its pre-image.

1.



2.

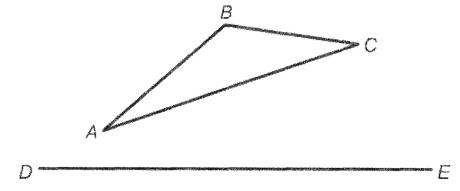




You will need a compass and a straightedge

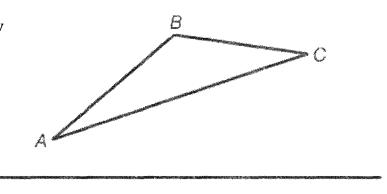
Using our knowledge of perpendicular bisectors we are going to reflect an object over a given line.

Reflect $\triangle ABC$ over DE.



Can we think of another way to do this same problem?

D



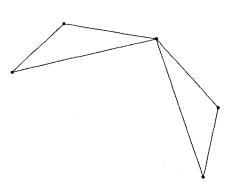
You will need a compass and cotraightedge

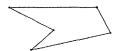
In 1-2, construct the line of rejection.

1.

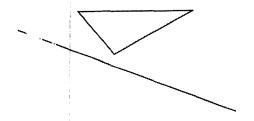


2.





3. Reflect the given figure across the line of reflection provided.



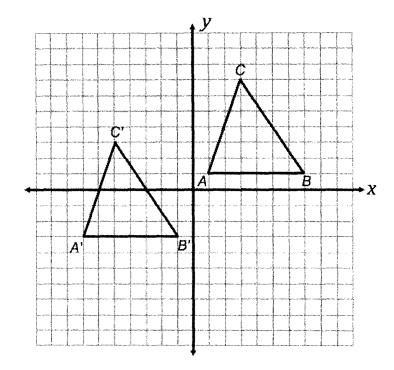
Lesson 3: Translations on the Coordinate Plane

Opening Exercise

Another type of rigid motion is called a *translation*, or slide, in which every point of a figure is moved the same distance in the same direction.

Describe how to translate $\triangle ABC$ to its image $\triangle A'B'C'$.

We can also write this translation using two different types of notation:



Is the size of the object preserved under a translation?

Is the order of the vertices the same?

Vocabulary

An **isometry** is a transformation that does not change in size. These include all of the rigid motions: reflections, translation and rotations.

A direct isometry preserves size and the order (orientation) of the vertices.

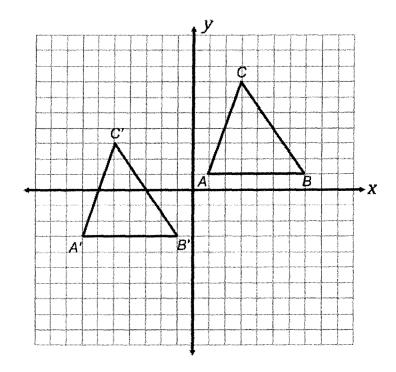
An **opposite isometry** preserves the size, but the order of the vertices changes.

A translation would be an example of which isometry?

A reflection would be an example of which isometry?

Using the same translation as the Opening Exercise, we are going to export the path the image follows.

Using rays, connect the pre-ingle points with the image points. What by ou notice about the rays you hav drawn?

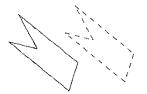


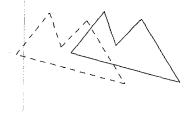
This ray is called a **vector**. A ector is a directed line segment that has both length and direction.

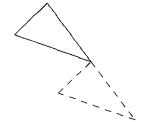
Exercises

You will need a straightedge

Draw the vector that defines such translation below.



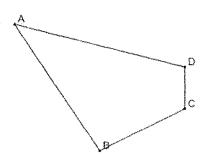




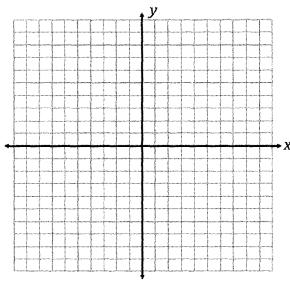
Translation Rule: $T_{a,b}(x,y) \rightarrow$

Exercises

- 1. Determine the coordinates of the image of the point (5, -3) under $T_{-2,-1}$.
- 2. Determine the coordinates of the image of the point (-8, -3) under the translation $(x,y) \rightarrow (x+4,y-1)$
- 3. Determine the translation that maps the point (-5, 5) to the point (7, 1).
- 4. A translation maps the point (-2, 5) to the point (-4, -4). What is the image of (1,4) under the same translation?
- 5. Translate the image one unit down and three units right. Draw the vector that defines the translation.



- 1. Determine the coordinates of the image of the point (-2, 2) under $T_{-2,6}$.
- 2. Determine the coordinates of the image of the point (2, -3) under the translation $(x,y) \rightarrow (x-4,y+2)$
- 3. If translation $T_{x,y}$ map point P(-3, 1) on to point P'(5, 5), find x and y.
- 4. A translation maps the point (3, 1) to the point (-4, 2). What is the image of (4,-1) under the same translation?
- 5. $\triangle ABC$ has vertices A = 1, 1, B (2, 3) and C (6, -2).
 - a. Graph $\triangle ABC$
 - b. Graph $\Delta A^{\dagger}B^{\dagger}C$, the image of ΔABC after the translation (x,y)-(x-2,y-6)
 - c. Draw the vect that defines the translation.

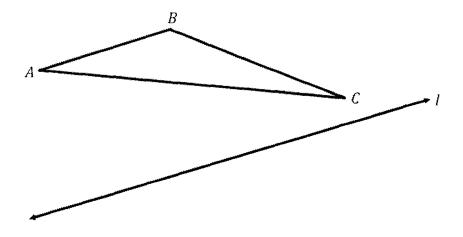


Lesson 4: Translations off the Coordinate Plane

Opening Exercise

You will need a compass and a straightedge

Construct $\Delta A'B'C'$, the image of ΔABC after a reflection over line l.

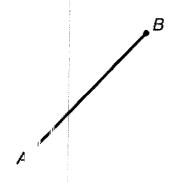


You will need a compass and a traightedge

As we learned in our last less is, a translated object follows the path of a vector. If we were to connect each pre-image point, we would have congruent and parallel segments.

In the diagram below, segmen AB is translated to produce A'B'.

- a. Draw the vector that d lines this translation.
- b. Using your compass, let ate B'
- c. Construct segment A'I



You will need a compass and a straightedge

Apply $T_{\overline{AB}}$ to segment \overline{CD} .

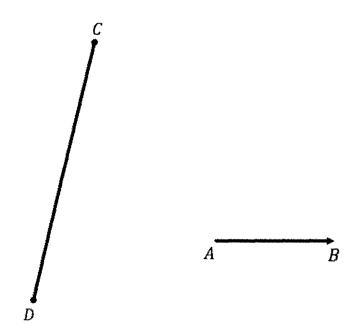
Vectors tell us 2 things: distance & direction

Step 1: Length

Using your compass, measure how far \overline{CD} will travel based on vector \overline{AB} .

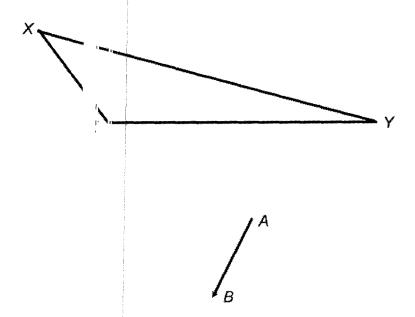
Step 2: Direction

How will you slide point *C* to its new point?



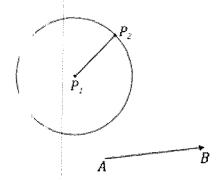
You will need a compass and a raightedge

Apply $T_{\overline{AB}}$ to ΔXYZ .



Example 4

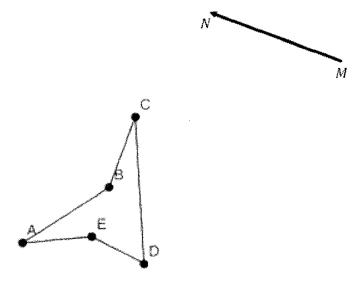
Use your compass and straig tedge to apply $T_{\overline{AB}}$ to the circle below:



- 1. a. Translate the figure 2 units down and 3 units left
 - *b.* Draw the vector that defines the translation



2. Use your compass apply $T_{\overline{MN}}$ to pentagon *ABCDE*.



Lesson 5: Rotations on Le Coordinate Plane

Opening Exercise

applied to create $\Delta A'B'C'$, the image of ΔABC .

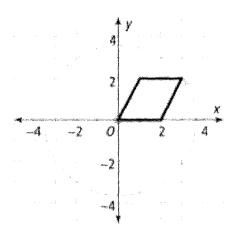
Fill in the table below to iden y the characteristics and types of rigid motions being

	-2 -2	A C C A A A A C C A A A A A A A A A A A	2 B X X -2 B X X -2 B X X X X X X X X X X X X X X X X X X	2 8 X -2 0 A 2 4 -2 -2 -4
Type of Rigid Motion:				
Is size preserved?	,			
Is orientation preserved?	,			
Which type of isometry?				

Vocabulary

A **rotation** is a rigid motion at turns a figure about a fixed point called the center frotation.

The angle of rotation is the umber of degrees the figure rotates. A positive an e of rotation turns the figure counterclockwise (a r gative angle of rotation can be used for clockwise ro tions).



Example 1

Take the point (4, 2) and rotate it as stated. Plot the new point and state its coordinates.

Rotation of 90 $^{\circ}$	Rotation of 180 $^{\circ}$	
•	•	
X	X	
Coordinates of the New Point:	Coordinates of the New Point:	
Rotation of 270°	Rotation of 360°	
•	•	
X	X	
Coordinates of the New Point:	Coordinates of the New Point:	

Summary of the Rules:

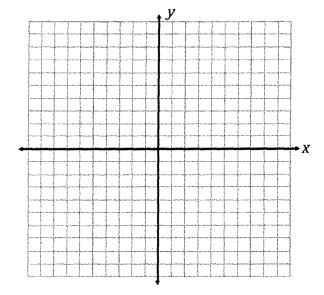
$$R_{0.90^{\circ}}: (x, y) \rightarrow$$

$$R_{0,180^{\circ}}: (x,y) \rightarrow$$

$$R_{O.270^{\circ}}: (x, y) \rightarrow$$

Given $\triangle COW$ with vertices C=1,2, O(-1,5) and W(-3,3).

- a. Graph $\triangle COW$ on the are provided.
- b. Graph and state the condinates of $\Delta C'O'W'$, the image of ΔCOW after a rotation of 180°

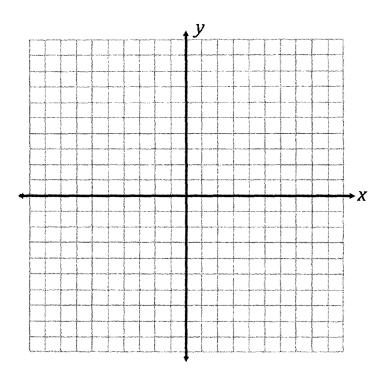


c. What type of isometry the image?

We are now going to rotate an image around a point other than the origin.

Given $\triangle ABC$ with vertices A(2, 3), B(0, 6) and C(2, 6).

- a. Graph $\triangle ABC$ on the axes provided below.
- b. Graph and state the coordinates of $\Delta A'B'C'$, the image of ΔABC after a rotation of 90° about the point (-1, 2).

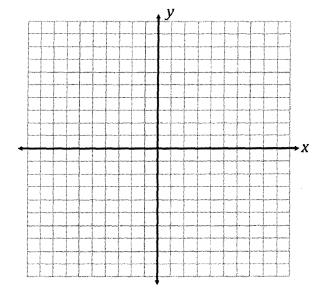


To rotate an image about a point other than the origin:

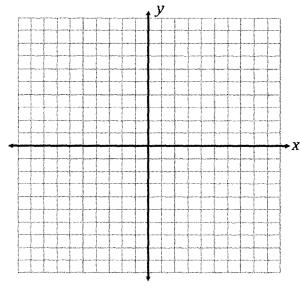
- Step 1: Translate the rotation point to the origin.
- Step 2: Translate the pre-image using the same translation as Step 1.
- Step 3: Rotate the image following the rules of rotations.
- Step 4: Translate the image the opposite direction as the translation from Step 1.

Exercises

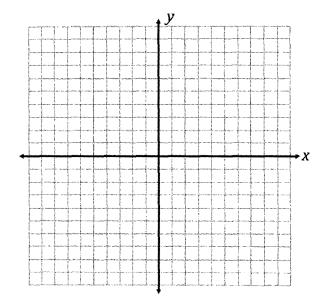
- 1. Given $\triangle ABC$ with vert $\Rightarrow A(4,0), B(2,3)$ and C(1,2).
 - a. Graph $\triangle ABC$ on the axes provided.
 - b. Graph and state the coordinates of $\Delta A^{\dagger}B^{\dagger}C^{\dagger}$, the integer of ΔABC after r_{y-axis} .
 - c. Graph and state the coordinates of ΔA "B"C", the tage of ΔA 'B'C' after r_{x-axis} .
 - d. This twice-refleted object is the same as which ngle transformation



- 2. Given $\triangle DOG$ with ve ices D (-3, 5), O (4, 6) and G (0, 2).
 - a. Graph ΔDOG 1 the axes provided.
 - b. Graph and star the coordinates of $\Delta D'O'G'$, the mage of ΔDOG after a rotation of 9 about the point (3, -2).



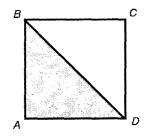
- 1. Given $\triangle CAT$ with vertices C(-1, 5), A(-3, 1) and T(-2, -2).
 - a. Graph ΔCAT on the axes provided.
 - b. Graph and state the coordinates of $\Delta C'A'T'$, the image of ΔCAT after $R_{O,270^{\circ}}$.
 - c. Graph and state the coordinates of ΔC "A"T", the image of ΔC 'A'T' after r_{x-axis} .
 - d. Which type of isometry is the image in part c? Explain your answer.



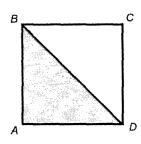
Lesson 6: Rotations off ne Coordinate Plane

Opening Exercise

Rotate the figure below about $\frac{1}{2}$ int $\frac{1}{2}$. Show the image of the figure after rotations of $\frac{90^{\circ}}{1}$, $\frac{1}{2}$ and $\frac{270^{\circ}}{1}$.



Now rotate the figure about 1 and D using the same rotations of 90° , 180° and 270° .

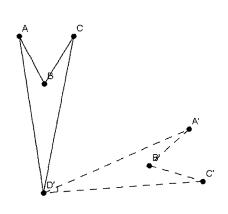


Notice the difference between the two images created!!!

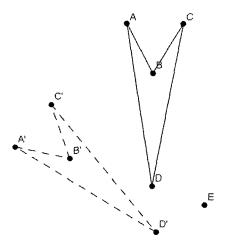
Given the two figures below, we are going to use a protractor to measure the angle of rotation.

To find the angle of rotation:

- Step 1: Identify the point that is the center of rotation.
- Step 2: Measure the angle formed by connecting corresponding vertices to the center point of rotation.
- Step 3: Check your answer using a different set of corresponding vertices.

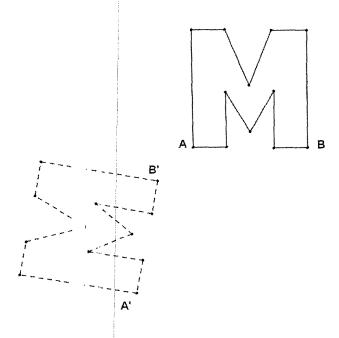


Center of Rotation: Angle of Rotation:



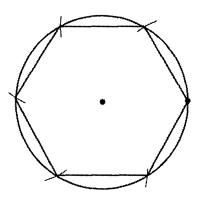
Center of Rotation: Angle of Rotation:

We are now going to locate the center of rotation.

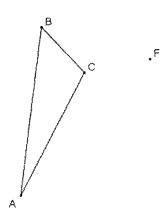


- Step 1: Construct the perper acular bisector of segment AA'. Step 2: Construct the perper acular bisector of segment BB'.
- Step 3: The point of intersec on of the perpendicular bisectors is the point of rotation. Label this point *P*.

In Unit 1, we looked at the construction of a hexagon by using equilateral triangles.



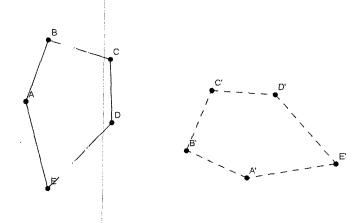
Using this same concept, we are going to rotate $\Delta ABC~60^\circ$ around point F using a compass and straightedge only.



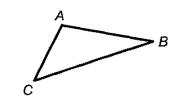
How could we rotate this image 120°?

How could we rotate this image 90°?

1. Find the center of rota on and the angle of rotation for the transformation below:



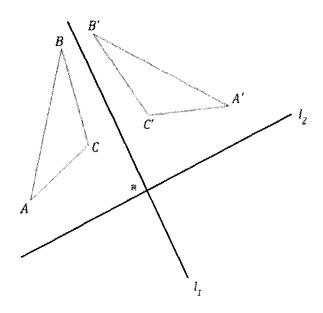
2. Rotate $\triangle ABC$ 120° ar and point R using a compass and straightedge only.



Lesson 7: Types of Symmetry

Opening Exercise

 $\Delta A^{\dagger}B^{\dagger}C^{\dagger}$ is the image of ΔABC after a reflection across line l_1 .

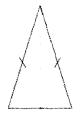


- a. Reflect the image across line l_2 .
- b. What is the relationship between the original triangle and the twice-reflected image?
- c. What does point *R* represent?
- *d.* How could we determine the angle of rotation?

Reflecting a figure twice over intersecting lines will give the same result as a rotation about the point of intersection!

Looking at the Opening Exerc , we can see that the lines of reflection are also lines of **symmetry**. The line of symmetry is equidistant from all corresponding pairs of points.

In the figures below, sketch al the lines of symmetry:









Example 2

Rotational Symmetry is a relation that maps a figure back on to itself.

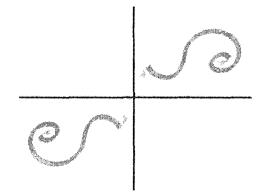
In regular polygons (polygo in which all sides and angles are congruent) the number of rotational symmetries is equito the number of sides of the figure.

How can we find the angles o otation?

:	Equilate al Trianş		Square	Regular Pentagon	Regular Hexagon
				
# of sides					
Angles of Rotation					

A rotation of 360° will alway map a figure back on to itself. This is called the **identity** transformation.

When reflecting an object through a point, the image and the pre-image create **point symmetry**. With point symmetry, the object will look exactly the same upside down! This can be seen by reflecting an object through the origin as pictured to the right.



Which of the objects below have point symmetry?





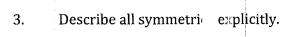




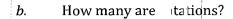
Exercises

Using regular pentagon *ABCD* pictured to the right, complete the following:

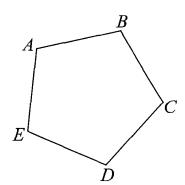
- 1. Draw all line of symme y.
- 2. Locate the center of ro tional symmetry.



a. What kinds are here?



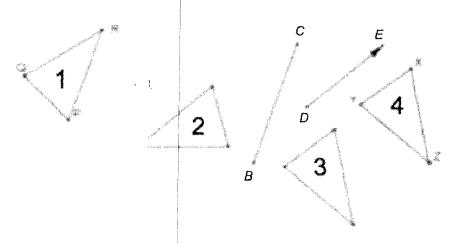
- c. What are the all les of rotation?
- d. How many are effections?



Home	work			
1.	Using	sing the figure to the right, describe all the symmetries explicitly.		
	a.	How many are rotations?		
	b.	What are the angles of rotation?	\ _	
	с.	How many are reflections?	_	
	d.	Shade the figure so that the resulting figure only has 3 possible rotational symmetries.		
2.	result Only horiz	he figures provided, shade exactly 2 of the 9 smaller squares so that the ng figure has: Only two lines of symmetry about the diagonals The horizontal line of symmetry and the diagonals about the diagonals are horizontal line of symmetry.		
	Only	ne horizontal line No line of symmetry of symmetry		

Lesson 8: Compositions of Rigid Motions

Opening Exercise



Looking at the picture above, escribe the type of rigid motion that takes place to go from:

$$\Delta 1 \rightarrow \Delta 2$$
:

$$\Delta 2 \rightarrow \Delta 3$$
:

$$\Delta 3 \rightarrow \Delta 4$$
:

When a series of rigid motion takes place with one rigid motion building off another (as shown above) this is called a proposition.

The symbol used for compos ons:

When performing or writing composition, you must work from right to left!

Two different ways to write e composition pictured above:

1.

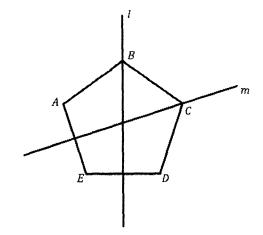
2.

Pentagon ABCDE is pictured to the right where l and m are lines of symmetry. Evaluate the following compositions:

a.
$$r_l \circ r_m(E)$$

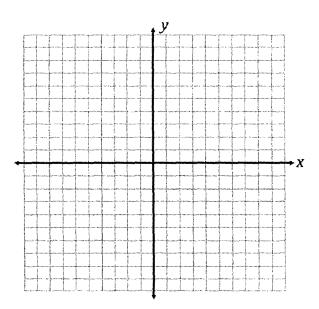
b.
$$r_m \circ r_l \left(\overline{BA} \right)$$

c.
$$r_m \circ R_{72^\circ}(B)$$



Example 2

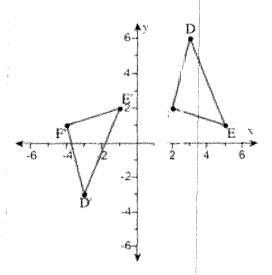
The coordinates of $\triangle ABC$ are A(-2,2), B(3,5) and C(4,2). Graph and state the coordinates of $\triangle A"B"C"$, the image of $\triangle ABC$ after the composition $r_{v-axis} \circ T_{2,-3}$.



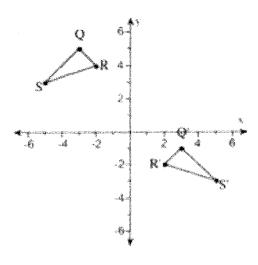
A composition of a translation and a reflection is called a **glide reflection**.

Using the diagrams, write the -ile of the composition:

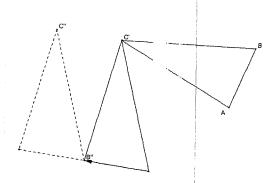
a.



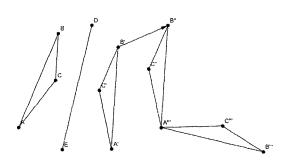
b.



с.

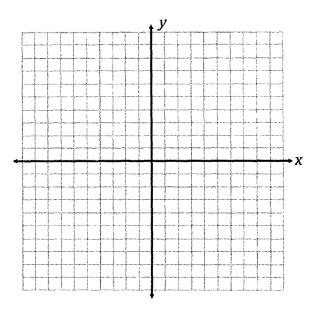


d.

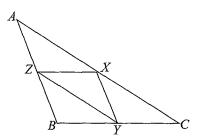


e. In each of the compose toms shown in parts *a-d*, is the image congruent to the pre-image? Explain.

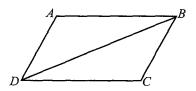
1. The coordinates of $\triangle ABC$ are A(-1, 1), B(-5, 3) and C(-2, 7). Graph and state the coordinates of $\triangle A"B"C"$, the image of $\triangle ABC$ after the composition $T_{-5,1} \circ R_{O,180^\circ}$.



2. In the diagram pictured, all of the smaller triangles are congruent to each other. What rigid motion(s) map *ZB* onto *AZ*?



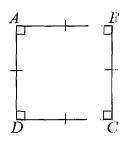
3. In the diagram pictured, what rigid motion(s) map CD onto AB?

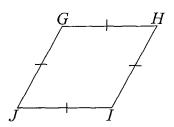


Lesson 9: Congruence it Terms of Rigid Motions

Opening Exercise

Pictured below are square $AB \rightarrow and rhombus GHIJ$. Are they congruent? Explain.





Vocabulary

congruence.

When figures are **congruent**, his means that there is a rigid motion (or a composition of rigid motions) that maps the e-image onto the image. This rigid motion is called a

Example 1

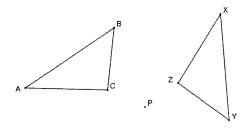
Under this definition of cong ence, describe why the figures in the Opening Exercise are not congruent.

Vocabulary

A **correspondence** between two triangles is a pairing of each vertex of one triangle with one and only one vertex of another triangle. This pairing can be expanded to figures other than triangles and could also involve sides.

Example 2

In the figure below, the triangle on the left has been mapped to the one on the right by a rotation of 240° about *P*. Identify all six pairs of corresponding parts (angles and sides).



Corresponding angles	Corresponding sides
$\angle A \rightarrow$	$AB \rightarrow$
$\angle B \rightarrow$	$AC \rightarrow$
$\angle C \rightarrow$	$BC \rightarrow$

- a. Is $\triangle ABC \cong \triangle XYZ$? Explain.
- b. What rigid motion mapped $\triangle ABC$ onto $\triangle XYZ$? Write the transformation in function notation.

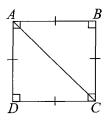
Important Discovery!

Rigid motions produce congruent figures and therefore, congruent parts (angles and sides). As a result, we can say that *corresponding parts of congruent figures are congruent*.

Exercises

reflection of $\triangle ADC$ at $\triangle SS$ segment AC.

ABCD is a square, and C is one diagonal of the square. $\triangle ABC$ is a



a. angles and side

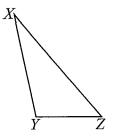
Complete the table le below identifying the corresponding

Correspondin ingles	Corresponding sides
$\angle BAC \rightarrow$	$AB \rightarrow$
$\angle ABC \rightarrow$	$BC \rightarrow$
$\angle BCA \rightarrow$	$AC \rightarrow$

- b. Are the corresponding sides and angles congruent? Justify your response.
- Is $\triangle ABC \cong \triangle Al$?? Justify your response. С.
- 2. Each side of $\triangle XYZ$ is like the length of each side of $\triangle ABC$.
 - Fill in the bland below so that each relationship between lengths of sides is a. true.

____×2 =





Is $\triangle ABC \cong \triangle X$? Justify your response. b.

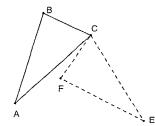
Important Discovery!

Corresponding parts do not: ways result in congruent figures.

- 1. Using your understanding of congruence, explain the following:
 - *a.* Why is a triangle not congruent to a quadrilateral?
 - b. Why is an isosceles triangle not congruent to a scalene triangle?

2. Draw a diagram with two triangles in which all three corresponding angles are congruent but the corresponding sides are not congruent.

3. In the figure below, the triangle on the left has been mapped to the one on the right by a rotation of 80° about vertex *C*. Identify all six pairs of corresponding parts (angles and sides).



Corresponding angles	Corresponding sides
$\angle A \rightarrow$	$AB \rightarrow$
$\angle B \rightarrow$	$AC \rightarrow$
$\angle C \rightarrow$	$BC \rightarrow$

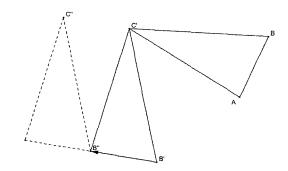
Write the rigid motion in function notation.

Lesson 10: Correspond nce and Transformations

Opening Exercise

The figure to the right shows series of rigid motions performed on ΔABC to produce the dotted triangle.

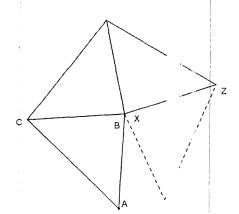
a. Identify the rigid moti



- b. Label vertex A".
- c. Is $\Delta A''B''C'' \cong \Delta ABC$: ustify your answer.

Example 1

Complete the table based on $\ \ \$ le series of rigid motions performed on ΔABC to produce the dotted triangle.



Sequence of rigid motions	
Composition in function notation	
Sequence of corresponding sides	
Sequence of corresponding angles	
Triangle congruence statement	